Modeling and Nonlinear Control of STATCOMs for Fast Voltage Regulation

A. K. Jain[‡], K. Joshi[†], A. Behal[†], and N. Mohan[‡]

[†]Electrical & Computer Engineering, Clarkson University, Potsdam, NY 13699

[‡]Electrical & Computer Engineering, University of Minnesota, Minneapolis, MN 55455

E-mail: abehal,joshik@clarkson.edu; akj,mohan@ece.umn.edu

Abstract

This paper presents a strategy for controlling a STATCOM to act as an instantaneous reactive current source. The STATCOM, consisting of a three phase inverter along with a capacitive filter for the dc bus and inductive line-side filters, is modeled with the overall objective of using it for load voltage regulation. Fast control of the reactive current is achieved using direct feedback linearization with respect to one control input. The other control input is used to indirectly regulate the dc bus capacitor voltage via regulation of the real current to a quasi-steady state value. Asymptotic stability of the system is rigorously established and a region of convergence is computed as a by-product of the stability analysis. Simulation results of the controlled STATCOM integrated with a distribution system model and a load voltage controller show the efficacy of the control strategy.

1 Introduction

In a power distribution system, fast load voltage regulation is required to compensate for time varying loads such as electric arc furnaces, fluctuating output power of wind generation systems, and transients on parallel connected loads [1]-[3]. A specially important problem is that of voltage flicker - a modulation of the phase voltage magnitude in the frequency range of 0.1 to 30 Hz. This is important due to the resulting variation in the light intensity of incandescent lamps. Due to their high control bandwidth, Static Compensators (STATCOMs) based on three phase pulse width modulated voltage source converters, have been proposed for voltage regulation [4]-[7]. For effecting fast control, the STATCOM is usually modeled using the $d-q$ axis theory for balanced three phase systems, which allows definition of instantaneous reactive current and instantaneous magnitude of phase voltages (*e.g.*, see [8]).

In [9], a load voltage magnitude controller was designed assuming the STATCOM to be a controlled reactive current source. This paper addresses control of the STATCOM internal dynamics and its integration with the voltage magnitude controller designed in [9]. A three phase inverter with line side inductive filters and a capacitor for the dc bus is used as the STATCOM. The control strategy accepts a reference reactive current input computed from the control and estimation strategy of [9] and utilizes the quadrature-axis inverter duty ratio to ensure that the STATCOM supplies the required reactive current to establish load voltage magnitude control. Since the direct-axis duty ratio cannot be used to directly control the dc bus voltage because of the non-minimum phase nature of the system, we propose an indirect technique which utilizes (i) a relationship between the steady state d-axis current and the dc bus voltage, and (ii) feedback linearization to regulate the d-axis current. In order to alleviate problems with steady-state errors due to control dependence on knowledge of model parameters, an adaptive update law is designed that guarantees parameter identification. Asymptotic stability of the system inside a computable region of attraction is established. Detailed simulation studies prove the effectiveness of the strategy and provide further insights into the nature of the integrated system.

Section 2 presents relevant background regarding system modeling and regulation of load voltage magnitude. The STATCOM model is presented in Section 3. Control objectives are stated in Section 4. The control strategy and estimator design are described in Section 5. Stability analysis is presented in Section 6. Simulation results and discussions are presented in Section 7.

2 Background

Since this paper aims to extend the work of the authors in modeling and voltage regulation for a load on a distribution system using STATCOMs, we present

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succinctly some of the background material previously published in a series of papers leading up and relevant to the issues addressed in this paper. We begin by describing the system dynamics on the distribution-side assuming that the STATCOM acts as a controlled reactive current source.[1] Similar to [9, 10], we use a simplified model for a load supplied on a power distribution system. It consists of the source modeled as an infinite bus ($v_{s,abc}$), the distribution line represented by an inductive impedance (R_s, L_s), the load modeled by a resistance[2] ($R_L = 1/g_L$), a STATCOM in parallel with the load modeled as an ideal current source, and a coupling capacitor[3] (C_c). It is assumed that the source, load, and the STATCOM are balanced three-phase systems. Thus the system dynamics can be represented in the following equivalent two phase $d-q$ system [10]

$$\begin{aligned} C_c \dot{v}_{Ld} &= -g_L v_{Ld} + i_{sd} + i_{SCd} && (1)\\ L_s \dot{i}_{sd} &= -v_{Ld} - R_s i_{sd} + \omega L_s i_{sq} + V_s \cos\alpha && (2)\\ L_s \dot{i}_{sq} &= -R_s i_{sq} - \omega L_s i_{sd} - V_s \sin\alpha && (3)\\ \dot{\alpha} &= \omega - \omega_s && (4)\\ \omega &= \frac{i_{sq} + i_{SCq}}{C_c v_{Ld}} && (5) \end{aligned}$$

where $v_{Ld}(t), i_{sd}(t), i_{sq}(t)$, and $\alpha(t) \in \Re^1$ are the states of the distribution-side system, $\omega(t) \triangleq d\theta/dt, V_s$ denotes the constant magnitude of the infinite bus voltage, while ω_s represents the constant frequency of the infinite bus voltage. Since $v_{Lq}(t) \equiv 0$ by the above choice for $\theta(t)$, $v_{Ld}(t)$ represents the instantaneous magnitude of the load phase voltages, while $i_{SCq}(t) \in \Re^1$ denotes the reactive current supplied by the STATCOM and is considered to be the control input to the system. If parasitic losses are ignored, a STATCOM only supplies reactive power so $i_{SCd} \equiv 0$ in (1).

The distribution system modeled by the dynamics of (1)-(5) is non-minimum phase when $v_{Ld}(t)$ is chosen as the output of the system with $i_{SCq}(t)$ as the control input [10]. Efficacious Lyapunov-based control strategies for voltage regulation and flicker mitigation were proposed in [9] and were shown to outperform the linear controller. To motivate the remainder of the paper, we present here the following readily implementable control strategy from [9]

$$\begin{aligned} i_{SCq} = &\ -i_{sq} + C_c v_{Ld}\Big(\omega_s + \frac{\partial \alpha^*}{\partial \hat{g}_L}\,\dot{\hat{g}}_L + k_\rho(\rho - \rho^*) \\ &+ \frac{v_{Ld}\sin\rho}{L_s i_{sm}}\Big) \\ \dot{\hat{g}}_L = &\ -k_g v_{Ld}^* \tilde{v}_{Ld} \end{aligned} \quad (6)$$

where $k_\rho, k_g \in \Re^1$ are positive control gains, $i_{sm} = \left(i_{sd}^2 + i_{sq}^2\right)^{1/2}$, $\rho = \arctan(i_{sq}/i_{sd}), \hat{g}_L(t)$ denotes a dynamic load estimate, v_{Ld}^* denotes the reference load voltage magnitude, $\tilde{v}_{Ld}$ denotes the voltage regulation error, while $\alpha^*(\hat{g}_L), \rho^*(\hat{g}_L)$ denote quasi-steady solutions for the system of (1)-(5).[4]

3 STATCOM Model

In this section, we present the internal dynamics of the STATCOM implemented by a three-phase inverter along with the dynamics of the dc bus capacitor voltage and the filter inductances. Under the assumption that the internal dynamics of a STATCOM are much slower than the switching frequency of the inverter [12], the STATCOM dynamics can be represented by the following set of equations

$$\begin{aligned} C_{dc}\dot{V}_{dc} &= -pV_{dc} - u_{abc}^T i_{SC,abc} && (7)\\ L_{sc}\dot{i}_{SC,abc} &= u_{abc}V_{dc} - v_{Ld,abc} - R_{sc} i_{SC,abc} && (8)\\ u_{abc} &= d_{abc} - (d_a + d_b + d_c)/3 && (9) \end{aligned}$$

where $C_{dc}, p, R_{sc}, L_{sc} \in \Re^1$ denote, respectively, the inverter dc bus capacitance, parasitic conductance in parallel with the capacitor, inverter and inductor parasitic resistance, and inverter filter inductance, $V_{dc}(t)$ is the inverter dc bus voltage, $i_{SC}(t)$ denotes inverter output current, $v_{Ld}(t)$ denotes the load voltage, $d_{abc}(t)$ denotes the duty ratios[5] for the three inverter legs while the subscript '*abc*' refers to vectors of individual phase quantities. Applying the global invertible $d-q$ transformation, one can transform the three phase dynamics of (7)-(9) to the $d-q$ reference frame as follows

$$\begin{aligned} C_{dc}\dot{V}_{dc} &= -pV_{dc} - u_d i_{SCd} - u_q i_{SCq} && (10)\\ L_{sc}\dot{i}_{SCd} &= -v_{Ld} - R_{sc} i_{SCd} + \omega L_{sc} i_{SCq} + u_d V_{dc} && (11)\\ L_{sc}\dot{i}_{SCq} &= -R_{sc} i_{SCq} - \omega L_{sc} i_{SCd} + u_q V_{dc} && (12) \end{aligned}$$

where $V_{dc}(t), i_{SCd}(t)$,and $i_{SCq}(t)$ are the states of the STATCOM, $u_d(t)$ and $u_q(t)$ are the control inputs, while $\omega(t)$ has been previously defined.

[1] Depending on the avaialble dc bus voltage, the instantaneous line voltage and filter inductance, the output current of the STATCOM can be controlled within limits of magnitude and response time, by using Pulse Width Modulation (PWM).

[2] As described in [9], the assumption of a purely resistive load allows for simplicity of analysis and does not involve any loss of generality.

[3] See [11] for details on inclusion of C_c.

[4] Note that this quasi-steady state solution changes as $\hat{g}_L$ changes.

[5] Duty ratio for an inverter leg is defined as the ratio of the time in a switching cycle that the output is connected to the top of the dc bus to the switching time period. Thus, $d_{abc} \in [0, 1]$.

4 Control Objective

Motivated by the desire to take advantage of the control design of (6) and the fact that the STATCOM internal dynamics are much faster than the distribution-side dynamics of the previous Section, we make the assumptions that (a) the signal $i_{SCq}(t)$ in the distribution-side dynamics can be replaced by i^*_{SCq} and (1)-(5) can be analyzed as was done in [9], (b) for the subsystem of (10)-(12), i^*_{SCq} can be considered to be a constant reference input computed via (6), (c) the load voltage $v_{Ld}(t)$ can be treated as a constant input into the STATCOM subsystem such that a quasi-steady solution for (10)-(12) is readily obtainable, and (d) the parasitic conductance $p \in [p_{\min}, p_{\max}]$, where $p_{\min}$, $p_{\max} > 0$ are known apriori.

Provided that the system states are measurable, and the model parameters R_{sc} and L_{sc} are known, the primary control objective is to rapidly regulate the reactive STATCOM current $i_{SCq}(t)$ to the reference value i^*_{SCq}. A secondary control objective is to bound the dc bus voltage $V_{dc}(t)$ around a desired value denoted by $V^*_{dc} > 0$. An additional control objective is to compensate for uncertainty in the knowledge of the parasitic conductance p. Motivated by our primary control objective, we begin by defining the reactive current regulation error $e_q(t) \in \Re^1$ as follows

$$e_q = i_{SCq} - i^*_{SCq} \tag{13}$$

Equation (12) readily suggests the utilization of $u_q(t)$ toward rapid convergence of $e_q(t)$ to the origin. However, the secondary control objective cannot be achieved directly via $u_d(t)$ through the dynamics of (10) because of the possibility of a vanishing $i_{SCd}(t)$ during the transient. To address this problem, we define the direct-axis current tracking error $e_d(t) \in \Re^1$ as follows

$$e_d = i_{SCd} - i^*_{SCd} \tag{14}$$

where $i^*_{SCd}\left(i^*_{SCq}, V^*_{dc}, v_{Ld}, \hat{p}(t)\right) \in \Re^1$ is a tracking trajectory for $i_{SCd}(t)$ which is explicitly obtained in the following manner

$$i^*_{SCd}(t) = \frac{-v_{Ld} + \sqrt{v^2_{Ld} - 4R_{sc}(\hat{p}{V^*_{dc}}^2 + R_{sc}{i^*_{SCq}}^2)}}{2R_{sc}} \tag{15}$$

where $\hat{p}(t) \in \Re^1$ is a yet to be designed dynamic estimate for p. We would like to note here that (15) is obtained from a quasi-steady solution[6] of the system dynamics of (10)-(12) as similarly done in [9]. Provided we can prove asymptotic convergence of $\hat{p}(t), i_{SCq}(t)$, and $i_{SCd}(t)$ to p, i^*_{SCq}, and $i^*_{SCd}(\cdot)$, respectively, the system states will converge to the desired operating point as specified by setting the left hand side of (10)-(12) to zero. To prove convergence for $\hat{p}(t)$, we define the parasitic conductance estimation error $\tilde{p}(t) \in \Re^1$, and $\tilde{V}_{dc}(t) \in \Re^1$, an auxiliary error signal, as follows

$$\begin{aligned} \tilde{p} &= p - \hat{p} &(16)\\ \tilde{V}_{dc} &= V_{dc} - \hat{V}_{dc} &(17)\end{aligned}$$

where $\hat{V}_{dc}(t) \in \Re^1$ denotes the yet to be designed estimator for the dc bus voltage.

[6] The quasi-steady solution gives two values of i^*_{SCd}. The value with lower magnitude is chosen since we want to minimize power dissipation in the STATCOM.

5 Control Strategy and Estimator Design

In this section, we motivate and present the design of the control signals $u_d(t), u_q(t)$ as well as estimation signals $\hat{p}(t)$ and $\hat{V}_{dc}(t)$. We begin by first designing the control signals in order to drive $e_d(t), e_q(t)$ to the origin. After taking the time derivative of the $q-$axis current regulatory error of (13) and substituting the dynamics for $\dot{i}_{SCq}(t)$ from (12), we obtain the open loop error dynamics for $e_q(t)$ as follows

$$L_{sc}\dot{e}_q = -R_{sc}i_{SCq} - \omega L_{sc}i_{SCd} + u_q V_{dc} \tag{18}$$

For exponential convergence of $e_q(t)$ to zero, we design the control input $u_q(t)$ as follows

$$u_q = \frac{1}{V_{dc}}\left(R_{sc}i_{SCq} + \omega L_{sc}i_{SCd} - k_q e_q\right) \tag{19}$$

where $k_q > 0$ is a constant control gain. After substitution of (19) into (18), we obtain the closed-loop error dynamics for $e_q(t)$ as

$$L_{sc}\dot{e}_q = -k_q e_q \tag{20}$$

Similarly as above, we obtain the open loop error dynamics for $e_d(t)$ as follows

$$\begin{aligned} L_{sc}\dot{e}_d = \; & -v_{Ld} - R_{sc}i_{SCd} + \omega L_{sc}i_{SCq} + u_d V_{dc} \\ & -L_{sc}\frac{\partial i^*_{SCd}}{\partial \hat{p}}\dot{\hat{p}} \end{aligned} \tag{21}$$

We design the control input $u_d(t)$ as

$$\begin{aligned} u_d = \; & \frac{1}{V_{dc}}\Big(v_{Ld} + R_{sc}i_{SCd} - \omega L_{sc}i_{SCq} \\ & + L_{sc}\frac{\partial i^*_{SCd}}{\partial \hat{p}}\dot{\hat{p}} - k_d e_d\Big) \end{aligned} \tag{22}$$

so that we obtain the closed-loop tracking error dynamics for $e_d(t)$ as

$$L_{sc}\dot{e}_d = -k_d e_d \tag{23}$$

where $k_d > 0$ is a constant control gain.

We now begin to design the estimators to address the issue of convergence for $\hat{p}(t)$. Motivated by the structure of (10), we design the auxiliary signal $\hat{V}_{dc}(t)$ as follows

$$\begin{array}{rl} C_{dc}\dot{\hat{V}}_{dc}= & -\hat{p}V_{dc} - u_d i_{SCd} - u_q i_{SCq} + k_v\tilde{V}_{dc}, \\ \hat{V}_{dc}(0) = & V_{dc}(0) \end{array} \quad (24)$$

where $k_v > 0$ is a constant estimator gain. Now, we can obtain the dynamics for the auxiliary error signal $\tilde{V}_{dc}(t)$ as follows

$$C_{dc}\dot{\tilde{V}}_{dc} = -V_{dc}\tilde{p} - k_v\tilde{V}_{dc} \quad (25)$$

where we have utilized (10), (17), and (24). Motivated by the dynamics above and our subsequent stability analysis, the parameter estimate $\hat{p}(t) \in \Re^1$ is designed as follows

$$\dot{\hat{p}} = -\dot{\tilde{p}} = \begin{cases} \Omega_1 & \text{if } \hat{p} \in [p_{\min}, p_{\max}) \text{ and } \Omega_1 \geq 0 \\ \Omega_1 & \text{if } \hat{p} \in (p_{\min}, p_{\max}] \text{ and } \Omega_1 \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (26)$$

where $\Omega_1(t) \in \Re^1$ is defined as follows

$$\Omega_1 = -k_p V_{dc}\tilde{V}_{dc} \quad (27)$$

where $k_p > 0$ is a constant adaptation gain. If we choose $\hat{p}(0) \in [p_{\min}, p_{\max}]$, the above update law for $\hat{p}(t)$ ensures that $\hat{p}(t) \in [p_{\min}, p_{\max}]\ \forall\ t$, and as a consequence, we can claim that $\hat{p}(t), \tilde{p}(t) \in \mathcal{L}_\infty$.

6 Stability Analysis

Before we get to the main result, we state and prove the following lemma.

Lemma 1 *The signals $i_{SCd}(t), i_{SCq}(t), V_{dc}(t) \in \mathcal{L}_\infty$.*

Proof: Let $V_0 = \frac{1}{2}z^T\Lambda z$ be a non-negative function. By differentiating $V_0(t)$ along the dynamics of (10)-(12), it is easy to see that $z(t) \triangleq \begin{bmatrix} i_{SCd} & i_{SCq} & V_{dc} \end{bmatrix} \in \Re^3$ can be upperbounded as follows

$$\|z(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}}\|z(0)\|\exp(-\frac{\lambda_3}{2\lambda_2}t) + \frac{\lambda_2}{\sqrt{\lambda_1}\lambda_3}\sup_t |v_{Ld}|$$

where $\lambda_i > 0$ depend on system parameters. Thus, $\|z(t)\| \in \mathcal{L}_\infty \Rightarrow i_{SCd}(t), i_{SCq}(t), V_{dc}(t) \in \mathcal{L}_\infty$.

6.1 Main Theorem

Theorem 2 *If the control gains k_d, k_q and the initial errors $e_d(0), e_q(0)$ are selected such that the following sufficient conditions hold for the nominal model of the STATCOM*

$$k_d > L_{sc}\beta_6 \qquad k_q > L_{sc}\beta_6 \quad (28)$$

$$\frac{e_d(0)\beta_2}{\tau_d^{-1} - \beta_6} + \frac{e_d^2(0)\beta_3}{2\tau_d^{-1} - \beta_6} + \frac{e_q(0)\beta_4}{\tau_q^{-1} - \beta_6} + \frac{e_q^2(0)\beta_5}{2\tau_q^{-1} - \beta_6} < \left[V_{dc}^{*\,2} p_{\min} - \left(\frac{2L_{sc}}{C_{dc}}\right)p_e^2\right]\frac{1}{\beta_6}, \quad (29)$$

then the errors $e_d(t), e_q(t)$ and $\tilde{p}(t)$ asymptotically converge to the origin and all plant and controller signals remain bounded. Here, $p_e, \beta_i\ \forall\ i = 2...6$ are model dependent constants defined in Appendices B and C, while $\tau_d \triangleq k_d^{-1}L_{sc}$ and $\tau_q \triangleq k_q^{-1}L_{sc}$ are the time constants associated with $e_d(t)$ and $e_q(t)$, respectively.

Proof: From (20) and (23), we already know that $e_d(t), e_q(t)$ asymptotically (exponentially) converge to the origin. We begin the remainder of the proof by defining a positive function $V_1(t) \in \Re^1$ as follows

$$V_1 = \frac{1}{2}C_{dc}\tilde{V}_{dc}^2 + \frac{1}{2}k_p^{-1}\tilde{p}^2 \quad (30)$$

After taking the time derivative of (30) along the trajectories of (25) and (26), we obtain the following expression

$$\dot{V}_1 = -k_v\tilde{V}_{dc}^2 \quad (31)$$

From (30) and (31), $\tilde{p}(t), \hat{p}(t) \in \mathcal{L}_\infty$ and $\tilde{V}_{dc}(t) \in \mathcal{L}_\infty \cap \mathcal{L}_2$. From (25) and Lemma 1, it is easy to see that $\dot{\tilde{V}}_{dc}(t) \in \mathcal{L}_\infty$. This implies from Barbalat's Lemma [13] that $\lim_{t\to\infty}\tilde{V}_{dc}(t) = 0$. From (42) and the boundedness of v_{Ld}, i_{SCq}^*, and $\hat{p}(t)$, it is easy to see that $i_{SCd}^* \in \mathcal{L}_\infty$. From the previous assertions and the expression of (19), it is easy to see that $u_q(t)$ is bounded if $V_{dc}(t)$ can be bounded away from zero. This observation leads us into the next stage of the proof.

In this stage, we prove that $V_{dc}(t) \geq \varepsilon > 0\ \forall\ t$ if the conditions given in (28) and (29) are satisfied. We start by defining a non-negative function $V_2(t) \in \Re^1$ as follows

$$V_2 = \frac{1}{2}C_{dc}V_{dc}^2 \quad (32)$$

After differentiating (32) with respect to time along the dynamics of (10), we obtain

$$\begin{array}{rl} \dot{V}_2 = & -\Big(v_{Ld} + R_{sc}i_{SCd} - \omega L_{sc}i_{SCq} + L_{sc}\dfrac{\partial i_{SCd}^*}{\partial\hat{p}}\dot{\hat{p}} \\ & -k_d e_d\Big)i_{SCd} - \Big(R_{sc}i_{SCq} + \omega L_{sc}i_{SCd} \\ & -k_q e_q\Big)i_{SCq} - V_{dc}^2 p \end{array} \quad (33)$$

where we have utilized the design equations of (19) and (22) to substitute for $u_d(t)$ and $u_q(t)$. After utilizing the definitions of (13) and (14) in (33), one can obtain the following convenient form for $\dot{V}_2(t)$

$$\begin{aligned}\dot{V}_2 = \; & V_{dc}^{*\,2}\hat{p} - 2R_{sc}\left(i_{SCd}^{*}e_d + i_{SCq}^{*}e_q\right) - R_{sc}(e_d^2 + e_q^2) \\ & + k_d e_d i_{SCd} + k_q e_q i_{SCq} - v_{Ld}e_d \\ & + \left[-pV_{dc}^2 - L_{sc}\frac{\partial i_{SCd}^{*}}{\partial \hat{p}}k_p V_{dc}\tilde{V}_{dc}i_{SCd}\right]\end{aligned} \tag{34}$$

where we have utilized the quasi-steady power balance equation

$$V_{dc}^{*\,2}\hat{p} = -v_{Ld}i_{SCd}^{*} - R_{sc}\left(i_{SCd}^{*^2} + i_{SCq}^{*^2}\right)$$

In (34) above, let us denote the bracketed term by $\gamma(t)$. After utilizing the definition of (14), we can lower bound $\gamma(t)$ as follows

$$\gamma \geq -\beta_1 V_{dc}^2 - L_{sc}k_p\left[\tilde{V}_{dc}^2 + \left|\frac{\partial i_{SCd}^{*}}{\partial \hat{p}}\right|\tilde{V}_{dc}^2 e_d^2\right] \tag{35}$$

where β_1 is a model dependent constant defined as

$$\beta_1 = p_{\max} + L_{sc}k_p\left(\left|\frac{\partial i_{SCd}^{*}}{\partial \hat{p}}\right|_{\max} + \left|\frac{\partial i_{SCd}^{*}}{\partial \hat{p}}i_{SCd}^{*}\right|_{\max}^2\right)$$

where the maximum is computed over the entire model and control parameter space. In order to prove that β_1 exists and is bounded, we need to prove the boundedness of $\frac{\partial i_{SCd}^{*}}{\partial \hat{p}}$. We refer the reader to Appendix A for the details of such a proof. We next target the bracketed term in (35); an upperbound for $\tilde{V}_{dc}(t)$ in terms of model parameters is derived in Appendix B. After utilizing (44) in (35), we can find a new lower bound for $\gamma(t)$ in the following fashion

$$\gamma \geq -\beta_1 V_{dc}^2 - \frac{2L_{sc}}{C_{dc}}p_e^2 - \frac{2L_{sc}}{C_{dc}}\left|\frac{\partial i_{SCd}^{*}}{\partial \hat{p}}\right|p_e^2 e_d^2 \tag{36}$$

After substituting (36) in (34), we obtain a lower bound for $\dot{V}_2(t)$ as follows

$$\begin{aligned}\dot{V}_2 \geq \; & -\beta_1 V_{dc}^2 + V_{dc}^{*\,2}p_{\min} - \frac{2L_{sc}}{C_{dc}}p_e^2 \\ & -|e_d|\left\{v_{Ld} + (k_d + 2R_{sc})\left|i_{SCd}^{*}\right|_{\max}\right. \\ & \left. +|e_d|\left(k_d + R_{sc} + \frac{2L_{sc}}{C_{dc}}\left|\frac{\partial i_{SCd}^{*}}{\partial \hat{p}}\right|_{\max}p_e^2\right)\right\} \\ & -|e_q|\left\{(k_q + 2R_{sc})\left|i_{SCq}^{*}\right|_{\max} + |e_q|(k_q + R_{sc})\right\}.\end{aligned} \tag{37}$$

Given the definitions of constants $\beta_i \; \forall \; i = 2,3,4,5,6$ in Appendix C and the expressions for $e_d(t), e_q(t)$ as obtained from the solution of (20) and (23), we can rewrite the inequality of (37) as follows

$$\begin{aligned}\dot{V}_2 \geq \; & -\beta_6 V_2 + V_{dc}^{*\,2}p_{\min} - \frac{2L_{sc}}{C_{dc}}p_e^2 \\ & -\beta_2|e_d(0)|\exp(-t/\tau_d) \\ & -\beta_3|e_d(0)|^2\exp(-2t/\tau_d) \\ & -\beta_4|e_q(0)|\exp(-t/\tau_q) \\ & -\beta_5|e_q(0)|^2\exp(-2t/\tau_q)\end{aligned} \tag{38}$$

where the time constants τ_d, τ_q have been previously defined in the statement of Theorem 2. From (38), a lower bound for $V_2(t)$ is easily obtainable as follows

$$\begin{aligned}V_2(t) \geq \; & \left[V_{dc}^{*\,2}p_{\min} - (\frac{2L_{sc}}{C_{dc}})p_e^2\right]\frac{1}{\beta_6} + \frac{|e_d(0)|\,\beta_2}{\beta_6 - \tau_d^{-1}} \\ & + \frac{e_d^2(0)\beta_3}{\beta_6 - 2\tau_d^{-1}} + \frac{|e_q(0)|\,\beta_4}{\beta_6 - \tau_q^{-1}} + \frac{e_q^2(0)\beta_5}{\beta_6 - \tau_q^{-1}} \\ & + \left[V_2(0)\exp(-\beta_6 t) - \frac{|e_d(0)|\,\beta_2}{(\beta_6 - \tau_d^{-1})}e^{-t/\tau_d}\right. \\ & - \frac{e_d^2(0)\beta_3}{\beta_6 - 2\tau_d^{-1}}\exp(-2t/\tau_d) \\ & - \frac{|e_q(0)|\,\beta_4}{\beta_6 - 2\tau_q^{-1}}\exp(-t/\tau_q) \\ & \left. - \frac{e_q^2(0)\beta_5}{\beta_6 - 2\tau_q^{-1}}\exp(-2t/\tau_q)\right]\end{aligned} \tag{39}$$

The first two rows denote the steady-state part of the solution while the remainder is the transient. An inspection of the transient reveals that choosing the time constants $\tau_{d,q} < \beta_6^{-1}$ is equivalent to the gain condition of (28) and ensures that the transient contribution to $V_2(t)$ is always non-negative. Now, it is easy to see that (29) is a sufficient condition on the initial conditions that ensures that the steady state terms in (39) are always net positive. Hence, we can say that $V_{dc}(t) \geq \varepsilon > 0 \; \forall \; t$ which now proves the boundedness of $u_q(t)$. From the previous assertions, it is a simple matter to show the boundedness of $u_d(t)$. A little signal chasing shows that all plant and controller signals stay bounded during closed-loop operation.

The final stage of the proof deals with proving that $\lim_{t\to\infty}\tilde{p}(t) = 0$. We begin by differentiating (25) to obtain the following equation

$$\begin{aligned}C_{dc}\ddot{\tilde{V}}_{dc} = \; & \tilde{p}C_{dc}^{-1}\left(u_d i_{SCd} + u_q i_{SCq} + V_{dc}p\right) \\ & - V_{dc}k_p V_{dc}\tilde{V}_{dc} + k_v C_{dc}^{-1}\left(V_{dc}\tilde{p} + k_v\tilde{V}_{dc}\right)\end{aligned} \tag{40}$$

From our previous assertions, all signals on the right hand side of (40) are bounded, so $\ddot{\tilde{V}}_{dc}(t) \in \mathcal{L}_\infty$ which establishes the uniform continuity of $\dot{\tilde{V}}_{dc}(t)$. From the fact that $\lim_{t\to\infty}\tilde{V}_{dc}(t) = 0$, we also know the following

$$\lim_{t\to\infty}\int_0^t \dot{\tilde{V}}_{dc}(\tau)\,d\tau = \tilde{V}_{dc}(\infty) - \tilde{V}_{dc}(0) = -\tilde{V}_{dc}(0) < \infty$$

From the above assertions and the integral form of Barbalat's Lemma [13], we know that $\lim_{t\to\infty}\dot{\tilde{V}}_{dc}(t) = 0$. From (25) and the strict positiveness of $V_{dc}(t)$, we can now state that $\lim_{t\to\infty}\tilde{p}(t) = 0$. Thus, all the errors asymptotically converge to zero. Hence, $i_{SCd}(t)$ and consequentially, $V_{dc}(t)$ are regulated to the correct reference as was hypothesized in Section 4.□

7 Simulation Results and Discussion

The simulation results will be made available from the authors upon request.

8 Conclusions

This paper has presented a non linear control strategy for controlling a STATCOM to act as an instantaneous reactive current source with the overall objective of using it for load voltage regulation. A detailed large signal stability analysis has been presented to justify the control scheme. Simulation results show that the controlled STATCOM acts as a reactive current source while compensating for internal losses and maintaining its dc bus voltage to a reference value. Using the controlled STATCOM, the voltage magnitude controller successfully regulates the load voltage. A comparative study of nonlinear and linear controllers to control a STATCOM to act as a reactive current source has been presented in [14].

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Appendices available upon request.