Generalized Output Regulation Problem for a Class of Nonlinear Systems Using Error Feedback

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Abstract— **We address the problem of output regulation for nonlinear systems driven by a partially unknown non-autonomous exosystem via error feedback. We generalize the classical output regulation problem in order to expand the class of reference or disturbance signals. Our study of the above problem, refereed to as the so-called generalized output regulation problem depends on the classical notion of the exact disturbance decoupling. A local necessary and sufficient conditions for the solvability of the problem are given.**

I. INTRODUCTION

The classical output regulation of nonlinear systems with known autonomous exosystem have been studied extensively. When disturbances are generated by known autonomous exosystems, local results have been shown in [10], [14] using "full information" which includes the measurements of exogenous signals as well as of the system state. The necessary and sufficient conditions for the existence of a local full information solution of the classical output regulation problem are given in [10], [14]; they basically mean that the linearized system is stabilizable and there exists a certain invariant manifold. The classical output regulation via error feedback has been solved in [2], [9], [13] by application of system immersion technique.

The plant uncertainty parametrized by unknown constant parameters is treated as a special case of exogenous signals and the solution, extended from the error feedback regulation, is referred to as the structurally stable regulation in [2]. However, the main limitation of the classical regulation scheme is that a precise model of the system that generates all exogenous inputs must be available, to be replicated in the control law. This limitation becomes immediately evident in the problem of rejecting a sinusoidal disturbances, not only of unknown amplitude and phase, but also of *unknown frequency*. Therefore, an alternative formulation would be to require asymptotic tracking of *known* reference trajectories in spite of *unmodelled* disturbances acting on the exosystem. More recently, in [21], the problem of handling parametric uncertainties affecting the autonomous system which generates the exogenous signals was also successfully addressed. To unify this alternative formulation with the classical output regulation concept, the so-called *generalized output regulation* may be considered.

The generalized output regulation was first posed and solved in [20] for linear systems both continuous and discrete time in terms of necessary and sufficient geometric conditions involving the classical notions of disturbance decoupling. The corresponding design procedure presented in [20] handles the unmodelled bounded disturbances generated by the known nonautonomous linear system driven by an unknown bounded reference signal.

In [17], we have presented a results in terms of sufficient conditions of the state feedback generalized output regulation problem for nonlinear systems. The state feedback generalized almost output regulation problem for a class of nonlinear systems is solved in [18]. In [19], we address the problem of generalized output regulation for a significant class of nonlinear systems, in the presence of unknown parameters.

The focus of this paper is on *generalized output regulation* of a class nonlinear systems using error feedback. We established a link between the two approaches; as a matter of fact, the classical notions of disturbance decoupling and the nonlinear regulation theory. We propose a local

*This work was supported by the PROMEP.
**Supported in part by the Grant Agency of the Czech Republic through the research grant No. 102/05/0011, and in part by the Ministry of Education of CR by project 1P05LA262
***Supported by the Ministry of Education of the Czech Republic under research program LN00B096.

solution in terms of necessary and sufficient condition to the generalized output regulation problem via error feedback for a particular class of nonlinear systems.

The paper is organized as follows. In Section II we outline the generalized nonlinear output regulation problem which, as anticipated above, is based on the introduction of a driving signal to the exosystem. In Section III, we state the assumptions necessary to the well-posedness of the problem while in Section IV we repeat results on disturbance decoupling. In Section V an controller solving the error feedback generalized output regulation problem for a specific class of nonlinear systems, is determined. We show how this specific problem is related to the standard problems of disturbance decoupling for nonlinear systems which have been recently studied in [13], [16]. Finally, Section VI draws conclusions and outlines some future research.

II. Generalized nonlinear output regulation

Following the linear concept of the generalized output regulation problem [20], the generalized output regulation problem for nonlinear systems [17] can be introduced via the configuration provided by the master-slave block diagram of Fig. 1. The master is a non-autonomous dynamical system driven by an external signal $r(t)$ that produces a desirable behavior for the slave while also modelling the available information about the external disturbances. The controller or regulator has the access to two sets of information, measured outputs of the slave (plant) as well as a certain output of the master, and it generates an input u for the slave. The slave (plant) controlled by this input u produces an output which tries to track the desired behavior dictated by the master. The task of the controller is to generate u so that the tracking error e is converging to zero for all initial conditions of both plant and exosystem and all external signals $r(t)$ from a suitable functional class. The master-slave configuration of Fig. 1 has applications in other areas of engineering as well, e.g. synchronization in communication systems [11].

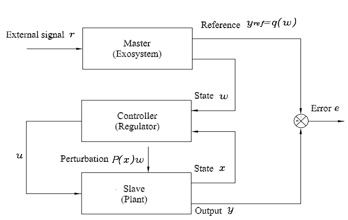

Fig. 1. Configuration of the output regulation schemes. (a) $r \not\equiv 0$, generalized output regulation. (b) $r \equiv 0$, classical output regulation.

The plants (slaves) we consider in the problem formulation are affine single-input single-output (SISO) nonlinear systems, described by the equations of the form

$$\dot{x} = f(x) + g(x)u + p(x)w, \tag{1}$$

$$y = h(x) \tag{2}$$

where (1) describes the plant with state x, defined on a neighborhood X of the origin of R^n, input $u \in R$ and output $y \in R$, subjected to the effect of a disturbance represented by the vector field $p(x)$. It is assumed that the vector field $f(\cdot)$, $g(\cdot)$ and $p(\cdot)$ are smooth vector fields, while $h(\cdot)$ is a smooth function, with $f(0) = 0$, $g(0) = 0$ and $h(0) = 0$. We only consider reference outputs to be tracked and perturbations to be rejected which both are generated by an unknown exosystem as follows.

Assumption 1: (Exosystem) Exosystem is the following nonautonomous system with output

$$\begin{aligned} \dot{w} &= Sw + Dr(t),\ r \in R^\rho, \\ y_{ref} &= q(w), \quad w \in R^l. \end{aligned} \tag{3}$$

Here, $D \in R^{\rho \times \rho}$, $S \in R^{l \times l}$ and $q,\ q(0) = 0$, is a smooth function, while $r(t)$ is an unknown external driving signal. Further, it is assumed that S has all its eigenvalues in the closed right-half plane and $r(t)$ is limited to a functional subclass of $\mathcal{L}_\infty^{R^\rho}$ where it holds for all solutions of (3) and some class-$\mathcal{K}$ functions α, β that

$$\|w(t)\|_{\mathcal{L}_\infty^{R^l}} \leq \alpha(\|w(0)\|_{R^l}) + \beta(\|r(t)\|_{\mathcal{L}_\infty^{R^\rho}}). \tag{4}$$

From the master-slave system point of view, the master system consists of the exosystem (3) driven by the signal r, whereas the slave consists of the plant (1)-(2). The controller is to be designed so that the slave obeys the master such that the so-called error signal e

$$e(t) = y(t) - q(w(t))$$

has certain desirable properties. Obviously, the proposed scheme of Fig. 1 and equations (1)-(3) are a generalization of the classical output regulation scheme as it introduces the driving signal r to the exosystem. In particular, it includes the case of the exogenous system $\dot{w} = Sw$ with unknown S putting $r(t) = S - S_{nominal}$. The problem we consider in this paper is the following:

Problem 1: (Error Feedback Generalized Output Regulation Problem (EFGORP)) Given the reference output $y_{ref}(t)$ generated by an exosystem (3), the EFGORP consists in finding a controller of the form

$$\dot{\xi} = \eta(\xi, e) \tag{5}$$

$$u = \theta(\xi, e) \tag{6}$$

with state $\xi \in R^\nu$, in which $\eta(0, 0) = 0$ and $\theta(0, 0) = 0$, such that:

S_{EI} The equilibrium $(x, \xi) = (0, 0)$ of

$$\dot{x} = f(x) + g(x)\theta(\xi, e) \tag{7}$$

$$\dot{\xi} = \eta(\xi, h(x)) \tag{8}$$

is asymptotically stable in the first approximation.

R_{EI} There exists a neighborhood $U \subset R^n \times R^\nu \times R^l$ of $(0, 0, 0)$ such that, for each initial condition on

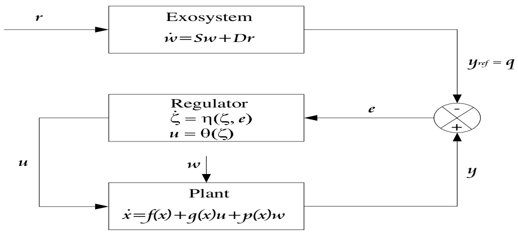

Fig. 2. Error feedback generalized output regulation.

U and for any signal r (piecewise continuous), the solution of the closed loop system:

$$\begin{aligned} \dot{x} &= f(x)+g(x)\theta(\xi,e)+p(x)w && (9)\\ \dot{\xi} &= \eta(\xi,h(x)-q(w)) && (10)\\ \dot{w} &= Sw+Dr && (11) \end{aligned}$$

satisfies

$$\lim_{t\to\infty} e(t)=0.$$

Our work can be viewed as an extension of the standard solutions of the classical regulator problem that have been given in [1], [6], [10].

III. Standing Assumptions

The class of systems we consider are those modeled by differential equations of the kind (1)-(2) which are transformed into the output feedback form (see [16] for the necessary and sufficient conditions)

$$\begin{aligned} \dot{\zeta} &= A\zeta+bu+Pw+\phi(y) && (12)\\ y &= C\zeta := h(\zeta) && (13) \end{aligned}$$

with (A,b,C) in observer canonical form, i.e.,

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 1\\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad b=\begin{bmatrix} 0\\ \vdots\\ b_\rho\\ \vdots\\ b_n \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$

where $\zeta\in R^n$ is the new vector state, $P\in R^{n\times m}$ is a constant matrix, ϕ, is a known nonlinear smooth vector field in R^n with $\phi(0)=0$. Condition $b_\rho\neq 0$ indicates that the original nonlinear system (1)-(2) has a constant relative degree of ρ. As in the classical case, we need the following assumptions:

Assumption 2: (Stabilizability) The pair (A,b) is stabilizable, i.e. $b_n\neq 0$.

Assumption 3: (Detectability) The pair

$$\left(\begin{bmatrix} C & 0 \end{bmatrix}, \begin{bmatrix} A & P\\ 0 & S \end{bmatrix} \right)$$

is detectable.

We will show that there exists a solution of the regulation equation for any exosystem that satisfies Assumption 1. The following Lemma summarizes the result for later use in the paper.

Lemma 1: For the system (slave) (12) with an exosystem satisfying Assumption 1 there exist mappings $\pi(w)\in R^n$, $\rho(w)\in R^\rho$ and $\gamma(w)\in R$ such that

$$\begin{aligned} \frac{\partial\pi}{\partial w}(Sw+D\rho(w)) &= A\pi(w)+b\gamma(w)+Pw\\ &\quad +\phi(q(w)) && (14)\\ C\pi(w)-q(w) &= 0 && (15) \end{aligned}$$

Proof: First we assume that $r=0$, and by virtue of Assumption 1 on the exosystem, center manifold theorem (see [3]) applies and, consequently, in $U\times W$, a neighborhood of the origin in $R^n\times R^l$ a center manifold

$$\{(x,w)\in U\times W : x=\pi(w);\quad \pi(0)=0\}$$

exist, i.e. there exist a global solution of the system of P.D.E.

$$\begin{aligned} \frac{\partial\pi_1}{\partial w}Sw &= \pi_2(w)+P_1w+\phi_1(q(w))\\ &\vdots\\ \frac{\partial\pi_{n-\rho-1}}{\partial w}Sw &= \pi_{n-\rho}(w)+P_{n-\rho-1}w+\phi_{n-\rho-1}(q(w))\\ \frac{\partial\pi_{n-\rho}}{\partial w}Sw &= \pi_{n-\rho+1}(w)+P_{n-\rho}w+\phi_{n-\rho}(q(w))\\ &\quad +b_\rho\gamma(w)\\ &\vdots\\ \frac{\partial\pi_n}{\partial w}Sw &= P_nw+\phi_n(q(w))+b_n\gamma(w) \end{aligned}$$

where P_i and ϕ_i denote the $i-$th row of P and $\phi(q(w))$, respectively. A solution of $\gamma(w)$ can always be found from the above equations. For the general case, $r\neq 0$ we define a transform of state as

$$\tilde{x}=\zeta-\pi(w).$$

which puts (12)-(14) in the so-called *error system* form

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x}+b(u-\gamma(w))+\phi(y)-\phi(C\pi(w))-\frac{\partial\pi}{\partial w}Dr\\ &:= A\tilde{x}+\chi(\tilde{x},w)\\ e &= \tilde{x} \end{aligned}$$

$$\chi(\tilde{x},w)=b(u-\gamma(w))+\phi(y)-\phi(C\pi(w))-\frac{\partial\pi}{\partial w}Dr$$

lemma has been proved. ◃

Since our study of the generalized output regulation problem depends on the classical notion of disturbance decoupling, first we will review and generalize these notions.

IV. Disturbance decoupling

Consider a general nonlinear system

$$\begin{aligned} \dot{x} &= f(x)+g(x)u+v(x)r && (16)\\ y &= h(x) && (17) \end{aligned}$$

where (16) describes the plant with state x, defined on a neighborhood X of the origin of R^n, and input $u \in R$, subject to the effect of a disturbance represented by the vector field $v(x)r$. The equation (17) defines the output plant $y \in R$. We consider the dynamic, output feedback, disturbance decoupling problem. The goal is the same as before, to insulate the output y from the input r, but now we allow the feedback functions α and β to depend dynamically on the output. We give a precise definition.

Definition 1: (The Dynamic, Output Feedback Disturbance Decoupling Problem (DOFDDP)) Consider the nonlinear system (16)-(17). The DOFDDP consists in finding a dynamics controller

$$\begin{aligned} \dot{\xi} &= \varphi(\xi, y) && (18)\\ u &= \alpha(\xi, y) + \beta(\xi, y)v_n && (19) \end{aligned}$$

where $\alpha(\cdot,\cdot)$ and $\beta(\cdot,\cdot)$ are smooth mappings, with $\beta(\cdot,\cdot)$ nonsingular for all x in a neighborhood X of the origin, and where v_n denotes the new input such that in the feedback modified dynamics

$$\begin{aligned} \dot{x} &= f(x) + g(x)(\alpha(\xi, y) + \beta(\xi, y) + v(x)r \\ \dot{\xi} &= \varphi(\xi, y) \end{aligned}$$

the disturbances r do not influence the output (17).
The following theorem gives sufficient conditions to solve the DOFDDP.

Theorem 1: [12] Consider the system (16)-(17) and suppose the following:

(a) there exists a regular (f, g) invariant distribution Δ^1
(b) there exists a regular (h, f) invariant distribution Δ^2
(c) $\Delta^2 \cap \ker dh$ is a regular distribution
(d) $span\{p\} \subseteq \Delta^2 \subseteq \Delta^1 \subseteq \ker dh$

Then the DOFDDP is solvable.
The following theorem is almost the converse of the last result.

Theorem 2: [12] Suppse the dynamic, output feedback, decoupling problem is solvable; there then exists a regular distribution Δ such that

(a) Δ is locally (f, g) invariant on R^n
(b) Δ is (h, f) invariant
(c) $span\{p\} \subseteq \Delta \subseteq \ker dh$

V. MAIN RESULTS

In this section, we design a controller that solves the problem of generalized output regulation via error feedback.

Theorem 3: Consider the system given in (12)-(13). Let Assumptions 1, 2 and 3 be satisfied. Then, the generalized output regulation problem via the error feedback regulators is solvable if and only if the following conditions are true:

(a) there exist C^k $(k \geq 2)$ mappings $\pi(w)$, $c(w)$, $\rho(w)$, $\pi : R^l \to R^n$, $c : R^l \to R^m$, $\rho : R^l \to R^\rho$, defined locally in a neighborhood of the origin $W^0 \subset R^l$, with $\pi(0) = 0$, $c(0) = 0$, $\rho(0) = 0$ satisfying the so-called generalized regulation equation

$$\begin{aligned} \frac{\partial \pi}{\partial w}(Sw + D\rho(w)) &= A\pi(w) + bc(w) + Pw \\ &\quad + \phi(q(w)) && (20)\\ C\pi(w) - q(w) &= 0 && (21) \end{aligned}$$

(b) there exists K and a regular (f_e, D_e) invariant distribution Δ^1 for the system

$$\underbrace{\begin{bmatrix} \dot{\tilde{x}} \\ \dot{w} \end{bmatrix}}_{\dot{p}} = \underbrace{\begin{bmatrix} A + bK & 0 \\ 0 & S \end{bmatrix}}_{A_e} \underbrace{\begin{bmatrix} \tilde{x} \\ w \end{bmatrix}}_{p} + \underbrace{\begin{bmatrix} -\frac{\partial \pi(w)}{\partial w} D \\ D \end{bmatrix}}_{D_e} r + \underbrace{\begin{bmatrix} \phi(C\tilde{x} + C\pi(w)) - \phi(C\pi(w)) \\ 0 \end{bmatrix}}_{\phi_e} \quad (22)$$

$$e = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{\tilde{C}} \begin{bmatrix} \tilde{x} \\ w \end{bmatrix} \quad (23)$$

$$y_e = \underbrace{\begin{bmatrix} C & C\pi(w) \end{bmatrix}}_{C_e} \begin{bmatrix} \tilde{x} \\ w \end{bmatrix} \quad (24)$$

such that $A + bK$ is Hurwits and $f_e = A_e p + \phi_e$.

(c) there exist Δ^2 a regular $(\tilde{C}p, f_e)$ invariant distribution.
(d) the distribution $\Delta^3 = \Delta^2 \bigcap \Delta^0$ is regular, where $\Delta^0 = \ker \tilde{C}$
(e) $span\{D_e\} \subset \Delta^2 \subset \Delta^1 \subset \ker \tilde{C}$.

Moreover, if conditions (a)-(e) are satisfied, then there exist a controller of the form

$$\begin{aligned} \dot{\tilde{\xi}} &= \tilde{A}_e\tilde{\xi} + \tilde{b}_e y + \phi_e && (25)\\ u_e &= \tilde{C}_e\tilde{\xi} + \tilde{d}_e y, && (26) \end{aligned}$$

that solves the disturbance decoupling problem via output feedback for the system

$$\begin{aligned} \dot{p} &= A_e p + D_e r + \phi_e + \begin{bmatrix} 0 & b \\ I & 0 \end{bmatrix} u_e && (27)\\ e &= \tilde{C}p && (28)\\ y_e &= C_e p. && (29) \end{aligned}$$

In that case the generalized output regulation problem via output feedback regulators is solved by the controller

$$\begin{aligned} \dot{\xi} &= \eta(\xi, e) && (30)\\ u &= \theta(\xi, e) && (31) \end{aligned}$$

with ξ, θ and $\eta(\xi, e)$ defined in the following way

$$\begin{aligned} \xi &= col(\xi_1, \xi_2) \\ \theta(\xi) &= \gamma(\xi_2) + H(\xi_1 - \pi(\xi_2)) \\ \eta(\xi_1, \xi_2, e) &= col(\eta_1(\xi_1, \xi_2, e), \eta_2(\xi_1, \xi_2, e)) \\ \eta_1(\xi_1, \xi_2, e) &= A\xi_1 + P\xi_2 + b(\gamma(\xi_2) + H(\xi_1 - \pi(\xi_2)) \end{aligned}$$

$$-G_1\left(C\xi_1 - q\left(\xi_2\right) - e\left(\xi_1,\xi_2\right)\right)$$
$$\eta_2\left(\xi_1,\xi_2,e\right) = S\xi_2 - G_2(C\xi_1 - q(\xi_2) - e(\xi_1,\xi_2)).$$

Here H, G_1 and G_2 are matrices such that

$$A + bH \quad \text{and} \quad \begin{bmatrix} A - G_1C & P - G_1Q \\ -G_2C & S - G_2Q \end{bmatrix}$$

have all eigenvalues with negative real part, where H and Q are matrices defined by

$$H = \left[\frac{\partial\theta}{\partial\xi}\right]_{\xi=0} \quad Q = \left[\frac{\partial q}{\partial w}\right]_{w=0}.$$

Proof: For necessity, we suppose that we have a controller of the form (30)-(31) that achieves generalized output regulation. First we assume that $r = 0$ then it is easy to check that $\lim_{t\to\infty} e = 0$ for all initial condition if and only if there exist C^r mappings $x = \pi(w)$ and $\xi = \sigma(w)$ such that

$$\begin{aligned}
\frac{\partial\pi(w)}{\partial w}Sw &= A\pi(w) + b\theta(\sigma(w)) + Pw + \phi(C\pi(w)), \\
\frac{\partial\sigma(w)}{\partial w}Sw &= \eta(\sigma(w), 0), \\
0 &= C\pi(w) - q(w)
\end{aligned}$$

if we choose $c(w) = Lw + K\pi(w)$ then it is obvious that (a) is satisfied. Next we consider the general case where r is not necessarily 0. Then for our controller (30)-(31), which is known to achieve exact output regulation. By virtue of Assumption 1 on the exosystem, center manifold theorem applies [3], thus, the system has a center manifold at $(0,0,0)$, the graph of the mappings $x = \pi(w)$ and $\xi = \sigma(w)$ with $\pi(w)$ and $\sigma(w)$ satisfying (d). Perform the change of coordinates $\tilde{x} = \zeta - \pi(w)$, differentiating the latter equality with respect to time, in view of (12)-(13) we obtain that the closed-loop system in the new coordinates w and $\tilde{x}_i = x_i - \pi_i(w)$, $1 \le i \le n$ can be written as

$$\begin{aligned}
\dot{\tilde{x}} &= (A + bK)\tilde{x} + (A + bK)\pi(w) + \phi(Cx) \\
&\quad + bc(w) - bK\pi(w) - \frac{\partial\pi}{\partial w}(Sw + Dr) \\
\dot{w} &= Sw + Dr \\
e &= C\tilde{x}
\end{aligned}$$

Obviously, $\forall\varepsilon > 0 \ \exists\delta(\varepsilon) > 0$ such that $\|\chi(\tilde{x}, w)\| \le \varepsilon\|\tilde{x}\| \ \ \forall\|(w,\tilde{x})^\top\| < \delta(\varepsilon)$. Moreover, the exosystem $\dot{w} = Sw + Dr$ generates by Assumption 1 small trajectories $w(t)$ for small $r(t)$ and small $w(0)$. Invoking the standard result on vanishing perturbations (see Lemma 9.1 on page 341 of [15]), it holds for the following system

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \chi(\tilde{x}, w), \quad \|\chi(\tilde{x}, w)\| < \varepsilon\|\tilde{x}\|, \tag{32}$$

$\tilde{A} = A + bK$ being Hurwitz due to Assumption 2, that there exists $b > 0$ such that for any $r(t)$ from a functional class specified in Assumption 1 and $\|r(t)\|_{\mathcal{L}_\infty^{\mathbb{R}^\rho}} < b$ it holds that $\tilde{x}(t) \to 0$ as $t \to \infty$ for $\tilde{x}(0), w(0)$ sufficiently small[1]. As $e = h(\zeta) - h(\pi(w)) = h(\tilde{x} + \pi(w)) - h(\pi(w))$, $\lim_{t\to\infty} e(t) = 0$, and Lemma (1), we finally obtain (22)-(24). It is clear then that the disturbance decoupling problem via error feedback for the system (22)-(24) is solved by the feedback law (25)-(26), and hence, using Theorem (1), we find that (c), (d) and (e) are satisfied. For sufficiency, by Assumption 2, there exist the matrices H, G_1 and G_2 are matrices such that

$$A + bH \quad \text{and} \quad \begin{bmatrix} A - G_1C & P - G_1Q \\ -G_2C & S - G_2Q \end{bmatrix}$$

have all eigenvalues with negative real part, where A, B, C, H, P and Q are matrices defined as before. Then, we set the dynamic stabilizer as (30)-(31) satisfies $(\mathrm{S_{EI}})$. ◃

VI. CONCLUSIONS

We have proposed a control algorithm of the generalized output regulation problem via error feedback for a particular class of nonlinear systems. It is known that in the classical case, where the exosystem is autonomous, one has to know explicitly all the frequency components of the signal that needs to be tracked in order to come up with a model for the exosystem. However, it is clear that, by utilizing a nonautonomous exosystem driven by a reference signal r, one does not necessarily need to have such knowledge. In fact, by an appropriate selection of the driven signal r, a nonautonomous exosystem can be constructed so that any arbitrarily specified signal can be modelled as a signal that needs to be tracked. Moreover, the class of external disturbances that could act on the given plant can be significantly broadened.

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