ON ADAPTIVE SAMPLING FOR MULTISENSOR TRACKING OF A MANEUVERING TARGET USING IMM/PDA FILTERING

Sumedh P. Puranik and Jitendra K. Tugnait

Dept. of Electrical & Computer Engineering, Auburn University, Auburn, Alabama 36849, USA.
Tel: (334)844-1846, Fax: (334)844-1809, Email: puransp, tugnajk@eng.auburn.edu

ABSTRACT

We present a novel adaptive sampling scheme for tracking a highly maneuvering target. During the maneuvering mode of a target, estimation accuracy of the tracker is typically worse than that during the constant velocity (benign motion) mode. Hence it is desired to have higher sampling rate during the maneuvering mode and relatively lower sampling rate for the benign motion mode. In the proposed adaptive sampling policy, we predict an expected cost function for a candidate sampling interval selected from a predefined set of sampling intervals. We then choose the largest sampling interval for which an expected performance metric is satisfied. We use a random sampling approach coupled with switching multiple kinematic models for target motion, to generate future (pseudo-)measurements which allows computation of the relevant performance metric. The algorithm is illustrated via a simulation example involving tracking of a maneuvering target in presence of clutter and two sensors (radar and sonar). Multisensor tracking is achieved by a suboptimal filtering algorithm developed by the interacting multiple model (IMM) filtering approach combined with the probabilistic data association (PDA) technique and with the proposed adaptive sampling scheme.

I INTRODUCTION

Two modes of operation of surveillance radars that track multiple targets are scan-while-track (SWT) and track-while-scan (TWS). In SWT, the search for new targets is accomplished by scanning sections of the surveillance region between track dwells. In TWS, tracking is accomplished by processing the detections from scanning the surveillance region. Phased-array radars typically employ SWT, while rotating radars typically employ TWS [16]. Sensor management, or multisensor resource allocation, includes techniques for adjusting the operations of electronically scanned sensors for the presence of other sensors [16]. For instance, in [14], instead of setting a quasiperiodic data radar for surveillance tracking, the time at which the next measurement is required to maintain the track is computed after every measurement, and then a measurement is scheduled for the required time. Thus, the sample period between consecutive measurements is adapted giving rise to aperiodic data. The focus of this paper is on measurement scheduling (as in [14]) for phased array radars to maintain the tracks with largest possible sampling intervals.

We consider the problem of multisensor tracking of a highly maneuvering target in the presence of clutter. Since the target is maneuvering, multiple model filtering is necessary. The switching multiple model approach has been found very effective for modeling maneuvering targets [4]-[7],[11],[13]. Among all the multiple model filtering algorithms, interacting multiple model (IMM) estimator has yielded the best performance vs computational cost trade-off [4],[7],[16]. In this approach various modes of target motion are represented by distinct kinematic models. In a Bayesian framework, the target maneuvers are modeled by switching among a predefined set of models where switching is controlled by a Markov chain (transition probability matrix). Presence of clutter introduces additional uncertainty in terms of the measurement origin problem: i.e. how to associate the data available at the sensor(s) with target and clutter (false measurements). This problem of data association has already been considered and effectively solved in the Bayesian framework by using probabilistic data association (PDA) technique [1],[2],[5]. The IMM estimator combined with the PDA technique is an effective

way to track a maneuvering target in clutter. In this paper we consider this algorithm with a variable sampling policy.

Prior work on adaptive sampling rate for IMM/PDAF has been reported in [5],[14],[15]. In these works, the next required revisit (sampling) time is computed as the latest time at which the target can be expected to be in the predicted track gate, which is defined by the radar beamwidth, the length of the range window used in the track mode, and maximum acceleration of the target. These methods are based on the predicted values of standard deviations of the azimuthal and elevation angles, which are obtained from the predicted innovation covariance matrix. A threshold is determined as some fraction of the radar-antenna beamwidth. The obtained standard deviations of the azimuthal and elevation angles are compared with the threshold and a sampling interval satisfying this criterion is chosen as the next sampling instance. In essence, from a predetermined set of sampling intervals, one chooses the largest value for which the track is likely to be maintained. Note that the decision for selecting a sampling interval is based on the predicted states.

In this paper we propose a new sampling interval selection approach based on random sampling where we generate sampling interval-dependent pseudo-measurements. The basic idea is to evaluate some cost function for the candidate sampling interval and then to choose the largest sampling interval that satisfies a certain performance metric. The proposed random sampling approach coupled with switching multiple kinematic models for target motion, to generate future (pseudo-)measurements, allows computation of the relevant performance metric.

The paper is organized as follows. The basic problem formulation and design parameters for adaptive sampling are presented in Sec. II. The proposed adaptive sampling strategy, the algorithm to evaluate the cost function based on pseudo-measurements and a review of the IMM/PDAF algorithm are presented in Sec. III. A computer simulation example is presented in Sec. IV.

II PROBLEM FORMULATION

Assume that the dynamics of the target can be modeled as one of the n hypothesized models. The model set is denoted as $\mathcal{M}_n := \{1, 2, \cdots, n\}$. For the target, the event that model j is in effect during the sampling period $(t_{k-1}, t_k]$ will be denoted by $M_{t_k}^j$. For the j-th hypothesized model (mode), the state dynamics and measurements of the target are modeled as

$$x_{t_k} = F^j(T_k)x_{t_{k-1}} + G^j(T_k)v_{t_{k-1}}^j \qquad (1)$$

and

$$z_{t_k}^l = h^{j,l}(x_{t_k}) + w_{t_k}^{j,l} \quad \text{for } l = 1, ..., q, \qquad (2)$$

where x_{t_k} is the system state of the target at time t_k and has dimension n_x, z_{t_k} is the (true) measurement vector from sensor l at t_k and has dimension n_{zl}, $F^j(T_k)$ and $G^j(T_k)$ are the system matrices when model j is in effect over the sampling period $(t_{k-1}, t_k]$ such that $T_k = t_k - t_{k-1}$, and $h^{j,l}$ is the nonlinear transformation of x_{t_k} to $z_{t_k}^l$ ($l = 1, 2, ..., q$) for model j. A first-order linearized version of (2) is given by

$$z_{t_k}^l = H_{t_k}^{j,l} x_{t_k} + w_{t_k}^{j,l} \qquad (3)$$

where $H_{t_k}^{j,l}$ is the Jacobian matrix of $h^{j,l}$ evaluated at some value of the estimate of state x_{t_k} (see Sec. III.1). The process noise $v_{t_{k-1}}^j$ and the measurement noise $w_{t_k}^{j,l}$ are mutually uncorrelated

zero-mean white Gaussian processes with covariance matrices $Q_{t_{k-1}}^j$ and $R_{t_k}^{j,l}$, respectively.

Note that, in general, at any time t_k, some measurements may be due to clutter and some due to the target, i.e. there can be more than a single measurement at time t_k. The measurement set (not yet validated) generated at time t_k is denoted as

$$Z_{t_k}^l := \{z_{t_k}^{l(1)}, z_{t_k}^{l(2)}, \cdots, z_{t_k}^{l(m_l)}\} \tag{4}$$

where m_l is the number of measurements generated at time t_k. Variable $z_{t_k}^{l(i)}$ $(i = 1, \cdots, m_l)$ is the ith measurement within the set. The validated set of measurements from sensor l at time t_k will be denoted by $Y_{t_k}^l$, containing $\overline{m}_l$ $(\le m_l)$ measurement vectors. The cumulative set of validated measurements from sensor l up to time t_k is denoted as

$$\mathcal{Z}_1^{t_k(l)} = \{Y_1^l, Y_2^l, \cdots, Y_{t_k}^l\}. \tag{5}$$

The cumulative set of validated measurements from all sensors up to time t_k is denoted as

$$\mathcal{Z}_1^{t_k} = \{\mathcal{Z}_1^{t_k(1)}, \mathcal{Z}_1^{t_k(2)}, \cdots, \mathcal{Z}_1^{t_k(q)}\} \tag{6}$$

where q is the number of sensors.

At the initial time t_0, the initial conditions for the system state of the target under each model j are assumed to be Gaussian random variables with the known mean $\bar{x}_0^j$ and the known covariance P_0^j. The probability of target being in model j at t_0, $\mu_0^j = P\{M_0^j\}$, is also assumed to be known. The switching from model $M_{t_{k-1}}^i$ to model $M_{t_k}^j$ is governed by a finite-state stationary Markov chain with known transition probabilities $p_{ij} = P\{M_{t_k}^j | M_{t_{k-1}}^i\}$.

The transition probability matrix is a function of the sampling interval and has to be selected based on the sampling interval. Following [5], the elements of this matrix are designed based on the expected sojourn time in each mode of the hybrid system. The relation is [5]

$$\tau_i = \frac{T_k}{1 - p_{ii}} \tag{7}$$

where τ_i is the expected sojourn time of the ith mode, p_{ii} is the transition probability from the ith mode to the same mode, and T_k is the sampling interval. For each model p_{ii} is calculated as [5]

$$p_{ii} = \min\left\{u_i, \max\left(l_i, 1 - \frac{T}{\tau_i}\right)\right\} \tag{8}$$

where l_i (say 0.2) and u_i (say 0.8) are lower and upper limits respectively for the ith mode transition probability. The expected sojourn times for the models are assumed to be as in (7). The off-diagonal elements of the transition probability matrix depends on the switching characteristics among the various modes. In a typical example, in which the target motion is modeled by three modes, evaluation of off-diagonal elements is obtained as follows:

$$\begin{array}{ll} p_{12} = 0.2\,(1 - p_{11}), & p_{13} = 0.8\,(1 - p_{11}), \\ p_{21} = 0.2\,(1 - p_{22}), & p_{23} = 0.8\,(1 - p_{22}), \\ p_{31} = 0.3\,(1 - p_{33}), & p_{32} = 0.7\,(1 - p_{33}). \end{array} \tag{9}$$

Typically to handle all possible target motions, three modes have been used [5],[8]: constant velocity mode, white-noise acceleration mode, and Wiener acceleration mode. For each mode of operation, selection of process noise level is determined based on the maximum acceleration or acceleration increment corresponding to that mode. For a constant velocity mode, standard deviation of the process noise is small. For a white noise acceleration mode (second order dynamic model), the standard deviation of the process noise is of the order of maximum acceleration [5]

$$\sigma_{v_2} = \alpha_2\, a_{\max} \tag{10}$$

where σ_{v_2} is the process noise standard deviation, a_{max} is the maximum acceleration, and $0 < \alpha_2 \le 1$. For Wiener process acceleration mode (third order model), the standard deviation

of the process noise is a function of maximum acceleration increment and is given by [5]

$$\sigma_{v_3} = \min\{\alpha_3\, \dot{a}_{\max} T_k, a_{\max}\} \tag{11}$$

where σ_{v_3} is the process noise standard deviation, $\dot{a}_{max}$ is the maximum acceleration per unit time, T_k is the sampling interval and $0 < \alpha_3 \le 1$.

Thus, when we consider adaptive sampling for the IMM estimator, transition probability matrix and process noise level for acceleration modes vary as a function of sampling interval and have to be chosen accordingly.

With this problem formulation, we present the IMM/PDA algorithm and adaptive sampling scheme in the next section.

III   ALGORITHM

This section is organized as follows. In Sec. III.1 we review the IMM/PDA filter algorithm, in Sec. III.2 we present the pseudo-measurement generation and cost function evaluation algorithm, and in Sec. III.3 we present the proposed adaptive sampling algorithm.

III.1   IMM/PDA filter for multisensor tracking

As the IMM/PDA filter for multiple sensors with sequential updating (that is what we use in this paper) is given in detail in [5, Sec. 4.5] (where all the underlying assumptions and approximations may be found in further detail), we will only briefly outline the basic steps in "one cycle" (i.e. processing needed to update for a new set of measurements). First we list the main underlying assumptions.

(1) It is assumed that the target track has been initiated, and our objective is to maintain the track.

(2) Any measurement is either associated with a single target or caused by clutter.

(3) Clutter is modeled as independently and identically distributed (i.i.d.) with uniform spatial distribution over the validation region.

Assumed available: Given the state estimate $\widehat{x}_{t_{k-1}|t_{k-1}}^j :=$ $E\{x_{t_{k-1}}|M_{t_{k-1}}^j, \mathcal{Z}_1^{t_{k-1}}\}$, the associated covariance $P_{t_{k-1}|t_{k-1}}^j$ and the conditional mode probability $\mu_{t_{k-1}}^j = P[M_{t_{k-1}}^j | \mathcal{Z}_1^{t_{k-1}}]$ at time t_{k-1} for each mode $j \in \mathcal{M}_n$, the goal is to find the filtered state estimate for the target

$$\widehat{x}_{t_k|t_k} = E\{x_{t_k} | \mathcal{Z}_1^{t_k}\} \tag{12}$$

and the associated error covariance matrix

$$P_{t_k|t_k} = E\{[x_{t_k} - \widehat{x}_{t_k|t_k}][x_{t_k} - \widehat{x}_{t_k|t_k}]' | \mathcal{Z}_1^{t_k}\} \tag{13}$$

where x_{t_k}' denotes the transpose of x_{t_k}.

Step 1. Interaction $-$ mixing of the estimate from the previous time $(\forall j \in \mathcal{M}_n)$: predicted mode probability:

$$\mu_{t_k}^{j-} := P\{M_{t_k}^j | \mathcal{Z}_1^{t_{k-1}}\} = \sum_{i=1}^n p_{ij} \mu_{t_{k-1}}^i. \tag{14}$$

mixing probability:

$$\mu^{i|j} := P\{M_{t_{k-1}}^i | M_{t_k}^j, \mathcal{Z}_1^{t_{k-1}}\} = p_{ij} \mu_{t_{k-1}}^i / \mu_{t_k}^{j-}. \tag{15}$$

mixed estimate:

$$\widehat{x}_{t_{k-1}|t_{k-1}}^{0j} := E\{x_{t_{k-1}} | M_{t_k}^j, \mathcal{Z}_1^{t_{k-1}}\} = \sum_{i=1}^n \widehat{x}_{t_{k-1}|t_{k-1}}^i \mu^{i|j}. \tag{16}$$

covariance of the mixed estimate:

$$P_{t_{k-1}|t_{k-1}}^{0j} :=$$

$$E\{[x_{t_{k-1}} - \widehat{x}_{t_{k-1}|t_{k-1}}^{0j}][x_{t_{k-1}} - \widehat{x}_{t_{k-1}|t_{k-1}}^{0j}]' | M_{t_k}^j, \mathcal{Z}_1^{t_{k-1}}\}$$

$$= \sum_{i=1}^{n} \{ P^i_{t_{k-1}|t_{k-1}} + [\widehat{x}^i_{t_{k-1}|t_{k-1}} - \widehat{x}^{0j}_{t_{k-1}|t_{k-1}}]$$
$$\times [\widehat{x}^i_{t_{k-1}|t_{k-1}} - \widehat{x}^{0j}_{t_{k-1}|t_{k-1}}]' \} \mu^{i|j}. \tag{17}$$

Step 2. Predicted state $(\forall j \in \mathcal{M}_n)$: State prediction:

$$\widehat{x}^j_{t_k|t_{k-1}} := E\{x_{t_k}|M^j_{t_k}, \mathcal{Z}^{t_{k-1}}_1\} = F^j(T_k)\widehat{x}^{0j}_{t_{k-1}|t_{k-1}}. \tag{18}$$

State prediction error covariance:

$$P^j_{t_k|t_{k-1}} = E\{[x_{t_k} - \widehat{x}^j_{t_k|t_{k-1}}][x_{t_k} - \widehat{x}^j_{t_k|t_{k-1}}]'|M^j_{t_k}, \mathcal{Z}^{t_{k-1}}_1\}$$
$$= F^j(T_k)P^{0j}_{t_{k-1}|t_{k-1}}F^{j'}(T_k) + G^j(T_k)Q^j_{t_{k-1}}G^{j'}(T_k). \tag{19}$$

Using (2) and (18), the mode-conditioned predicted measurement is

$$\widehat{z}^{j,1}_{t_k} := h^{j,1}(\widehat{x}^j_{t_k|t_{k-1}}). \tag{20}$$

Using the linearized version (3), the covariance of the mode-conditioned residual $\nu^{j,1(i)}_{t_k} := z^{1(i)}_{t_k} - \widehat{z}^{j,1}_{t_k}$ is given by

$$S^{j,1}_{t_k} := E\{\nu^{j,1(i)}_{t_k} \nu^{j,1(i)'}_{t_k}|M^j_{t_k}, \mathcal{Z}^{t_{k-1}}_1\} = H^{j,1}_{t_k} P^j_{t_k|t_{k-1}} H^{j,1'}_{t_k} + R^{j,1}_{t_k} \tag{21}$$

where $H^{j,1}_{t_k}$ is the first order derivative (Jacobian matrix) of $h^{j,1}(.)$ at $\widehat{x}^{j(0)}_{t_k|t_{k-1}}$. Note that (21) assumes that $z^{1(i)}_{t_k}$ originates from the target.

Step 3. Measurement validation for sensor 1: The validation region is taken to be the same for all models, i.e., as the largest of them. Let $(|A| = \det(A))$

$$\overline{j} := \arg \left\{ \max_{j \in \mathcal{M}_n} |S^{j,1}_{t_k}| \right\}. \tag{22}$$

Then measurement $z^{1(i)}_{t_k}$ ($i = 1, 2, \cdots, m(t_k)$) is validated if and only if

$$[z^{1(i)}_{t_k} - \widehat{z}^{\overline{j},1}_{t_k}]'[S^{\overline{j},1}_{t_k}]^{-1}[z^{1(i)}_{t_k} - \widehat{z}^{\overline{j},1}_{t_k}] < \gamma \tag{23}$$

where γ is an appropriate threshold. The volume of the validation region with the threshold γ is

$$V^1_{t_k} := c_{n_{z1}} \gamma^{n_{z1}/2} |S^{\overline{j},1}_{t_k}|^{1/2} \tag{24}$$

where n_{z1} is the dimension of the measurement and $c_{n_{z1}}$ is the volume of the unit hypersphere of this dimension ($c_1 = 2, c_2 = \pi, c_3 = 4\pi/3$, etc.). Choice of γ is discussed in [5, Sec. 2.3.2].

Step 4. State estimation with validated measurements from sensor 1 $(\forall j \in \mathcal{M}_n)$: From among all the raw measurements at time t_k, i.e., $Z^1_{t_k} := \{z^{1(1)}_{t_k}, z^{1(2)}_{t_k}, \cdots, z^{1(m(t_k))}_{t_k}\}$, define the set of validated measurement for sensor 1 at time t_k as $Y^1_{t_k} := \{y^{1(1)}_{t_k}, y^{1(2)}_{t_k}, \cdots, y^{1(\overline{m}(t_k))}_{t_k}\}$ where $\overline{m}(t_k)$ is the total number of validated measurement at time t_k and $y^{1(i)}_{t_k} := z^{1(l_i)}_{t_k}$ where $1 \le l_1 < l_2 < \cdots < l_{\overline{m}(t_k)} \le m(t_k)$ when $\overline{m}(t_k) \ne 0$.

Define the events

$$\theta^i_{t_k} =$$
$$\begin{cases} y^{1(i)}_{t_k} \text{is the target originated measurement,} & i = 1, ..., \overline{m}(t_k) \\ \text{none of the measurements is target originated,} & i = 0. \end{cases} \tag{25}$$

Define the mode conditioned association event probabilities $\beta^{j,i,1}_{t_k} := P\{\theta^i_{t_k}|M^j_{t_k}, \mathcal{Z}^{t_{k-1}}_1, Y^1_{t_k}, \overline{m}(t_k)\}$ and mode conditioned innovations $\nu^{j,i,1}_{t_k} := y^{1(i)}_{t_k} - \widehat{z}^{j,1}_{t_k}$ for $i = 1, ..., \overline{m}(t_k)$. Using a diffused prior mode for clutter [1],[5], it turns out that

$$\beta^{j,i,1}_{t_k} = \begin{cases} \frac{1}{c'} \left\{ (P_G)^{-1} \mathcal{N} \left[\nu^{j,i,1}_{t_k}; 0, S^{j,1}_{t_k} \right] \right\} \\ \qquad \text{for } i = 1, ..., \overline{m}(t_k) \\ \frac{1}{c'} \left\{ \overline{m}(t_k)(1 - P_D P_G)(P_D P_G V^1_{t_k})^{-1} \right\} & i = 0, \end{cases} \tag{26}$$

where P_D is the detection probability of sensor 1, P_G is probability that target is in the validation region, and c' is a normalization constant such that $\sum_{i=0}^{\overline{m}(t_k)} \beta^{j,i,1}_{t_k} = 1$ and

$$\mathcal{N}(x; y, P) := |2\pi P|^{-1/2} \exp[-\frac{1}{2}(x - y)'P^{-1}(x - y)]. \tag{27}$$

One can evaluate the likelihood function for each mode j as [5]

$$\Lambda^{j,1}_{t_k} := p[Y^1_{t_k}|M^j_{t_k}, \mathcal{Z}^{t_{k-1}}_1]$$
$$= \left[V^1_{t_k}\right]^{-\overline{m}(t_k)} \gamma_0(\overline{m}(t_k)) + \left[V^1_{t_k}\right]^{-\overline{m}(t_k)+1}$$
$$\times \sum_{i=1}^{\overline{m}(t_k)} P_G^{-1} \mathcal{N} \left[\nu^{j,i,1}_{t_k}; 0, S^{j,1}_{t_k} \right] \gamma_i(\overline{m}(t_k)) \tag{28}$$

where

$$\gamma_i(\overline{m}(t_k)) := \begin{cases} \frac{1}{\overline{m}(t_k)} P_D P_G & i = 1, ..., \overline{m}(t_k) \\ 1 - P_D P_G & i = 0. \end{cases} \tag{29}$$

Using $\widehat{x}^j_{t_k|t_{k-1}}$ (from (18)), its covariance $P^j_{t_k|t_{k-1}}$ (from (19)), and $\nu^{j,1}_{t_k} = \sum_{i=1}^{m(t_k)} \beta^{j,i,1}_{t_k} \nu^{j,i,1}_{t_k}$ the combined mode-conditioned innovations, one computes the partial state update $\widehat{x}^{j,1}_{t_k|t_k}$ and its covariance $P^{j,1}_{t_k|t_k}$ according to the standard PDAF [2],[5]:

Kalman gain:

$$W^j_{t_k} = P^j_{t_k|t_{k-1}} H^{j,1'}_{t_k}[S^{j,1}_{t_k}]^{-1}. \tag{30}$$

State estimate update:

$$\widehat{x}^{j,1}_{t_k|t_k} := E\{x_{t_k}|M^j_{t_k}, \mathcal{Z}^{t_{k-1}}_1, Y^1_{t_k}\} = \widehat{x}^j_{t_k|t_{k-1}} + W^j_{t_k} \nu^{j,1}_{t_k} \tag{31}$$

Covariance of $\widehat{x}^{j,1}_{t_k|t_k}$:

$$P^{j,1}_{t_k|t_k} = P^j_{t_k|t_{k-1}} - (1 - \beta^{j,0,1}_{t_k})W^j_{t_k} S^{j,1}_{t_k} W^{j'}_{t_k}$$
$$+ W^j_{t_k}(\sum_{i=1}^{\overline{m}(t_k)} \beta^{j,i,1}_{t_k} \nu^{j,i,1}_{t_k} \nu^{j,i,1'}_{t_k} - \nu^{j,1}_{t_k} \nu^{j,1'}_{t_k})W^{j'}_{t_k}. \tag{32}$$

Step 5. Mode-conditioned predicted measurements for sensor 2 $(\forall j \in \mathcal{M}_n)$: This step is similar to part of Step 2. The predicted measurement is given by

$$\widehat{z}^{j,2}_{t_k} := h^{j,2}(\widehat{x}^{j,1}_{t_k|t_k}). \tag{33}$$

Using the linearized version (3), the covariance of the mode-conditioned residual $\nu^{j,2(i)}_{t_k} := z^{2(i)}_{t_k} - \widehat{z}^{j,2}_{t_k}$ is given by

$$S^{j,2}_{t_k} := E\{\nu^{j,2(i)}_{t_k} \nu^{j,2(i)'}_{t_k}|M^j_{t_k}, \mathcal{Z}^{t_{k-1}}_1, Y^1_{t_k}\} = H^{j,2}_{t_k} P^{j,1}_{t_k|t_k} H^{j,2'}_{t_k} + R^{j,2}_{t_k} \tag{34}$$

where $H^{j,2}_{t_k}$ is the first order derivative (Jacobian matrix) of $h^{j,2}(.)$ at $\widehat{x}^{j,1}_{t_k|t_k}$. Note that (34) assumes that $z^{2(i)}_{t_k}$ originates from the target .

Step 6. Measurement validation for sensor 2: This is similar to Step 3 where superscript 1 now becomes 2 and n_{z1} is replaced with n_{z2}.

Step 7. State update with validated measurements for sensor 2 $(\forall j \in \mathcal{M}_n)$: This step is similar to Step 4. Using the validated measurements obtained in Step 6 and starting from $\widehat{x}^{j,1}_{t_k|t_k}$ and $P^{j,1}_{t_k|t_k}$ one computes the final state update $\widehat{x}^j_{t_k|t_k}$ and associated covariance $P^j_{t_k|t_k}$. Likelihood function $\Lambda^{j,2}_{t_k}$ is

obtained in the similar manner of $\Lambda_{t_k}^{j,1}$. Finally we have the combined likelihood [5] $\Lambda_{t_k}^j = \Lambda_{t_k}^{j,1}\Lambda_{t_k}^{j,2}$.

Step 8. Update of mode probabilities ($\forall j \in \mathcal{M}_n$):

$$\mu_{t_k}^j := \frac{1}{c}\mu_{t_k}^{j-}\Lambda_{t_k}^{j,1}\Lambda_{t_k}^{j,2} \qquad (35)$$

where c is a normalization constant such that $\sum_{j=1}^n \mu_{t_k}^j = 1$.

Step 9. Combination of the mode-conditioned estimates: The final state estimate update at time t_k is given by

$$\widehat{x}_{t_k|t_k} = \sum_{j=1}^n \widehat{x}_{t_k|t_k}^j \mu_{t_k}^j \qquad (36)$$

and its covariance is given by

$$P_{t_k|t_k} = \sum_{j=1}^n \left\{ P_{t_k|t_k}^j + [\widehat{x}_{t_k|t_k}^j - \widehat{x}_{t_k|t_k}][\widehat{x}_{t_k|t_k}^j - \widehat{x}_{t_k|t_k}]' \right\} \mu_{t_k}^j. \qquad (37)$$

III.2 Adaptive Sampling: Pseudo-measurement Generation and Cost function Evaluation

State estimates obtained by the filter represents the best inference made about the true state of the target from the noisy measurements obtained by the sensors. The associated error covariance matrix represents the estimation accuracy and is a measure of how close the estimated value is to the true state of the target. This estimation accuracy increases as the sampling rate at which the observations are made is increased and decreases if the sampling rate is reduced. This is reflected in the error covariance matrix. We utilize this fact to select the future sampling intervals. Using this error covariance control approach, we can select the future sampling interval such that it will maintain the state covariance matrix below a certain desired level. Let $P_{desired}$ be the desired level for the state covariance submatrix for a subvector of target states; e.g. suppose we desire to control the covariance of the target position estimates only, rather than that of the entire state vector. Let Ax_{t_k} denote the desired $n_A \times 1$ subvector of the target state x_{t_k}; here A is a $n_A \times n_x$ matrix with entries of ones or zeros. Let $\hat{x}_{t_{k+1}|t_{k+1}}^{(\delta)}$ denote the state estimate and $P_{t_{k+1}|t_{k+1}}^{(\delta)}$ denote the associated error covariance matrix at time t_{k+1} for the selected sampling interval δ. Let the difference matrix be obtained as:

$$\triangle P = P_{desired} - AP_{t_{k+1}|t_{k+1}}^{(\delta)}A'. \qquad (38)$$

Then we may pick the maximum sampling interval δ such that the minimum eigenvalue of the difference matrix $\triangle P$ is positive. Since neither $x_{t_{k+1}}$ nor the measurement at time t_{k+1} (hence $P_{t_{k+1}|t_{k+1}}^{(\delta)}$) is available, the cost function cannot be evaluated. Therefore, a pseudo-state and pseudo-measurement generation scheme based on a random sampling approach (Monte Carlo methods) is presented next.

At the end of time t_k, the IMM/PDA filter updates the state using sequential update of actual measurements obtained from multiple sensors. Now the objective is to generate the pseudo-measurements at time $t_{k+1} = t_k + T_{k+1}$ (T_{k+1} is a variable sampling interval), when given the updated mode probabilities ($\mu_{t_k}^j$), the mode conditioned state estimates ($\hat{x}_{t_k|t_k}^j$) and the associated mode-conditioned state covariance matrices ($P_{t_k|t_k}^j$). The updated mode probabilities at time t_k give us the probabilistic information about the mode in which the target most likely was for the duration $(t_{k-1}, t_k]$. On the other hand, evolution of the mode from time t_k to t_{k+1} is governed by the transition probability matrix. Since we know these two probability distributions of modes and distribution of mode-conditioned state, pseudo-state and pseudo-measurement can be obtained by a random sampling approach as outlined next. The evolution of the

state, modeled as a Gauss-Markov process is governed by eqn.(1). Pseudo-measurement for each sensor is then obtained by nonlinear measurement model given by eqn.(2). More details are given below.

Step 1. Randomly sample a mode considered to be active during $(t_{k-1}, t_k]$ as $\tilde{j}^{(m)}$ (m denotes the mth random sample) from the distribution of modes governed by $\mu_{t_k}^j$.

Step 2. Having selected mode $\tilde{j}^{(m)}$ for time $(t_{k-1}, t_k]$, randomly select mode $\tilde{i}^{(m)}$ for time $(t_k, t_{k+1}]$ from the mode distribution governed by the transition probabilities $p_{\tilde{j}^{(m)}i}$.

Step 3. Randomly sample state x_{t_k} from the distribution $\mathcal{N}\left(\hat{x}_{t_k|t_k}^{\tilde{j}}, P_{t_k|t_k}^{\tilde{j}}\right)$ as $\tilde{x}_{t_k}^{(m)}$. Similarly, generate the process noise $v_{t_k}^{\tilde{i}(m)}$ from the distribution $\mathcal{N}\left(0, Q_{t_k}^{\tilde{i}}\right)$.

Step 4. Generate a sample of the pseudo-state $x_{t_{k+1}}$ denoted by:

$$\tilde{x}_{t_{k+1}}^{(m)} = F_{(t_{k+1}, t_k)}^{\tilde{i}(m)} \tilde{x}_{t_k}^{(m)} + G_{(t_{k+1}, t_k)}^{\tilde{i}(m)} v_{t_k}^{\tilde{i}(m)} \qquad (39)$$

Step 5. Generate a pseudo-measurement sample $\tilde{z}_{t_{k+1}}^{l(m)}$ for each sensor by the equation

$$\tilde{z}_{t_{k+1}}^{l(m)} = h^l(\tilde{x}_{t_{k+1}}^{(m)}) + \tilde{w}_{t_{k+1}}^{l(m)} \qquad (40)$$

where the measurement noise sample $w_{t_{k+1}}^{l(m)}$ is obtained from $\mathcal{N}\left(0, R_{t_{k+1}}^l\right)$ with $l = 1, 2, ...q$ sensor indices and $m = 1, 2, ...N$ sampling indices.

Step 6. Carry out state estimation at time t_{k+1} to obtain sequentially updated state $\hat{x}_{t_{k+1}|t_{k+1}}^{(\delta)(m)}$ and associated error covariance $\tilde{P}_{t_{k+1}|t_{k+1}}^{(\delta)(m)}$, by using the pseudo-measurements $\tilde{z}_{t_{k+1}}^{l(m)}$ and the IMM/PDA filter.

Step 7. Execute steps 1-6 for $m = 1, 2, \cdots, N$. The expected cost function for the selected sampling interval is then obtained by:

$$\widehat{\triangle P} = P_{desired} - (1/N) A \left(\sum_{m=1}^N \tilde{P}_{t_{k+1}|t_{k+1}}^{(\delta)(m)} \right) A'. \qquad (41)$$

Selection of Desired Covariance Matrix: A straightforward approach to select the desired covariance matrix $P_{desired}$ can be to use raw measurement errors of the state components. Since the purpose of the estimator is to provide better inferred value of the state than the directly measured value, this fact can be used to set the upper limit on the estimation error. It is desired to have the estimation error of the filter smaller than the raw measurement errors. Then the desired covariance matrix is a diagonal matrix with squared raw measurement errors being the variances of the state components.

III.3 Adaptive Sampling Policy:

As in [5],[14]-[16], we select sampling intervals T_{k+1} from among a predeteremined set, $\{\delta_i\}_1^L$ where $\delta_1 < \delta_2 < \cdots < \delta_L$. The sampling interval for time $t_{k+1} = t_k + T_{k+1}$ (where $T_{k+1} \in \{\delta_i\}_1^L$) is selected as follows:

(1) At the end of the IMM/PDA state update at time t_k, select the largest sampling interval $T_{k+1} = \delta_L$ for time t_{k+1}.

(2) Evaluate the cost function $\widehat{\triangle P}$ for this sampling interval by using the algorithm described in the Subsection III.2. The transition probability matrix and the process noise level are selected according to the sampling interval, as discussed in sec.II.

(3) If the minimum eigenvalue of $\widehat{\triangle P}$ is positive, then select the sampling interval δ_L for time t_{k+1}. Carry out state estimation with the IMM/PDA filter and actual measurements.

(4) If the minimum eigenvalue of $\widehat{\triangle P}$ is negative, then select the next maximum sampling interval and repeat the steps (1) to (3). If the minimum eigenvalue of $\widehat{\triangle P}$ is negative for all the sampling intervals (which might happen during onset and termination of the maneuvering mode), then select the minimum sampling interval.

IV SIMULATION EXAMPLE

We now consider a example of highly maneuvering target in presence of clutter. The set of candidate sampling rates was taken to be $\{\delta_1 = 1s, \delta_2 = 2s, \delta_3 = 3s\}$.

The True Trajectory: The target starts at location [21689 10840 40] in Cartesian coordinates in meters. The initial velocity is [-8.3 -399.9 0] in m/s. Target stays at constant altitude with a constant speed of 400 m/s. Its trajectory is a straight line with constant velocity between 0 and 20 sec., a coordinated turn of 0.15 rad/s with a constant acceleration of 60 m/s² between 20 and 35 s, a straight line with a constant velocity between 35 and 55 s, a coordinated turn of 0.1 rad/s with a constant acceleration of 40 m/s² between 55 and 70 s, and finally a straight line with a constant velocity between 70 and 90 s.

The Target Motion Models: In each mode, target dynamics are modeled in Cartesian coordinates with state of the target being position, velocity and acceleration in each of the three Cartesian coordinates (x, y and z). Thus x_{t_k} is of dimension 9 (n_x=9). Three models are considered and the system matrices F and G are defined as

$$F = \begin{bmatrix} F_b & 0 & 0 \\ 0 & F_b & 0 \\ 0 & 0 & F_b \end{bmatrix} \quad \text{and} \quad G = \begin{bmatrix} G_b & 0 & 0 \\ 0 & G_b & 0 \\ 0 & 0 & G_b \end{bmatrix}.$$

- Model 1: nearly constant velocity model with zero mean perturbation in acceleration.

$$F_b^1 = \begin{bmatrix} 1 & T_k & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad G_b^1 = \begin{bmatrix} \frac{T_k^2}{2} \\ T_k \\ 0 \end{bmatrix}$$

where T_k is the sampling period. The standard deviation of the process noise of M^1 is 5m/s².

- Model 2: Wiener process acceleration (nearly constant acceleration motion)

$$F_b^2 = \begin{bmatrix} 1 & T_k & \frac{T_k^2}{2} \\ 0 & 1 & T_k \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad G_b^2 = \begin{bmatrix} \frac{T_k^2}{2} \\ T_k \\ 1 \end{bmatrix}.$$

The standard deviation of the process noise of M^2 is 7.5m/s².

- Model 3: Wiener process acceleration (model with large acceleration increments, for the onset and termination of maneuvers), with $F_b^3 = F_b^2$ and $G_b^3 = G_b^2$. The standard deviation of the process noise of M^3 is selected as a function of the sampling interval discussed in Sec. II and is set to $min\{30T_k, 70\}$m/s² with maximum acceleration being set to 70m/s².

The set of sampling interval used is $T_k = \{1, 2, 3\}$ s and the mean sojourn times $\{\tau_i\}_1^3$ assumed for three modes are 15,4 and 2 sec. respectively. The initial model probabilities for the target are: $\mu_0^1 = 0.8$ and $\mu_0^2 = 0.1$ and $\mu_0^3 = 0.1$. The mode switching probability matrix for the target is obtained as per the discussion in Sec.II and is given by

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.04 & 0.16 \\ 0.05 & 0.75 & 0.2 \\ 0.15 & 0.35 & 0.5 \end{bmatrix} \quad \text{:for } T_k\text{=1s}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.04 & 0.16 \\ 0.1 & 0.5 & 0.4 \\ 0.24 & 0.56 & 0.2 \end{bmatrix} \quad \text{:for } T_k\text{=2s}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.04 & 0.16 \\ 0.15 & 0.25 & 0.6 \\ 0.24 & 0.56 & 0.2 \end{bmatrix} \quad \text{:for } T_k\text{=3s}$$

The Sensors: Two sensors, (radar and infrared) are used to obtain the measurements. The measurements are range and azimuth from radar. Azimuth and elevation angles from the infrared. The range, azimuth and elevation transformations, respectively, are given by

$$r = (x^2 + y^2 + z^2)^{1/2}, \ a = \tan^{-1}(y/x), \ e = \tan^{-1}[z/(x^2 + y^2)^{1/2}].$$

The measurement noise $w_k^{j,l}$ for sensor $l(l = 1, 2)$ is assumed to be zero-mean white Gaussian with known covariance matrices $R^1 = $ diag[400m², 49mrad²] and $R^2 = $ diag[4mrad², 4mrad²]. The sensors are assumed to be located at the origin of the coordinate system. It was assumed that the probability of detection $P_D = 1$ for both sensors.

The Clutter: For generating false measurements in simulations, the clutter was assumed to be Poisson distributed with expected number of $\lambda_1 = 5 \times 10^{-6}$/m mrad for sensor 1 and $\lambda_2 = 3.5 \times 10^{-4}$/mrad² for sensor 2. These statistics were used for generating the clutter in all simulations. However, a nonparametric clutter model was used for implementing all the algorithms for target tracking.

Other Parameters: The gates for setting up the validation regions for the sensor were based on the threshold $\gamma = 16$. With the measurement vector of dimension 2, this leads to a gate probability $P_G = 0.9997$ (see p. 96 of [5]). The desired state covariance matrix used in the proposed adaptive sampling scheme, was set to $P_{desired} = 10^4 I_{3\times 3}$ where $I_{3\times 3}$ is a 3×3 identity matrix for x, y and z position components of the state vector. That is, while evaluating the cost function, we consider only the covariance matrix of the position components of the state vector. We allow the maximum of 100m (selected based on raw measurement error) mean square error (MSE) in position in each direction (x, y and z).

Simulation Results: The results were obtained from 200 Monte Carlo runs for actual state estimation (based on actual data) and 100 Monte Carlo runs for random sampling ($N = 100$). During the initial phase of tracking (for first 5 sec period) sampling rate of 1 sec was used and then adaptive sampling scheme was applied. Fig. 1 shows the true trajectory of the target. Figs. 2 and 3 show the RMSE (root mean-square error) for the filtered position estimate for the target as a function of time with and without adaptive sampling rate. It is seen from Fig. 4 that during non-maneuvering mode the average sampling rate chosen by the adaptive sampling scheme is higher (close to 2sec) whereas during maneuvering mode it is close to minimum sampling rate (1sec). The average sampling interval for tracking with adaptive sampling was found to be 1.4364 s, which is 43.64 percent higher than the minimum sampling rate. Tracking with fixed 1 sec sampling rate has 0/200 track loss and with the adaptive sampling track losses were found to be 2/200. For 2 sec sampling rate, track losses were found to be 12/200 and for 3 sec sampling rate it were 64/200.

V CONCLUSIONS

We proposed a new adaptive sampling scheme for tracking a highly maneuvering target using IMM/PDA filter. In the proposed scheme, we first predict the cost function for the candidate sampling interval and then choose the largest sampling interval that satisfies a certain performance metric. We used a random sampling approach coupled with switching multiple kinematic models for target motion, to generate future (pseudo-)measurements which allows computation of the relevant performance metric. The proposed scheme is applicable to the multisensor tracking scenario. The multisensor IMM/PDAF algorithm combined with the proposed adaptive sampling scheme was illustrated via a simulation example involving tracking of a maneuvering target in presence of clutter and two sensors (radar and infrared).

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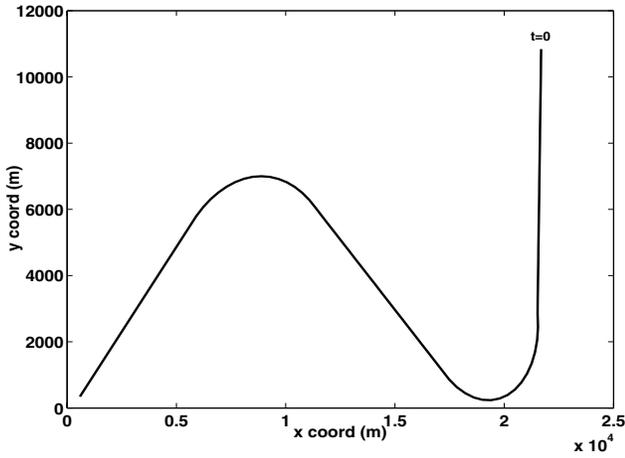
Figure 1. True target trajectory: position in xy plane.
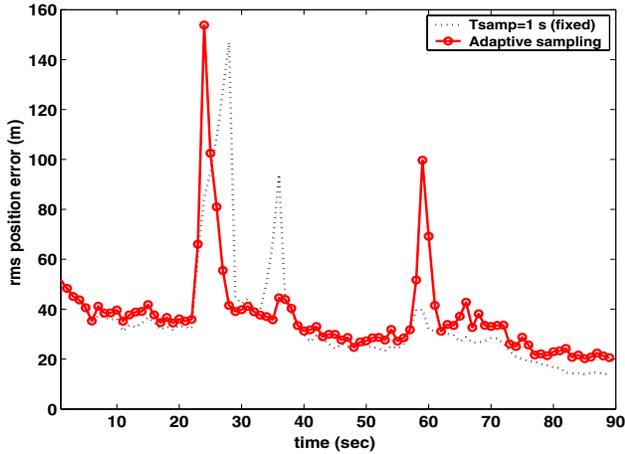
Figure 2. Root mean-square error (RMSE) in position for IMM/PDAF using 1 sec. and adaptive sampling rates.
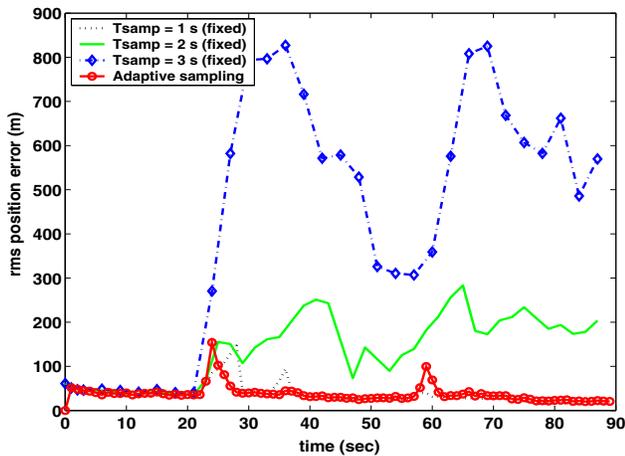
Figure 3. Root mean-square error (RMSE) in position for IMM/PDAF using 1, 2 and 3 sec. (fixed), and adaptive sampling rates.
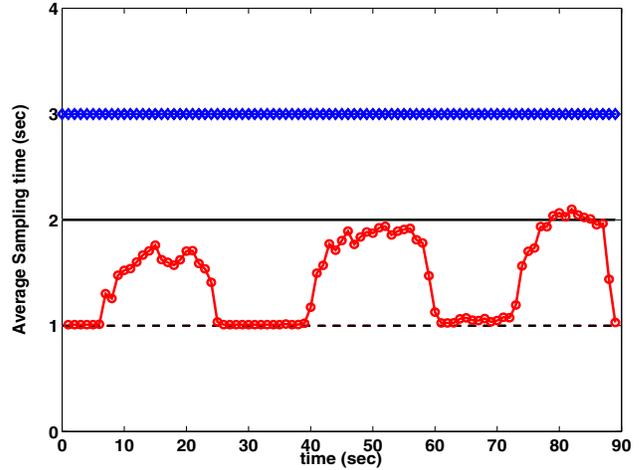
Figure 4. Average sampling rate selection for fixed sampling rates 1, 2 and 3 sec. vs. adaptive sampling rate. (Averaged over 200 runs)

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