An Adaptive Filtering Approach to Target Tracking

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ABSTRACT

A method is presented for augmenting an extended Kalman filter with an adaptive element. The resulting estimator provides robustness to parameter uncertainty and unmodeled dynamics. The design of the adaptive element employs a linearly parameterized neural network. The network weights are adjusted on line using the filter error residuals. Boundedness of signals is proven using Lyapunov's direct method and a backstepping argument. Simulations illustrate the theoretical results.

I. INTRODUCTION

The problem of state estimation of nonlinear stochastic systems is a widely encountered problem in science and engineering and has received a considerable amount of attention since the early development of methods for linear state estimation [1]–[5]. In the case of linear dynamical systems with white process and measurement noise, the Kalman filter is known to be an optimal estimator [6], while in linear systems with deterministic disturbances the Luenberger observer offers a complete and comprehensive answer to the problem of state estimation [7]. Of the numerous attempts being made for the development of nonlinear estimator theory, the most popular one is the extended Kalman filter (EKF), whose design is based on a first order local linearization of the system around a reference trajectory at each time step [8]–[10]. In addition to its application in the field of nonlinear state estimation, EKFs are also used to estimate the unknown parameters of stochastic linear dynamical systems as reported in [11]–[15]. While parameter estimation of linear and nonlinear systems using EKFs has received a fair amount of attention, nonlinear state estimation using EKFs has become one of the most researched problems [16]–[20].

EKFs are extensively applied to problems related to target tracking and to target rendezvous and interception. In [21], the authors have developed a set of tracking algorithms that are applicable for ballistic reentry vehicles, tactical missiles and airplanes. In [22]–[24], the feasibility of target tracking is studied from a point of view of range-only measurements. However in some situations it may be impractical to measure the range, and state estimation using measurements of the line-of-sight or bearing angle is highly desirable. Hence, designing EKFs for target trackers with bearings-only measurement has been a widely studied subject [25]–[31]. In the case of bearing measurements the process may be unobservable unless the sensing vehicle executes a maneuver [26], which further complicates the bearings-only problem.

A key to successful target tracking lies in the effective extraction of useful information about the target's state from observations. In the setting of estimation, this necessitates adding additional states to model the target dynamics. Consequently, the accuracy of the estimator depends on the accuracy to which the target behavior has been characterized. Target behavior not captured by modelling introduces estimation bias, and can even cause divergence in the estimate.

To account for modelling errors in the process, neural network (NN) based adaptive identification and estimation schemes have been proposed in [32]–[36]. In [32], an approach is developed that augments a linear time invariant filter with an NN while in [33]–[35] schemes for augmenting an EKF with an NN are provided. However the approaches in [32]–[35] all require the knowledge of the full dimension of the system. In [36], an approach that does not require the knowledge of the full dimension of the system is developed. However, this approach only permits augmentation of a linear time invariant state estimator with an NN. In this paper we develop an adaptive design in a form that is useful for augmenting an EKF. The NN is trained directly by the residuals produced by the EKF.

We illustrate a typical application in which the range between two aircrafts is regulated, by feeding back estimates of the target velocity obtained by processing camera images. The goal is to accomplish a task in an unmanned system that is commonly performed in a manned system

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that relies primarily on visual information. This problem is approached from the point of view of using (i) bearings-only measurement and (ii) bearing angle and the angle subtended by the target in the image plane as measurements, which will be referred to as the two-angles problem.

The paper is structured as follows: Section II provides basic definitions and theorems which are required for the stability analysis. Section III sets up the problem under consideration and summarizes several important approximation properties of NNs. Section IV presents the form of the adaptive estimator and the formulation of the error dynamics. Section V provides the stability analysis needed to prove the main theorem in the paper and the resulting ultimate bounds. In Section VI the performance of the proposed NN based EKF is illustrated by considering the application of an aircraft called the **follower**, tracking another aircraft called the **target**. Section VII presents the concluding remarks. Throughout the report bold symbols are used for vectors, capital letters for matrices, small letters for scalars, and $\|\cdot\|_F$, $\|\cdot\|$ stand for Frobenius norm and for $2-$norm correspondingly unless otherwise specified.

II. Mathematical Preliminaries

Consider the nonlinear dynamical system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}), \qquad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{1}$$

where $\boldsymbol{f} : [0, \infty) \times \mathcal{D} \to \mathcal{R}^n$ is continuously differentiable, $\mathcal{D} = \{\boldsymbol{x} \in \mathcal{R}^n | \ \|\boldsymbol{x}\|_2 < r\}$, and the Jacobian matrix $\left[\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right]$ is bounded and Lipschitz on $\mathcal{D}$, uniformly in t.

Theorem 1: [37] Let $\boldsymbol{x} = 0$ be an equilibrium point for the nonlinear system in (1). Let k, λ and r_0 be positive constants with $r_0 < \frac{r}{k}$. Let $\mathcal{D}_0 = \{\boldsymbol{x} \in \mathcal{R}^n | \ \|\boldsymbol{x}\| < r_0\}$. Assume that the trajectory of the system satisfies

$$\|\boldsymbol{x}(t)\| \leq \ k\|\boldsymbol{x}(t_0)\| e^{-\lambda(t-t_0)}, \ \forall \ \boldsymbol{x}(t_0) \in \mathcal{D}_0, \ \forall \ t \geq t_0 \geq 0$$

Then, there is a $\mathcal{C}^1$ function $V : [0, \infty) \times \mathcal{D}_0 \to \mathcal{R}$ that satisfies the inequalities

$$\begin{aligned} c_1\|\boldsymbol{x}\|^2 &\leq V(t, \boldsymbol{x}) \leq c_2\|\boldsymbol{x}\|^2 \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \boldsymbol{x}}\boldsymbol{f}(t, \boldsymbol{x}) &\leq -c_3\|\boldsymbol{x}\|^2 \\ \left\|\frac{\partial V}{\partial \boldsymbol{x}}\right\| &\leq c_4\|\boldsymbol{x}\| \end{aligned} \tag{2}$$

for some positive constants $c_1, \ c_2, \ c_3$ and c_4.

In [38], [39], it has been shown that for an *observable* system such an approximation can be achieved using a finite sample of the output history. We recall the main theorem from [39] in the form of the following existence theorem.

Theorem 2: [39] Assume that an n-dimensional state vector $\boldsymbol{x}(t)$ of an *observable* time-invariant system

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}) \\ \boldsymbol{y} &= \boldsymbol{h}(\boldsymbol{x}) \end{aligned} \tag{3}$$

evolves on an n-dimensional ball of radius $\bar{r}$ in $\mathcal{R}^n$, $\mathcal{B}_{\bar{r}} = \{\boldsymbol{x} \in \mathcal{R}^n, \|\boldsymbol{x}\| \leq \bar{r}\}$. Also assume that the system output $\boldsymbol{y}(t) \in \mathcal{R}^m$ and its derivatives up to the order $(n-1)$ are bounded. Then given arbitrary $\epsilon^* > 0$, there exists a set of bounded weights M and a positive time delay $d > 0$, such that the function $\boldsymbol{f}(\boldsymbol{x})$ in (3) can be approximated over the compact set $\mathcal{B}_{\bar{r}}$ by a linearly parameterized NN

$$\boldsymbol{f}(\boldsymbol{x}) = M^T\boldsymbol{\sigma}(\boldsymbol{\mu}) + \boldsymbol{\epsilon}(\boldsymbol{\mu}), \quad \|M\|_F \leq M^*, \quad \|\boldsymbol{\epsilon}(\boldsymbol{\mu})\|_F \leq \epsilon^*$$

using the input vector:

$$\boldsymbol{\mu}(\boldsymbol{y}(t), d) = \left[\begin{array}{ccc} \Delta_d^{(0)}\boldsymbol{y}^T(t) & \cdots & \Delta_d^{(n-1)}\boldsymbol{y}^T(t) \end{array} \right]^T \in \mathbb{R}^{nm}$$

where $\Delta_d^{(0)}\boldsymbol{y}^T(t) \triangleq \boldsymbol{y}^T(t)$, $\Delta_d^{(k)}\boldsymbol{y}^T(t) \triangleq \frac{\Delta_d^{(k-1)}\boldsymbol{y}^T(t) - \Delta_d^{(k-1)}\boldsymbol{y}^T(t-d)}{d}$, $k = 1, 2, \cdots,$ $\|\boldsymbol{\mu}\| \leq \mu^*, \ \mu^* > 0$ is a uniform bound on $\mathcal{B}_{\bar{r}}$.

Remark 1: Notice that when the dimension n of the system is not known, and only an upper bound $n_1 > n$ for its dimension is available, then, provided that the $(n_1 - 1)$ derivatives of the output are bounded, one can use an input vector, comprised of $(n_1 - 1)$ quotients, while not sacrificing on the bound of the approximation.

III. Problem Formulation

Let the dynamics of an *observable* and *bounded* nonlinear process be given by the following equations [1] :

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{z}) = \boldsymbol{f}_0(\boldsymbol{x}) + Bz_1, & \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \dot{\boldsymbol{z}} &= \boldsymbol{h}(\boldsymbol{z}), & \boldsymbol{z}(0) = \boldsymbol{z}_0 \\ \boldsymbol{y} &= C^T\boldsymbol{x} \end{aligned} \tag{4}$$

where $\boldsymbol{x} \in \mathcal{D}_{\boldsymbol{x}} \subseteq \mathcal{R}^{n_{\boldsymbol{x}}}$, $\boldsymbol{z} \in \mathcal{D}_{\boldsymbol{z}} \subseteq \mathcal{R}^{n_{\boldsymbol{z}}}$, are the states of the system, $\mathcal{D}_{\boldsymbol{x}}$ and $\mathcal{D}_{\boldsymbol{z}}$ are compact sets, $\boldsymbol{f}_0(\boldsymbol{x}) : \mathcal{D}_{\boldsymbol{x}} \to \mathcal{R}^{n_{\boldsymbol{x}}}$ is a known smooth function which can be expressed as a Taylor series expansion for all the values of $\boldsymbol{x}$ in the domain of interest $\mathcal{D}_{\boldsymbol{x}}$, B and C are known matrices, $\boldsymbol{h}(\boldsymbol{z}) : \mathcal{D}_{\boldsymbol{z}} \to \mathcal{R}^{n_{\boldsymbol{z}}}$ is an unknown function, $\boldsymbol{z}_1 \in \mathcal{R}^m$ has a known upper bound $\bar{z}_1$ and $\boldsymbol{y} \in \mathcal{R}^m$ is a vector of available measurements. The dimension $n_{\boldsymbol{z}}$ of the vector $\boldsymbol{z}$ is unknown and hence the dimension $n = n_{\boldsymbol{x}} + n_{\boldsymbol{z}}$ is also unknown. The relative degree of $\boldsymbol{y}$ is defined to be the vector $[\ r_1 \quad r_2 \quad \cdots \quad r_m\ ]$ such that $\nabla y_j^{(i)}B = 0, \quad i = 0, 1, \cdots, r_j - 1, \quad j = 1, \cdots, m$ and $\nabla y_j^{(i)}B \neq 0, \quad i = r_j, \quad j = 1, \cdots, m$

IV. Adaptive Estimator and Error Dynamics

Using Theorem 2, consider the following NN approximation of $\boldsymbol{z}_1$

$$\boldsymbol{z}_1 = M^T\boldsymbol{\sigma}(\boldsymbol{\mu}) + \boldsymbol{\epsilon}(\boldsymbol{\mu}), \|M\|_F \leq M^*, \|\boldsymbol{\epsilon}(\boldsymbol{\mu})\|_F \leq \epsilon^* \tag{5}$$

[1] For the definition on observability of nonlinear systems, refer to [40].

where M^* denotes a known upper bound for the Frobenius norm of the weight in (5), $\boldsymbol{\mu}$ is a vector of the difference quotients of the measurement $\boldsymbol{y}$ as defined in (4). We propose the following *adaptive* estimator for the dynamics in (4)

$$\begin{aligned}\dot{\hat{\boldsymbol{x}}} &= \boldsymbol{f}_0(\hat{\boldsymbol{x}}) + B\boldsymbol{\nu}_{ad} + K(t)(\boldsymbol{y} - \hat{\boldsymbol{y}}),\ \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_0 \\ \hat{\boldsymbol{y}} &= C^T\hat{\boldsymbol{x}} \end{aligned} \quad (6)$$

where the gain history $K(t)$ depends on the history of the past measurements, which are all uniquely defined by $\boldsymbol{x}_0$ and $\hat{\boldsymbol{x}}_0$, and $\boldsymbol{\nu}_{ad}$ is the output of the adaptive element and is designed as

$$\boldsymbol{\nu}_{ad} = \hat{M}^T\boldsymbol{\sigma}(\boldsymbol{\mu}) \quad (7)$$

where $\hat{M}$ is the estimate of the weight that is adjusted on line. Denoting the tracking error signals $\boldsymbol{e} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ and $\tilde{\boldsymbol{y}} = \boldsymbol{y} - \hat{\boldsymbol{y}}$, we can formulate the error dynamics as follows

$$\begin{aligned}\dot{\boldsymbol{e}} &= \Big(\boldsymbol{f}_{\boldsymbol{e}}(t, \boldsymbol{e}) - K(t)C^T\Big)\boldsymbol{e} + B\big(\boldsymbol{z}_1 - \boldsymbol{\nu}_{ad}\big) \\ \tilde{\boldsymbol{y}} &= C^T\boldsymbol{e} \end{aligned} \quad (8)$$

Assumption 1: We assume that for $\boldsymbol{z}_1 = 0$, and in the absence of the NN, the equilibrium point $\boldsymbol{e} = 0$ of the error dynamics in (8) is exponentially stable regardless of the measurement history.

Remark 2: When $\boldsymbol{z}_1 \neq 0$, and in the absence of the NN, (8) is not necessarily input-to-state stable with the $\boldsymbol{z}_1$ viewed as the input.

V. Stability Analysis - Backstepping Approach

In this section we show through Lyapunov's direct method that the tracking error $\boldsymbol{e}$ and the NN weight error $\tilde{M} \triangleq M - \hat{M}$ are ultimately bounded. The arguments in the proof will be based on the idea of *backstepping* which enables a choice of a particular form of the Lyapunov function [37]. By substituting (5) and (7) into (8), the error dynamics along with the adaptation law in (10) can be written as

$$\begin{aligned}\dot{\boldsymbol{e}} &= \Big(\boldsymbol{f}_{\boldsymbol{e}}(t, \boldsymbol{e}) - K(t)C^T\Big)\boldsymbol{e} + B\tilde{M}^T\boldsymbol{\sigma}(\boldsymbol{\mu}) + B\boldsymbol{\epsilon}(\boldsymbol{\mu}) \\ \dot{\tilde{M}} &= \Gamma_M\Big(\boldsymbol{\sigma}(\boldsymbol{\mu})\tilde{\boldsymbol{y}}^T + k_{\boldsymbol{\sigma}}\|\tilde{\boldsymbol{y}}\|\hat{M}\Big) \\ \tilde{\boldsymbol{y}} &= C^T\boldsymbol{e} \end{aligned} \quad (9)$$

where Γ_M specifies the learning rate of the NN and $k_{\boldsymbol{\sigma}}$ denotes the σ-modification gain [41]. Defining $\boldsymbol{F}_{\boldsymbol{e}}(t, \boldsymbol{e}) \triangleq (\boldsymbol{f}_{\boldsymbol{e}}(t, \boldsymbol{e}) - K(t)C^T)\boldsymbol{e}$ and using Assumption 1 and Theorem 1, when $\boldsymbol{z}_1 = 0$, and in the absence of the NN, we are guaranteed the existence of a Lyapunov function $V_{\boldsymbol{e}}(t, \boldsymbol{e})$ that satisfies conditions of Theorem 1.

Consider the composite error vector, $\boldsymbol{\zeta} = \begin{bmatrix} \boldsymbol{e}^T & \tilde{M}^T \end{bmatrix}^T$. Introduce the largest ball $\mathcal{B}_R \triangleq \{\boldsymbol{\zeta} \mid \|\boldsymbol{\zeta}\| \leq R\}$, $R > 0$, that lies in $\Omega_{\boldsymbol{\zeta}} \triangleq \Big\{(\boldsymbol{e}, \tilde{M})\Big\}$.

Theorem 3: Let the initial errors, $\boldsymbol{e}(0)$ and $\tilde{M}(0)$, belong to the set Ω_α in Fig. 1. Let Assumption 1 hold and let the NN adaptation law be given by

$$\dot{\hat{M}} = -\Gamma_M\Big(\boldsymbol{\sigma}(\boldsymbol{\mu})\tilde{\boldsymbol{y}}^T + k_{\boldsymbol{\sigma}}\|\tilde{\boldsymbol{y}}\|\hat{M}\Big) \quad (10)$$

where $\frac{\gamma^2}{R^2 2c_1} < \Gamma_M < \frac{R^2}{\gamma^2 2c_1}$, $k_\sigma > 0$. Then the tracking error $\boldsymbol{e}(t)$ and the NN weight error $\tilde{M}(t)$ are uniformly ultimately bounded with the ultimate bound given by the right hand side of (16) and (17).

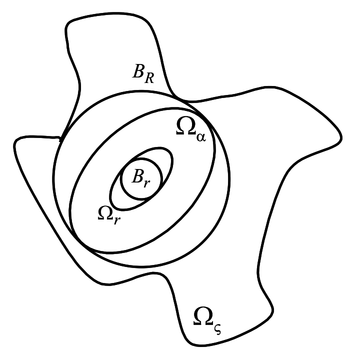

Fig. 1. Geometric Representation of the sets in the Error Space.

Proof: When $\boldsymbol{z}_1 \neq 0$, and in the presence of the NN, we choose the following Lyapunov function candidate to arrive at boundedness of the error signals

$$V(t, \boldsymbol{\zeta}) = V_{\boldsymbol{e}}(t, \boldsymbol{e}) + \frac{1}{2}\mathrm{tr}(\tilde{M}^T\Gamma_M^{-1}\tilde{M}) \quad (11)$$

The derivative of $V(t, \boldsymbol{\zeta})$ along (9) can be written as

$$\begin{aligned}\dot{V} &= \frac{\partial V_{\boldsymbol{e}}}{\partial t} + \frac{\partial V_{\boldsymbol{e}}}{\partial \boldsymbol{e}}\Big(\boldsymbol{F}_{\boldsymbol{e}}(t, \boldsymbol{e}) + B\tilde{M}^T\boldsymbol{\sigma}(\boldsymbol{\mu}) + B\boldsymbol{\epsilon}(\boldsymbol{\mu})\Big) \\ &\quad + \mathrm{tr}\Big(\tilde{M}^T\Big(\boldsymbol{\sigma}(\boldsymbol{\mu})\tilde{\boldsymbol{y}}^T + k_{\boldsymbol{\sigma}}\|\tilde{\boldsymbol{y}}\|\hat{M}\Big)\Big) \end{aligned} \quad (12)$$

Using (5), Theorem 1 and the inequality $\|\boldsymbol{\sigma}(\boldsymbol{\mu})\| \leq 1$, the derivative can be upper bounded as

$$\begin{aligned}\dot{V} &\leq -c_3\|\boldsymbol{e}\|^2 + c_4\|\boldsymbol{e}\|\ \|B\|\ \|\tilde{M}\|_F + c_4\|\boldsymbol{e}\|\ \|B\|\epsilon^* \\ &\quad + \|\tilde{\boldsymbol{y}}\|\|\tilde{M}\|_F + k_{\boldsymbol{\sigma}}\|\tilde{\boldsymbol{y}}\|\mathrm{tr}(\tilde{M}^T\hat{M}) \end{aligned} \quad (13)$$

Further using the inequalities $\|\tilde{\boldsymbol{y}}\| \leq \|C\|\ \|\boldsymbol{e}\|$ and $\mathrm{tr}\Big[\tilde{M}^T(M - \tilde{M})\Big] \leq \|\tilde{M}\|_F M^* - \|\tilde{M}\|_F^2$ and denoting $c_5 \triangleq c_4\|B\| + \|C\|$, we can upper bound the Lyapunov derivative as

$$\begin{aligned}\dot{V} &\leq -\|\boldsymbol{e}\|\Big(c_3\|\boldsymbol{e}\| + \|\tilde{M}\|_F\Big(k_{\boldsymbol{\sigma}}\|C\|\|\tilde{M}\|_F \\ &\quad - k_{\boldsymbol{\sigma}}\|C\|M^* - c_5\Big) - c_4\|B\|\epsilon^*\Big) \end{aligned} \quad (14)$$

Denoting $\kappa \stackrel{\Delta}{=} k_{\boldsymbol{\sigma}}\|C\|$ and completing the squares on $\|\tilde{M}\|_F\Big(\kappa\|\tilde{M}\|_F - \kappa M^* - c_5\Big)$ we finally obtain

$$\dot{V} \leq -\|\boldsymbol{e}\|\Big(c_3\|\boldsymbol{e}\| + \Big(\sqrt{\kappa}\|\tilde{M}\|_F - \Big(\frac{\kappa M^* + c_5}{2\sqrt{\kappa}}\Big)\Big)^2 - \frac{(\kappa M^* + c_5)^2}{4\kappa} - c_4\|B\|\epsilon^*\Big) \quad (15)$$

Thus either of the conditions

$$\|\boldsymbol{e}\| > \frac{(\kappa M^* + c_5)^2}{4\kappa c_3} + \frac{c_4\|B\|\epsilon^*}{c_3} \quad (16)$$

$$\|\tilde{M}\|_F > \frac{\kappa M^* + c_5}{2\kappa} + \sqrt{\frac{(\kappa M^* + c_5)^2}{4\kappa^2} + \frac{c_4\|B\|\epsilon^*}{\kappa}} \quad (17)$$

will guarantee $\dot{V}(t, \boldsymbol{\zeta}) < 0$ outside the compact set $\mathcal{B}_\gamma = \{\boldsymbol{\zeta} \in \mathcal{B}_R \mid \|\boldsymbol{\zeta}\| \leq \gamma\}$. ■

VI. Simulation Results

We consider the problem of target-follower tracking in two dimensions where the goal is to regulate the range between the target and the follower to 2 wing spans. Simulation runs were performed for the case of bearings-only measurement and the two-angles case. The two-angles case uses measurements of the bearing angle and the angle subtended by the target in the image plane. An advantage of using two angles as measurements is that observability is preserved even when the follower aircraft is not maneuvering.

The process dynamics can be described in polar coordinates by the following set of nonlinear differential equations [27]:

$$\frac{d}{dt}\begin{bmatrix} \dot{\beta} \\ \frac{\dot{r}}{r} \\ \beta \\ \frac{1}{r} \\ b \end{bmatrix} = \begin{bmatrix} -2\dot{\beta}\frac{\dot{r}}{r} + \frac{1}{r}(a_y\cos\beta - a_x\sin\beta) \\ \dot{\beta}^2 - \left(\frac{\dot{r}}{r}\right)^2 + \frac{1}{r}(a_y\sin\beta + a_x\cos\beta) \\ \dot{\beta} \\ -\frac{\dot{r}}{r}\frac{1}{r} \\ 0 \end{bmatrix} \quad (18)$$

and the measurements are given by the following equations:

$$\begin{aligned} \beta_m(t) &= \beta(t) + \nu_\beta(t) \\ \alpha_m(t) &= 2\tan^{-1}\left(\frac{b}{2r}\right) + \nu_\alpha(t) \end{aligned} \quad (19)$$

where β is the bearing angle, α the angle subtended by the target in the image plane, r represents the range between the target aircraft and the follower aircraft, b denotes the size of the target which is assumed to be constant and a_x and a_y are the horizontal relative acceleration components in a Cartesian frame. The measurement noise ν_β and ν_α are band limited zero mean white noise processes with a standard deviation of 0.01. The initial covariance matrix was chosen as $P_0 = \text{diag}\Big[\ 0.1^2 \quad 0.3 \quad 0.1^2 \quad 0.1 \quad 0.1^3\ \Big]$.

Figs. 2 and 3 compare the performances of the EKF and EKF+NN for the bearings-only case when the target maneuvers in a sinusoidal manner with a target acceleration of $0.3g$. As seen from Fig. 2 the estimates of the EKF exhibit severe performance degradation. Fig. 3 shows that there is a remarkable improvement when the EKF is augmented with the NN based adaptive element. The inset in Fig. 3(b) shows the true and the estimated range on a magnified scale in the time interval $40 \leq t \leq 120$.

Figs. 4 and 5 are generated for the two-angles case when the target maneuvers in a sinusoidal manner with a target acceleration of $0.3g$. Fig. 4 shows that the range estimate of the EKF is biased while the NN augmented EKF provides a nearly unbiased estimate. The inset in Fig. 4 shows the true and the estimated range on a magnified scale in the time interval $40 \leq t \leq 120$. Fig. 5 shows the target size estimate error for the EKF and the EKF+NN. The NN augmented EKF greatly reduces the bias in the error of the EKF estimate.

VII. Conclusions

In this paper we address the problem of augmenting an EKF with an adaptive element. A key application area is that of tracking a maneuvering target. The approach is applicable to uncertain multivariable nonlinear systems with uncertain parameters and unmodeled dynamics coupled to the process. The adaptive law is trained by an error signal that is generated from the residuals of the EKF. Boundedness of error signals is shown through Lyapunov's direct method and a backstepping argument. Simulations are used to show that augmenting the EKF with an NN helps in removing the bias in the range estimates and also improves the estimate of the target size. In the bearings-only case the NN was able to correct for the divergent estimates that were produced by the EKF.

Acknowledgements

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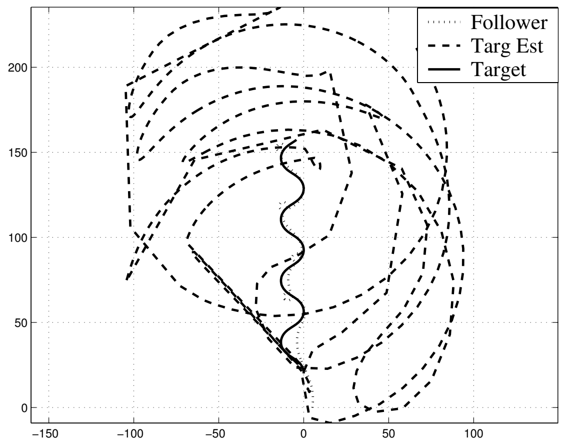
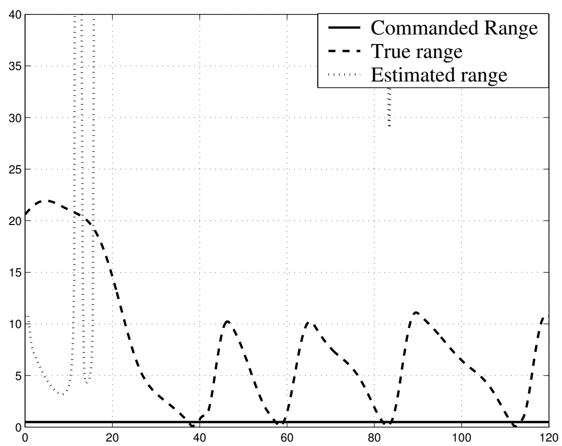
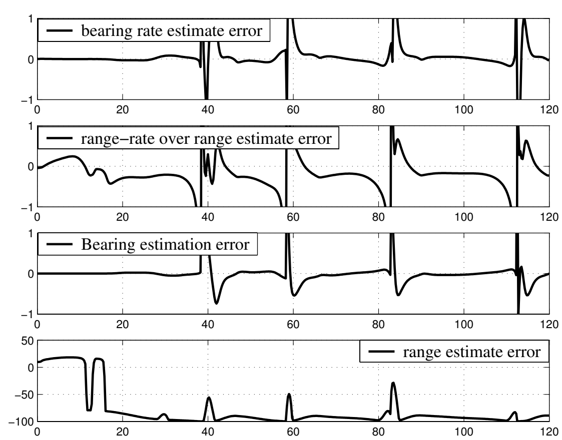


(a) Sinusoidal Target Maneuver

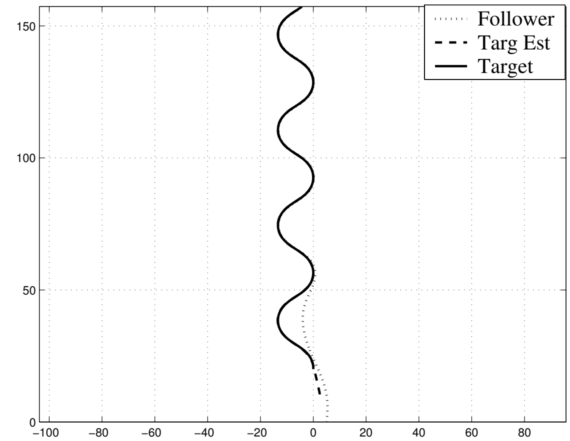


(b) True and Estimated LOS Range

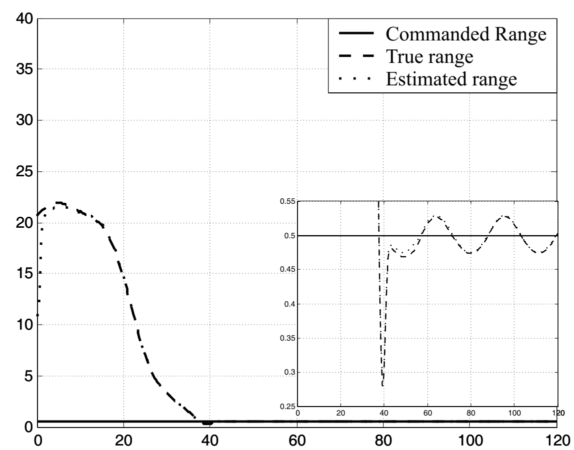


(c) State Estimation Errors

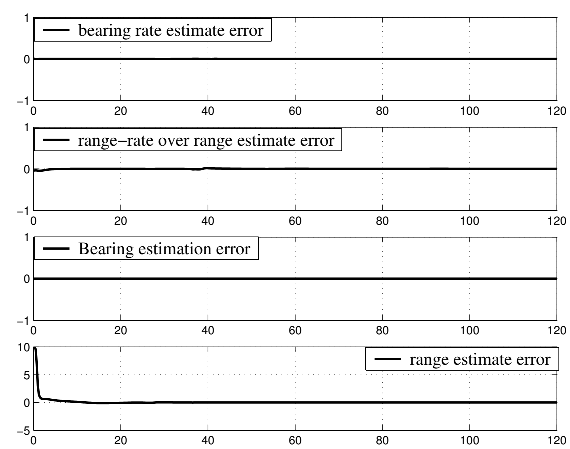
Fig. 2. EKF performance for the bearings-only case.



(a) Sinusoidal Target Maneuver



(b) True and Estimated LOS Range



(c) State Estimation Errors

Fig. 3. EKF+NN performance for the bearings-only case.

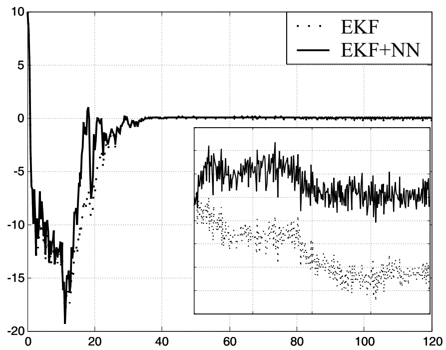


Fig. 4. Range estimate error - two angles measurement case

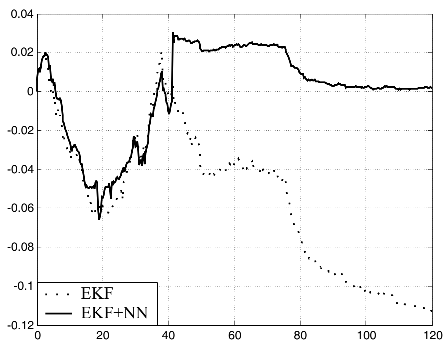


Fig. 5. Target size estimate error - two angles measurement case

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