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TRACKING OF MULTIPLE MANEUVERING TARGETS IN CLUTTER WITH POSSIBLY UNRESOLVED MEASUREMENTS USING IMM AND JPDAM COUPLED FILTERING

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ABSTRACT

We present a (suboptimal) filtering algorithm for tracking multiple highly maneuvering targets in a cluttered environment using multiple sensors. We concentrate on the case of two targets which may temporarily move in close formation, giving rise to a single detection due to the resolution limitations of the sensor. The proposed filtering algorithm is developed by applying the basic interacting multiple model (IMM) approach and the joint probabilistic data association with merged measurements (JPDAM) technique and coupled target state estimation to a Markovian switching system. The algorithm is illustrated via a simulation example.

1. INTRODUCTION

When two targets are "closely" spaced, they may give rise to a single detection due to the resolution limitations of the sensor. For instance, in radar ranging, returns from multiple targets could fall in the same range cell, resulting in one unresolved detection only [7],[8]. Standard tracking algorithms that ignore such a phenomenon, can lead to poor performance in multiple target tracking [7],[8]. Despite its importance, prior work on tracking with unresolved measurements in general and modeling of resolution capability of a sensor in particular, is sparse. Prior work includes [7] and [8] and references therein. In [7] the resolution phenomena related to tracking have been treated on the basis of a grid of resolution cells "frozen" in space. In [8] the resolution capability of a sensor is described in terms of a conditional probability of the event that two targets are unresolved, conditioned on the relative distance between the two targets in terms of the measured variables (range, azimuth etc.). A simple Gaussian shape is assumed which captures the sensor behavior in a mathematically tractable way. While [7] considers JPDA for data association, [8] exploits multiple hypothesis tracking (MHT).

In this paper we propose to use sensor resolution modeling of [8] in conjunction with JPDA coupled filtering and interacting multiple model (IMM) approach (see e.g. [5] for tracking with resolved measurements). In general, IMM/PDA filter is superior to IMM/MHT filter when the associated computational cost and performance are considered. Therefore, our emphasis will be on IMM/JPDA techniques. Neither [7] nor [8] consider multiple switching kinematic models for maneuvering targets; rather they are limited to single (nonswitching) kinematic model per target.

2. PROBLEM FORMULATION

Assume that there are total two targets with the target set denoted as $\mathcal{T}_2$. Assume that the target dynamics can be modeled by one of n hypothesized models. The model set is denoted as $\mathcal{M}_n := \{1, \cdots, n\}$ and there are total q sensors. For target r ($r \in \mathcal{T}_2$), the event that model j is in effect during the sampling period $(t_{k-1}, t_k]$ will be denoted by $M_k^j(r)$.

2.1. Target Dynamics

For the j-th hypothesized model (mode), the state dynamics of target r ($r \in \mathcal{T}_2$), are modeled as

$$x_k(r) = F_{k-1}^j(r)x_{k-1}(r) + G_{k-1}^j(r)v_{k-1}^j(r) \tag{1}$$

where $x_k(r)$ is the system state of target r at t_k and of dimension n_x (assuming all targets share a common state space), $F_{k-1}^j(r)$ and $G_{k-1}^j(r)$ are the system matrices when model j is in effect over the sampling period $(t_{k-1}, t_k]$ for target r. The process noise $v_{k-1}^j(r)$ is a zero-mean white Gaussian process with covariance matrix Q_{k-1}^j (same for all targets). At the initial time t_0, the initial conditions for the system state of target r under each model j are assumed to be Gaussian random variables with the known mean $\bar{x}_0^j(r)$ and the known covariance $P_0^j(r)$. The probability of model j at t_0, $\mu_0^j(r) = P[M_0^j(r)]$, is also assumed to be known. The switching from model $M_{k-1}^i(r)$ to model $M_k^j(r)$ is governed by a finite-state stationary Markov chain (same for all targets) with known transition probabilities $p_{ij} = P[M_k^j(r)|M_{k-1}^i(r)]$. Henceforth, time t_k will be denoted by k.

In coupled state estimation the states of two targets are estimated jointly [4]. To this end define the "global coupled" state

$$x_k := \mathrm{col}\{x_k(1), x_k(2)\} \tag{2}$$

and $J := \mathrm{col}\{j_1, j_2\}$ where $j_r \in \mathcal{M}_n$ is model j for target r, $F_k^J := \mathrm{block}-\mathrm{diag}\{F_k^{j_1}(1), F_k^{j_2}(2)\}$, $G_k^J := \mathrm{block}-\mathrm{diag}\{G_k^{j_1}(1), G_k^{j_2}(2)\}$, $v_k^J := \mathrm{col}\{v_k^{j_1}(1), v_k^{j_2}(2)\}$. Then we have the state equation for two targets as

$$x_k = F_{k-1}^J x_{k-1} + G_{k-1}^J v_{k-1}^J \tag{3}$$

where $E\{v_k^J v_k^{J\prime}\} = Q_k^J := \mathrm{block}-\mathrm{diag}\{Q_k^{j_1}, Q_k^{j_2}\}$. Define the global mode

$$M_k^J := \{M_k^{j_1}(1), M_k^{j_2}(2)\}. \tag{4}$$

The two targets are assumed to evolve independently of each other. Therefore, the transition probability for the global modes are given by

$$p_{IJ} := P\{M_k^{j_1}(1), M_k^{j_2}(2)|M_{k-1}^{i_1}(1), M_{k-1}^{i_2}(2)\} = \prod_{r=1}^{2} p_{i_r j_r}. \tag{5}$$

Similarly we have $\mu_0^J := P\{M_0^{j_1}(1), M_0^{j_2}(2)\} = \prod_{r=1}^{2} \mu_0^{j_r}(r)$.

2.2. Measurements

For the j-th hypothesized model (mode), measurements of target r ($r \in \mathcal{T}_2$), are modeled, when resolved, as

$$z_k^l(r) = h^l(x_k(r)) + w_k^l(r) \quad \text{for} \quad l = 1, \cdots, q \tag{6}$$

where $z_k^l(r)$ is the (true) measurement vector (i.e., due to target r) from sensor l at t_k and of dimension n_{zl}, and h^l is the nonlinear transformation of $x_k(r)$ to $z_k^l(r)$ ($l = 1, \cdots, q$). The measurement noise $w_k^l(r)$ is a zero-mean white Gaussian process with covariance matrix $\tilde{R}_k^l$ (same for all targets) and is mutually uncorrelated with the process noise $v_{k-1}^j(r)$. Similarly define the global measurement vector at sensor l as

$$z_k^l := \mathrm{col}\{z_k^l(1), z_k^l(2)\} \tag{7}$$

and related vectors $h^l(x_k) = \mathrm{col}\{h^l(x_k(1)), h^l(x_k(2))\}$, $w^l = \mathrm{col}\{w^l(1), w^l(2)\}$ where

$$E\{w_k^l w_k^{l\prime}\} = R_k^l := \mathrm{block}-\mathrm{diag}\{\tilde{R}_k^l, \tilde{R}_k^l\}. \tag{8}$$

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Then the measurement equation for two targets at sensor l (assuming no clutter and perfect detections) is given by

$$z_k^l = h^l(x_k) + w_k^l \qquad \text{for} \quad l = 1, \cdots, q. \tag{9}$$

Note that, in general, at any time k, some measurements may be due to clutter and some due to the target(s). The measurement set (not yet validated) generated by sensor l at time k is denoted as $Z_k^l := \{z_k^{l(1)}, z_k^{l(2)}, \cdots, z_k^{l(m_l)}\}$ where m_l is the number of measurements generated by sensor l at time k. Variable $z_k^{l(i)}$ ($i = 1, \cdots, m_l$) is the ith measurement within the set. The validated set of measurements of sensor l at time k will be denoted by $Y_k^l := \{y_k^{l(1)}, y_k^{l(2)}, \cdots, y_k^{l(\bar{m}_l)}\}$ where $\bar{m}_l$ is total number of validated measurement for sensor l at time k. And $y_k^{l(i)} := z_k^{l(l_i)}$ where $1 \leq l_1 < l_2 < \cdots < l_{\bar{m}_l} \leq m_l$ when $\bar{m}_l \neq 0$. The cumulative set of validated measurements from sensor l up to time k is denoted as $Z^{k(l)} := \{Y_1^l, Y_2^l, \cdots, Y_k^l\}$. The cumulative set of validated measurements from all sensors up to time k is denoted as $Z^k := \{Z^{k(1)}, Z^{k(2)}, \cdots, Z^{k(q)}\}$ where q is the number of sensors.

Our goal is to find the state estimate $\hat{x}_{k|k} := E\{x_k|Z^k\}$ and its error covariance matrix $P_{k|k} := E\{[x_k - \hat{x}_{k|k}][x_k - \hat{x}_{k|k}]'|Z^k\}$ where x_k' denotes the transpose of x_k.

3. MODELING FOR THE MERGED MEASUREMENTS

3.1. Modeling Assumptions

Prior work on modeling of merged measurements includes Trunk [2, 3] and Chang and Bar-Shalom [7]. Here we will follow Koch [8]. Based upon these earlier works, we assume the following: Only two targets were considered. The merged measurements arise when two targets are so close that the noisy measurements (rather than the predicted measurement) fall in the same resolution cell due to a lack of resolution of the sensor. The relative strength β_k (see (11)) of the signal is assumed to be Gaussian with mean 0.5 and a small variance ($0 < \beta_k < 1$); however, in our simulation examples, we set β_k=0.5 for all k. False detections (clutter) not related to the targets are uniformly distributed in the surveillance region and their number is assumed to be Poisson distributed. The detection probabilities for resolved targets P_D and unresolved targets P_D^a are the same.

3.2. Measurement Model

Due to a lack of resolution at the sensor, a detection may correspond to both targets. Let $s_k^l(i)$ denote the signal power of target i at sensor l and time k. Following [7], for unresolved targets at time k, the measurement equation for the two targets at sensor l (assuming no clutter and perfect detections with merged measurements of targets r_1 and r_2, ($r_1, r_2 \in \mathcal{T}_2$)), are modeled as ($l = 1, \cdots, q$)

$$z_k^{l,a} = \beta_k z_k^l(1) + (1-\beta_k) z_k^l(2) = h^{l,a}(x_k) + w_k^{l,a} \tag{10}$$

$$\text{where } \beta_k = s_k^l(1)/[s_k^l(1) + s_k^l(2)], \tag{11}$$

$$h^{l,a}(x_k) = \beta_k h^l(x_k(1)) + (1-\beta_k) h^l(x_k(2)), \tag{12}$$

$$w_k^{l,a} = \beta_k w_k^l(1) + (1-\beta_k) w_k^l(2), \tag{13}$$

$$E\{w_k^{l,a} w_k^{l,a\prime}\} = R_k^{l,a}. \tag{14}$$

3.3. Sensor Resolution Model

Let $\mathcal{A}$ denote the event that both targets are unresolved. Following [8], for sensor l, we introduce a related conditional probability P_a^l as

$$P_a^l := P(\mathcal{A}|x_k) = \left|2\pi R^{l,d}\right|^{1/2} \mathcal{N}\left(0; h^{l,d}(x_k), R^{l,d}\right) \tag{15}$$

where

$$h^{l,d}(x_k) = h^l(x_k(1)) - h^l(x_k(2)), \tag{16}$$

$$\mathcal{N}(x; y, P) := |2\pi P|^{-1/2} \exp\left[-\frac{1}{2}(x-y)' P^{-1} (x-y)\right]. \tag{17}$$

The positive definite $n_{zl} \times n_{zl}$ matrix $R^{l,d}$ is determined by corresponding sensor resolution (measurement accuracy). Later we illustrate our model by a 2-D radar measuring range and azimuth as well as an infrared sensor measuring azimuth and elevation angle. In this case, the range resolution is essentially determined by the length of the emitted pulse and the angular resolution is limited by the beam width. Following [8] we assume that the different measurement resolutions are independent of each other so that $R^{l,d}$ is diagonal.

4. IMM/JPDAM COUPLED FILTERING ALGORITHM

We now modify the IMM/JPDA coupled filtering algorithm of [5] to apply to the coupled system (1)-(9); it will be called IMM/JPDAMCF (CF stands for coupled filter). We will only briefly outline the basic steps in "one cycle" (i.e., processing needed to update for a new set of measurements) of the IMM/JPDAM coupled filter.

Assumed available: Given the state estimate $\hat{x}_{k-1|k-1}^J := E\{x_{k-1}|M_{k-1}^J, Z^{k-1}\}$, the associated covariance $P_{k-1|k-1}^J$, and the conditional mode probability $\mu_{k-1}^J := P[M_{k-1}^J|Z^{k-1}]$ at time $k-1$ for each mode $J \in \bar{\mathcal{M}}_n := \mathcal{M}_n \times \mathcal{M}_n$.

Step 1. Interaction - mixing of the estimate from the previous time ($\forall J \in \bar{\mathcal{M}}_n$): predicted mode probability:

$$\mu_k^{J-} := P[M_k^J|Z^{k-1}] = \sum_I p_{IJ} \mu_{k-1}^I. \tag{18}$$

mixing probability:

$$\mu^{I|J} := P[M_{k-1}^I|M_k^J, Z^{k-1}] = p_{IJ}\mu_{k-1}^I/\mu_k^{J-}. \tag{19}$$

mixed estimate:

$$\hat{x}_{k-1|k-1}^{0J} := E\{x_{k-1}|M_k^J, Z^{k-1}\} = \sum_I \hat{x}_{k-1|k-1}^I \mu^{I|J}. \tag{20}$$

covariance of the mixed estimate:

$$P_{k-1|k-1}^{0J} := E\{[x_{k-1} - \hat{x}_{k-1|k-1}^{0J}][x_{k-1} - \hat{x}_{k-1|k-1}^{0J}]'|M_k^J, Z^{k-1}\}$$

$$= \sum_I \{P_{k-1|k-1}^I + [\hat{x}_{k-1|k-1}^I - \hat{x}_{k-1|k-1}^{0J}][\hat{x}_{k-1|k-1}^I - \hat{x}_{k-1|k-1}^{0J}]'\}\mu^{I|J}. \tag{21}$$

Step 2. Predicted state and measurements for sensor 1 ($\forall J \in \bar{\mathcal{M}}_n$): state prediction:

$$\hat{x}_{k|k-1}^J := E\{x_k|M_k^J, Z^{k-1}\} = F_{k-1}^J \hat{x}_{k-1|k-1}^{0J}. \tag{22}$$

state prediction error covariance:

$$\begin{aligned} P_{k|k-1}^J &:= E\{[x_k - \hat{x}_{k|k-1}^J][x_k - \hat{x}_{k|k-1}^J]'|M_k^J, Z^{k-1}\} \\ &= F_{k-1}^J P_{k-1|k-1}^{0J} F_{k-1}^{J\prime} + G_{k-1}^J Q_{k-1}^J G_{k-1}^{J\prime}. \end{aligned} \tag{23}$$

For two resolved targets: Using (6) and (22), the global mode-conditioned predicted measurement for sensor 1 is

$$\hat{z}_k^{J,1} := h^1(\hat{x}_{k|k-1}^J). \tag{24}$$

Using the linearized (9), the covariance of the mode-conditioned residual $\nu_k^{J,1(I)} := z_k^{1(I)} - \hat{z}_k^{J,1}$ where $z_k^{1(I)} := \mathrm{col}\{z_k^{1(i_1)}, z_k^{1(i_2)}\}$, is given by

$$S_k^{J,1} := E\{\nu_k^{J,1(I)} \nu_k^{J,1(I)\prime}|M_k^J, Z^{k-1}\} = H_k^{J,1} P_{k|k-1}^J H_k^{J,1\prime} + R_k^{J,1}, \tag{25}$$

$$\begin{aligned} S_k^{j,1} &:= E\{\nu_k^{j,1(I)} \nu_k^{j,1(I)\prime}|M_k^J, Z^{k-1}\} \\ &= H_k^{j,1} P_{k|k-1}^J H_k^{j,1\prime} + R_k^{j,1}, \quad (j = j_1 \text{ or } j_2) \end{aligned} \tag{26}$$

where $H_k^{J,1} :=$ block $-$ diag$\{H_k^{j_1,1}, H_k^{j_2,1}\}$ is the Jacobian matrix of $h^{J,1}(.)$ evaluated at the state prediction $\hat{x}^J_{k|k-1}$.
For unresolved targets: Using (10) and (22), the global mode-conditioned predicted measurement for sensor 1 is

$$\hat{z}_k^{J,1,a} := h^{1,a}(\hat{x}^J_{k|k-1}). \tag{27}$$

Introduce a pseudo-measurement (as in [8]; $l = 1, 2$ and I is an identity matrix)

$$z_k^{l,d} := z_k^l(1) - z_k^l(2) \text{ and define } \hat{z}_k^{J,1,d} := [I \ \ -I]\hat{z}_k^{J,1}. \tag{28}$$

Using linearization around $\hat{x}^J_{k|k-1}$, the covariance of the mode-conditioned residual ($z_k^{l,d}$ set to zero [8])

$$\nu_k^{J,1,a} := \begin{bmatrix} z_k^{1,a} \\ 0 \end{bmatrix} - \begin{bmatrix} \hat{z}_k^{J,1,a} \\ \hat{z}_k^{J,1,d} \end{bmatrix} \tag{29}$$

is given by

$$S_k^{J,1,a} := E\left\{\nu_k^{J,1,a}\nu_k^{J,1,a'}|M_k^J, Z^{k-1}\right\}$$

$$= \begin{bmatrix} H_k^{J,1,a} \\ H_k^{J,1,d} \end{bmatrix} P^J_{k|k-1} \begin{bmatrix} H_k^{J,1,a'} & H_k^{J,1,d'} \end{bmatrix} + \begin{bmatrix} R_k^{1,a} & 0 \\ 0 & R^{1,d} \end{bmatrix} \tag{30}$$

where $H_k^{J,1,a} = [\beta_k H^{j_1,1} \ \ (1-\beta_k)H^{j_2,1}]$ and $H_k^{J,1,d} = [H_k^{j_1,1} \ \ -H_k^{j_2,1}]$. In (29) the measurement residual $\nu_k^{J,1,a}$ is the formulation of both, the mean position and the distance of the targets. Therefore, an assumed resolution conflict results in a fictitious measurement of $z_k^{1,d}$ with value 0 and error covariance $R^{1,d}$ [8].

Step 3. Measurement validation for sensor 1 ($\forall J \in \bar{\mathcal{M}}_n$): We first perform measurement validation for each target r ($r \in \mathcal{T}_2$) separately. For target r, the validation region is taken to be the same for all models, i.e., as the largest of them.
For two resolved targets: Just follow Step 3.3 in [5].
For unresolved targets: For unresolved targets at time k, $S_k^{J,1,a}$ is a $n_{zl} \times n_{zl}$ matrix and is based on the information relevant to the merged targets. Let $\hat{z}_k^{J,1,a}$ denote the $n_{zl} \times 1$ column matrix. That is, $\hat{z}_k^{J,1,a}$ is the mode-conditioned predicted measurement of the merged targets for sensor 1. Let $\bar{J}_a := \arg\{\max_{J \in \bar{\mathcal{M}}_n} \left|S_k^{J,1,a}\right|\}$.
Then measurement for unresolved targets $z_k^{1(i)}$ (i=1,2,$\cdots$,m_l) is validated if and only if

$$[z_k^{1(i)} - \hat{z}_k^{\bar{J}_a,1,a}]'[S_k^{\bar{J}_a,1,a}]^{-1}[z_k^{1(i)} - \hat{z}_k^{\bar{J}_a,1,a}] < \gamma \tag{31}$$

where γ is an appropriate threshold. The volume of the validation region with the threshold γ is $V_k^1(a) := c_{n_{zl}}\gamma^{n_{zl}/2}|S_k^{\bar{J}_a,1,a}|^{1/2}$. The volume of validation region for the whole target set is $V_k^1 = V_k^1(a)$.

Step 4. State estimation with validated measurement from sensor 1 ($\forall J \in \bar{\mathcal{M}}_n$): From among all the raw measurements from sensor 1 at time k, i.e., $Z_k^1 := \{z_k^{1(1)}, z_k^{1(2)}, \cdots, z_k^{1(m_1)}\}$, define the set of validated measurement for sensor 1 at time k as $Y_k^1 := \{y_k^{1(1)}, y_k^{1(2)}, \cdots, y_k^{1(\bar{m}_1)}\}$ where $\bar{m}_1$ is the total number of validated measurement for sensor 1 at time k and

$$y_k^{1(i)} := z_k^{1(l_i)} \tag{32}$$

with $1 \le l_1 < l_2 < \cdots < l_{\bar{m}_1} \le m_1$ when $\bar{m}_1 \ne 0$. We now consider joint probabilistic data association across targets with possibly unresolved measurements following [4], [5]. A marginal association event θ_{ir} is said to be effective at time k when the validated measurement $y_k^{1(i)}$ is associated with (i.e. originates from) target r ($r = 0, 1, 2$ where $r = 0$ means that the measurement is caused by clutter). Assuming that two targets can be possibly unresolved and detected as a single target, a joint association event Θ is effective when a set of marginal events $\{\theta_{ir}\}$ holds true simultaneously. That is, $\Theta = \bigcap_{i=1}^{\bar{m}_1} \theta_{ir_i}$ where r_i is the index of the target to which measurement $y_k^{1(i)}$ is associated in the event under consideration. Define the validation matrix (as in [4])

$$\Omega = [\omega_{ir}] \qquad i = 1, \cdots, \bar{m}_1, \qquad \text{for} \quad r = 0, 1, 2 \tag{33}$$

where $\omega_{ir} = 1$ if the measurement i lies in the validation gate of target r, else it is zero. A joint association event Θ is represented by the event matrix

$$\hat{\Omega}(\Theta) = [\hat{\omega}_{ir}(\Theta)] \qquad i = 1, \cdots, \bar{m}_1, \qquad \text{for} \quad r = 0, 1, 2 \tag{34}$$

where $\hat{\omega}_{ir} = 1$ if $\omega_{ir} \subset \Theta$, and $\hat{\omega}_{ir} = 0$ otherwise. A feasible association event is one where a measurement can have either only one source $\Sigma_{r=0}^2 \hat{\omega}_{ir}(\Theta) = 1$ or two sources (e.g. two targets) $\Sigma_{r=0}^2 \hat{\omega}_{ir}(\Theta) = 2$, and where at most one measurement can originate from a target

$$\delta_r(\Theta) = \Sigma_{i=1}^{\bar{m}_1} \hat{\omega}_{ir}(\Theta) \le 1 \qquad \text{for} \quad r = 1, 2. \tag{35}$$

The feasible association joint events Θ are mutually exclusive and exhaustive.

Following the definitions in [4], define the binary measurement association indicator

$$\tau_i(\Theta) = \Sigma_{r=1}^2 \hat{\omega}_{ir}(\Theta) \le 2 \qquad \text{for} \quad i = 1, \cdots, \bar{m}_1 \tag{36}$$

to indicate whether the validated measurement $y_k^{1(i)}$ is associated with target(s) in event Θ. Further, the number of false (unassociated) measurements in event Θ is

$$\phi(\Theta) = \Sigma_{i=1}^{\bar{m}_1}[1 - \min(1, \tau_i(\Theta))]. \tag{37}$$

A resolution indicator, $\rho(\Theta)$, is defined to be one when $\tau_i(\Theta) \le 1 \ \ \forall i$ and zero otherwise. We will limit our discussion to nonparametric JPDA [4]. The likelihood function for the global mode J can be evaluated as $\Lambda_k^{J,1}$

$$:= p\left[Y_k^1|M_k^J, Z^{k-1}\right] = \sum_{\Theta} p\left[Y_k^1|\Theta, M_k^J, Z^{k-1}\right] P\{\Theta|M_k^J, Z^{k-1}\} \tag{38}$$

When the targets are merged, the second term (apriori joint association probabilities) in the last line of (38) can be evaluated as [8]

$$P\{\Theta|M_k^J, Z^{k-1}\} := \int P\{\Theta|x_k, M_k^J, Z^{k-1}\} p[x_k|M_k^J, Z^{k-1}]dx_k$$

$$= D(\Theta) \int P(\mathcal{A}|x_k)\mathcal{N}(x_k; \hat{x}^J_{k|k-1}, P^J_{k|k-1})dx_k = D(\Theta)P_k^{J,1,a} \tag{39}$$

where

$$D(\Theta) = \frac{\phi(\Theta)!\epsilon}{\bar{m}_1!} \prod_{r=1}^{2} (P_D)^{\delta_r(\Theta)}(1 - P_D)^{1-\delta_r(\Theta)}, \tag{40}$$

P_D is the detection probability at sensor 1 (assumed to be the same for all targets), $\epsilon > 0$ is a "diffuse" prior (for nonparametric modeling of clutter) whose exact value is irrelevant and $P_k^{J,1,a}$

$$= \left|2\pi R^{1,d}\right|^{1/2} \mathcal{N}\left(0; H_k^{J,1,d} x^J_{k|k-1}, H_k^{J,1,d} P^{J,1}_{k|k-1} H_k^{J,1,d'} + R^{1,d}\right). \tag{41}$$

It then follows that

$$P\{\Theta|M_k^J, Z^{k-1}\} = \begin{cases} D(\Theta)(1 - P_k^{J,1,a}) \text{ for resolved target(s)} \\ D(\Theta)P_k^{J,1,a} \text{ for merged targets.} \end{cases} \tag{42}$$

The first term in (38) is

$$p\left[Y_k^1|\Theta, M_k^J, Z^{k-1}\right] = V_1^{-\phi(\Theta)} p\left[\tilde{Y}_k^1(\Theta)|M_k^J, Z^{k-1}\right] \tag{43}$$

where $\tilde{Y}_k^1(\Theta) \subset Y_k^1$ is a subset of the validated measurements Y_k^1, consisting of the measurements associated with the targets as specified by Θ. The number of measurements in $\tilde{Y}_k^1(\Theta)$ is equal $\bar{m}_1 - \phi(\Theta)$ where $\phi(\Theta)$ is the number of false alarms. Define a $\bar{m}_1 \times [\bar{m}_1 - \phi(\Theta)]$ matrix $\underline{\hat{\Omega}}(\Theta)$ as a submatrix of $\hat{\Omega}(\Theta)$ obtained by deleting the first column and all null columns of $\hat{\Omega}(\Theta)$. Then for a given Θ, we have a measurement vector $\tilde{Y}_k^1(\Theta)$ of dimension $(\Sigma_{i=1}^{\bar{m}_1}\min[1, \tau_i(\Theta)])n_{z1}$ given by

$$\tilde{Y}_k^1(\Theta) = (I_{n_{z1}} \otimes \underline{\hat{\Omega}}'(\Theta))\text{col}\{y_k^{1(i)}, i = 1, 2, \cdots, \bar{m}_1\} \tag{44}$$

where we stack up all target-associated validated measurements in Θ in ascending order of targets, I_n is the $n \times n$ identity matrix, and the symbol $\otimes$ denotes the Kronecker product. Define a $[(\bar{m}_1 - \phi(\Theta))z_{z1}] \times [2n_x]$ matrix $H_k^{J,1}(\Theta)$ as a submatrix of $H_k^{J,1}$ obtained by deleting all i-th block rows $(n_{z1}\times .)$ of $H_k^{J,1}$ for which $\delta_i(\Theta) = 0$. That is, we have modified $H_k^{J,1}$ to keep only the block elements associated with target-associated measurements in Θ.

To further simplify the equation for $\tilde{Y}_k^1(\Theta)$, one has to consider all the possible joint association events Θ. Define a related set of mutually exclusive and exhaustive data interpretations Ψ as follows (here we follow [8])

- Ψ_{11}: Both targets were resolved and detected ($\phi(\Theta) = \bar{m}_1 - 2$),
- Ψ_{10}: Both targets were resolved and only target 1 was detected ($\phi(\Theta) = \bar{m}_1 - 1, \delta_1(\Theta) = 1, \delta_2(\Theta) = 0$),
- Ψ_{01}: Both targets were resolved and only target 2 was detected ($\phi(\Theta) = \bar{m}_1 - 1, \delta_1(\Theta) = 0, \delta_2(\Theta) = 1$),
- Ψ_1: Both targets were detected but merged as a single measurement ($\phi(\Theta) = \bar{m}_1 - 1, \delta_1(\Theta) = 1, \delta_2(\Theta) = 1$),
- Ψ_0: No target was detected ($\phi(\Theta) = \bar{m}_1$).

It then follows that the linearized measurement equation for $\tilde{Y}_k^1(\Theta)$ is given by

$$\tilde{Y}_k^1(\Theta) = \begin{cases} H_k^{J,1}(\Theta)x_k + w_k^{J,1}, \text{ for } \Theta \in \Psi_{11} \\ H_k^{j_1,1}(\Theta)x_k + w_k^{j_1,1}(1), \quad \text{for } \Theta \in \Psi_{10} \\ H_k^{j_2,1}(\Theta)x_k + w_k^{j_2,1}(2), \quad \text{for } \Theta \in \Psi_{01} \\ \begin{bmatrix} H_k^{J,1,a}(\Theta) \\ H_k^{J,1,d}(\Theta) \end{bmatrix} x_k + \begin{bmatrix} w_k^{J,1,a} \\ w_k^{J,1,d} \end{bmatrix}, \text{ for } \Theta \in \Psi_1. \end{cases} \tag{45}$$

Conditioned on the joint association event Θ and mode J, the "coupled" innovation is given by

$$\nu_k^{J,1}(\Theta) = \begin{cases} \nu_k^{J,1}(\Theta) = \tilde{Y}_k^1(\Theta) - \hat{z}_k^{J,1}(\Theta), \quad \text{for } \Theta \in \Psi_{11} \\ \nu_k^{j_1,1}(\Theta) = \tilde{Y}_k^1(\Theta) - \hat{z}_k^{j_1,1}(\Theta), \quad \text{for } \Theta \in \Psi_{10} \\ \nu_k^{j_2,1}(\Theta) = \tilde{Y}_k^1(\Theta) - \hat{z}_k^{j_2,1}(\Theta), \quad \text{for } \Theta \in \Psi_{01} \\ \nu_k^{J,1,a}(\Theta) = \tilde{Y}_k^1(\Theta) - \begin{bmatrix} \hat{z}_k^{J,1,a}(\Theta) \\ \hat{z}_k^{J,1,d}(\Theta) \end{bmatrix}, \text{ for } \Theta \in \Psi_1, \\ \mathbf{0}, \text{ for } \Theta \in \Psi_0 \end{cases} \tag{46}$$

where $\hat{z}_k^{J,1}(\Theta)$ ($\hat{z}_k^{J,1,a}(\Theta)$, $\hat{z}_k^{J,1,d}(\Theta)$) are subvector(s) of $\hat{z}_k^{J,1}$ ($\hat{z}_k^{J,1,a}$, $\hat{z}_k^{J,1,d}$) obtained by deleting all i-th block rows $(n_{z1} \times 1)$ of $\hat{z}_k^{J,1}$ ($\hat{z}_k^{J,1,a}$, $\hat{z}_k^{J,1,d}$) for which $\delta_i(\Theta) = 0$. The covariance of mode-conditioned residual conditioned on the joint association event Θ is given by

$$\begin{aligned} &S_k^{J,1}(\Theta) = H_k^{J,1}(\Theta)P_{k|k-1}^J H_k^{J,1'}(\Theta) + R_k^{J,1}, \text{ for } \Theta \in \Psi_{11} \\ &S_k^{j_1,1}(\Theta) = H_k^{j_1,1}(\Theta)P_{k|k-1}^{j_1}(1) H_k^{j_1,1'}(\Theta) + R_k^{j_1,1}, \text{ for } \Theta \in \Psi_{10} \\ &S_k^{j_2,1}(\Theta) = H_k^{j_2,1}(\Theta)P_{k|k-1}^{j_2}(2) H_k^{j_2,1'}(\Theta) + R_k^{j_2,1}, \text{ for } \Theta \in \Psi_{01} \\ &S_k^{J,1,a}(\Theta) \text{ is given by (30) for } \Theta \in \Psi_1 \end{aligned} \tag{47}$$

where $P_{k|k-1}^{j_1}(1)$ and $P_{k|k-1}^{j_2}(2)$ are diagonal submatrices of $P_{k|k-1}^J$.

There are a total of $(\bar{m}_1 + 1) \times (\bar{m}_1 + 1)$ possible association hypotheses, each of which has an association probability. Then we have

$$p\left[Y_k^1|\Theta, M_k^J, Z^{k-1}\right] = \begin{cases} V_1^{2-\bar{m}_1} p[\tilde{Y}_k^1(\Theta)|M_k^J, Z^{k-1}], \text{ for } \Theta \in \Psi_{11} \\ V_1^{1-\bar{m}_1} p[\tilde{Y}_k^1(\Theta)|M_k^J, Z^{k-1}], \text{ for } \Theta \in \Psi_{10} \\ V_1^{1-\bar{m}_1} p[\tilde{Y}_k^1(\Theta)|M_k^J, Z^{k-1}], \text{ for } \Theta \in \Psi_{01} \\ V_1^{1-\bar{m}_1} p[\tilde{Y}_k^1(\Theta)|M_k^J, Z^{k-1}], \text{ for } \Theta \in \Psi_1 \\ V_1^{-\bar{m}_1}, \text{ for } \Theta \in \Psi_0 \end{cases} \tag{48}$$

where the conditional pdf (probability density function) of the validated measurements $\tilde{Y}_k^1(\Theta)$ given their origins (specified by Θ) and the global mode J, is given by

$$p\left[\tilde{Y}_k^1(\Theta)|M_k^J, Z^{k-1}\right] = \begin{cases} \mathcal{N}(\tilde{Y}_k^1(\Theta); \hat{z}_k^{J,1}(\Theta), S_k^{J,1}(\Theta)), \text{ for } \Theta \in \Psi_{11} \\ \mathcal{N}(\tilde{Y}_k^1(\Theta); \hat{z}_k^{j_1,1}(\Theta), S_k^{j_1,1}(\Theta)), \text{ for } \Theta \in \Psi_{10} \\ \mathcal{N}(\tilde{Y}_k^1(\Theta); \hat{z}_k^{j_2,1}(\Theta), S_k^{j_2,1}(\Theta)), \text{ for } \Theta \in \Psi_{01} \\ \mathcal{N}\left(\tilde{Y}_k^1(\Theta); \begin{bmatrix} \hat{z}_k^{J,1,a}(\Theta) \\ \hat{z}_k^{J,1,d}(\Theta) \end{bmatrix}, S_k^{J,1,a}(\Theta)\right), \text{ for } \Theta \in \Psi_1 \end{cases} . \tag{49}$$

The probability of the joint association event Θ given that global mode J is effective from time k-1 through k is $\beta_k^{J,1}(\Theta)$

$$:= P\{\Theta|M_k^J, Z^{k-1}, Y_k^1\} = \frac{1}{c} p[Y_k^1|\Theta, M_k^J, Z^{k-1}] P\{\Theta|M_k^J, Z^{k-1}\} \tag{50}$$

where the first term can be calculated from (43)-(47), the second term from (40)-(42), and c is a normalization constant such that $\Sigma_\Theta P\{\Theta|M_k^J, Z^{k-1}, Y_k^1\} = 1$.

Using $\hat{x}_{k|k-1}^J$ (from 22) and its covariance $P_{k|k-1}^J$ (from 23), one computes the partial update $\hat{x}_{k|k}^J$ and its covariance $P_{k|k}^J$ following the standard PDAF [4], except that the global state is conditioned on Θ, not the marginal events θ_{ir}; details follow. Kalman gain:

$$\begin{aligned} &W_k^J(\Theta) = P_{k|k-1}^J H_k^{J,1}(\Theta)'[S_k^{J,1}(\Theta)]^{-1}, \text{ for } \Theta \in \Psi_{11} \\ &W_k^{j_1}(\Theta) = P_{k|k-1}^{j_1}(1) H_k^{j_1,1}(\Theta)'[S_k^{j_1,1}(\Theta)]^{-1}, \text{ for } \Theta \in \Psi_{10} \\ &W_k^{j_2}(\Theta) = P_{k|k-1}^{j_2}(2) H_k^{j_1,2}(\Theta)'[S_k^{j_2,1}(\Theta)]^{-1}, \text{ for } \Theta \in \Psi_{01} \\ &W_k^{J,a}(\Theta) = P_{k|k-1}^J \begin{bmatrix} H_k^{J,1,a}(\Theta) \\ H_k^{J,1,d}(\Theta) \end{bmatrix}' [S_k^{J,1,a}(\Theta)]^{-1}, \text{ for } \Theta \in \Psi_1. \end{aligned} \tag{51}$$

Partial update of the state estimate:
$\hat{x}_{k|k}^{J,1}(\Theta) := E\left\{x_k|\Theta, M_k^J, Z^{k-1}, Y_k^1\right\}$

$$= \begin{cases} \hat{x}_{k|k-1}^J + W_k^J(\Theta)\nu_k^{J,1}(\Theta), \text{ for } \Theta \in \Psi_{11} \\ \hat{x}_{k|k-1}^J + \begin{bmatrix} W_k^{j_1}(\Theta)\nu_k^{j_1,1}(\Theta) \\ 0_{n_x \times 1} \end{bmatrix}, \text{ for } \Theta \in \Psi_{10} \\ \hat{x}_{k|k-1}^J + \begin{bmatrix} 0_{n_x \times 1} \\ W_k^{j_2}(\Theta)\nu_k^{j_2,1}(\Theta) \end{bmatrix}, \text{ for } \Theta \in \Psi_{01} \\ \hat{x}_{k|k-1}^J + W_k^{J,a}(\Theta)\nu_k^{J,1,a}(\Theta), \text{ for } \Theta \in \Psi_1 \\ \hat{x}_{k|k-1}^J, \text{ for } \Theta \in \Psi_0. \end{cases} \quad (52)$$

Mode-conditioned update of the state estimate:

$$\hat{x}_{k|k}^{J,1} := E\left\{x_k|M_k^J, Z^{k-1}, Y_k^1\right\} = \sum_{\Theta} \beta_k^{J,1}(\Theta)\hat{x}_{k|k}^{J,1}(\Theta). \quad (53)$$

Covariance of $\hat{x}_{k|k}^J$,
$P_{k|k}^{J,1} := E\left\{(x_k - \hat{x}_{k|k}^{J,1})(x_k - \hat{x}_{k|k}^{J,1})'|Y_k^1, Z^{k-1}, M_k^J\right\}$, can be derived following [5] and [4, (3.4.2-10)]; details are omitted for lack of space.

Step 5. The mode-conditioned predicted measurements for sensor 2 ($\forall J \in \bar{\mathcal{M}}_n$):
For two resolved targets: Follow Step 3.5 of [5] (see also Step 2 earlier).
For unresolved targets: The "predicted" measurement for sensor 2 is given by $\hat{z}_k^{J,2,a} := h^{2,a}(\hat{x}_{k|k}^{J,1})$. Define $\hat{z}_k^{J,2,d} := [I \ - I]h^2(\hat{x}_{k|k}^{J,1})$. Using linearization around $\hat{x}_{k|k}^{J,1}$, the covariance of the mode-conditioned residual

$$\nu_k^{J,2,a} := \begin{bmatrix} z_k^{2,a} \\ 0 \end{bmatrix} - \begin{bmatrix} \hat{z}_k^{J,2,a} \\ \hat{z}_k^{J,2,d} \end{bmatrix} \quad (54)$$

is given by

$$S_k^{J,2,a} := E\left\{\nu_k^{J,2,a}\nu_k^{J,2,a'}|M_k^J, Z^{k-1}, Y_k^1\right\}$$

$$= \begin{bmatrix} H_k^{J,2,a} \\ H_k^{J,2,d} \end{bmatrix} P_{k|k}^{J,1} \begin{bmatrix} H_k^{J,2,a'} & H_k^{J,2,d'} \end{bmatrix} + \begin{bmatrix} R_k^{2,a} & 0 \\ 0 & R^{2,d} \end{bmatrix} \quad (55)$$

where $H_k^{J,2,a} = [\beta_k H^{j_1,2} \quad (1-\beta_k)H^{j_2,2}]$ and $H_k^{J,2,d} = [H_k^{j_1,2} \quad -H_k^{j_2,2}]$.

Step 6. Measurement validation measurements for sensor 2 ($\forall J \in \bar{\mathcal{M}}_n$): This is similar to Step 3 where we replace $S_k^{J,1}$ with $S_k^{J,2}$, $z_k^{1(i)}$ with $z_k^{2(i)}$, m_1 with m_2, $V_k^1(r)$ with $V_k^2(r)$, and V_k^1 with V_k^2. Details are similar to that in Step 3, hence omitted.

Step 7. Update with validated measurements for sensor 2 ($\forall J \in \bar{\mathcal{M}}_n$): This is similar to Step 3.4. Using the validated measurements obtained from Step 3.6 and starting from $\hat{x}_{k|k}^{J,1}$ and $P_{k|k}^{J,1}$, one computes the final updates $\hat{x}_{k|k}^J$ and $P_{k|k}^J$, and the likelihood

$$\Lambda_k^{J,2} := p\left[Y_k^2, |M_k^J, Z^{k-1}, Y_k^1\right]$$

$$= \sum_{\Theta} p\left[Y_k^2|\Theta, M_k^J, Z^{k-1}, Y_k^1\right] P\{\Theta|M_k^J, Z^{k-1}, Y_k^1\}. \quad (56)$$

Details are similar to that in Step 3.4, hence omitted.

Step 8. Update of mode probabilities ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_2$):

$$\mu_k^J := P[M_k^J|Z^k] = \frac{1}{c}\mu_k^{J-}\Lambda_k^{J,1}\Lambda_k^{J,2} \quad (57)$$

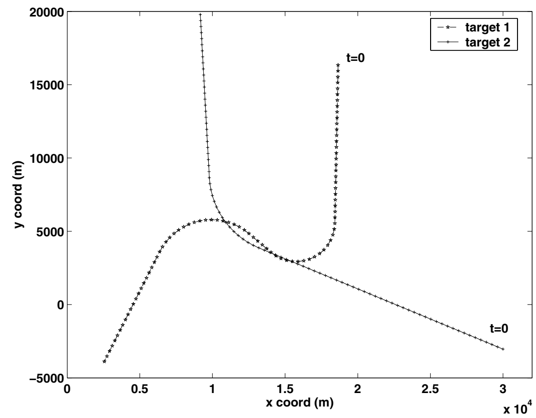

Figure 1. *The true trajectories of the maneuvering targets: position in xy plane.*

where c is a normalization constant such that $\Sigma_J \mu_k^J = 1$. For individual targets we have

$$\mu_k^{j_1}(1) := P[M_k^{j_1}(1)|Z^k] = \sum_{j_2=1}^{n} \mu_k^{j_1,j_2}, \ \ \mu_k^{j_2}(2) = \sum_{j_1=1}^{n} \mu_k^{j_1,j_2} \quad (58)$$

with $J = (j_1, j_2)$ in (57).

Step 9. Combination of the mode-conditioned estimates ($\forall r \in \mathcal{T}_2$): The final global state estimate update at time k is given by

$$\hat{x}_{k|k} := E\{x_k|Z^k\} = \sum\nolimits_J \hat{x}_{k|k}^J \mu_k^J \quad (59)$$

and its covariance is given by

$$P_{k|k} = \sum\nolimits_J \left\{P_{k|k}^J + \left[\hat{x}_{k|k}^J - \hat{x}_{k|k}\right]\left[\hat{x}_{k|k}^J - \hat{x}_{k|k}\right]'\right\}\mu_k^J. \quad (60)$$

The state estimate $\hat{x}_{k|k}(r)$ for target r is the n_x-subvector of $\hat{x}_{k|k}$ consisting of elements $(r-1)n_x + m$, $m = 1, 2$.

5. SIMULATION EXAMPLE

The following example of tracking two highly maneuvering targets in clutter is patterned after [5].

The True Trajectory: We consider a scenario similar to that in [5]. Target 1 starts at location [21689+d_x 10840+d_y 40] with d_x=-3040m and d_y=5500m in Cartesian coordinates in meters. The initial velocity (in m/s) is [-8.3 -399.9 0] and the target stays at constant altitude with a constant speed of 400 m/s. Its trajectory is: a straight line with constant velocity between 0 and 27s, a coordinated turn (0.15 rad/s) with constant acceleration of 60 m/s^2 between 27 and 42s, a straight line with constant velocity between 42 and 47s, a coordinated turn (0.1 rad/s) with constant acceleration of 40 m/s^2 between 47 and 65s, and a straight line with constant velocity between 65 and 87s. Target 2 starts at location [30000 -3040 40] in Cartesian coordinates in meters. The initial velocity (in m/s) is [-382 157 0] and the target stays at constant altitude with a constant speed of 413 m/s. Its trajectory is: a straight line with constant velocity between 0 and 44s, a coordinated turn (0.075 rad/s) with constant acceleration of 30 m/s^2 between 44 and 59s, and a straight line with constant velocity between 59 and 87s.

The Target Motion Models: There are 3 models exactly as in [5]. In each mode the target dynamics are modeled in Cartesian coordinates as $x_k(r) = F(r)x_{k-1}(r) + G(r)v_{k-1}(r)$ where the state of the target is position, velocity, and acceleration in each of the 3 Cartesian coordinates (x, y, and z). The details regarding these models may be found in [6]. The initial model probabilities for two targets are identical: $\mu_0^1 = 0.8$, $\mu_0^2 = 0.1$ and $\mu_0^3 = 0.1$. The mode switching probability matrix for two targets is also identical and is as in [6].

The Sensors: Two sensors (we assume collocation, and time synchronization of observations, etc.) are used to obtain the measurements. The measurements from sensor l are

No. of	IMM/JPDAMCF (proposed)	IMM/JPDACF ([5])
lost tracks	44/1000	94/1000
swapped tracks	3/1000	4/1000
successful tracks	953/1000	902/1000

Table 1. *Simulation results summary based on 1000 runs.*

$z_k^l = h^l(x_k) + w_k^l$, $l = 1, 2$, reflecting range and azimuth angle for sensor 1 (radar) and azimuth and elevation angles for sensor 2 (infrared). The range, azimuth, and elevation angle transformations would be given by $r = (x^2 + y^2 + z^2)^{1/2}$, a=$\tan^{-1}[y/x]$, $e = \tan^{-1}[z/(x^2 + y^2)^{1/2}]$, respectively. The measurement noise w_k^l for sensor l is assumed to be zero-mean white Gaussian with known covariances $R^1 = \text{diag}[q_r, q_{a1}] = \text{diag}[400\text{m}^2, 49\text{mrad}^2]$ with q_r and q_{a1} denoting the variances for the radar range and azimuth measurement noises, respectively, and $R^2 = \text{diag}[q_{a2}, q_e] = \text{diag}[4\text{mrad}^2, 4\text{mrad}^2]$ with q_{a2} and q_e denoting the variances for the infrared sensor azimuth and elevation measurement noises, respectively. Resolutions of both sensors are selected after from [8] (twice of the standard deviations for the corresponding sensor measurement noise): a range resolution of sensor 1 (α^R)=2$\times\sqrt{q_r}$=40m, a angular resolution of sensor 1 (α^{ϕ_1})=2$\times\sqrt{q_{a1}}$=14$\times$10^{-3}rad, a angular resolution of sensor 2 (α^{ϕ_2})=2$\times\sqrt{q_{a2}}$=4 $\times$ 10^{-3}rad and a elevation angle resolution of sensor 2 (α^θ)=2$\times\sqrt{q_e}$=4 $\times$ 10^{-3}rad. The noise for merged measurements $w_k^{l,a}$ for sensor l is assumed to be the same with resolved measurement noise w_k^l for sensor l. The measurement distance noise $w_k^{l,d}$ for sensor l is assumed to be zero-mean white Gaussian with known covariances $R^{1,d} = \text{diag}[\alpha^R, \alpha^{\phi_1}]$, and $R^{2,d} = \text{diag}[\alpha^{\phi_2}, \alpha^\theta]$. To generate the true target trajectories, following (15), given the measurements of the two targets at a given sensor, the targets are unresolved with the conditional probability $P_k^{l,a}$ and they are merged into one by the linear combination model (10) with the signal strength ratio β_k=0.5 for all time k. (The tracking algorithm does not have this knowledge of how $P_k^{l,a}$ is used to generate data.) Both sensors are assumed to be located at the coordinate system origin. The sampling interval was T=1s and it was assumed that the probability of detection P_D=0.997 for both sensors.

The Clutter: For generating false measurements in simulations, the clutter was assumed to be Poisson distributed with expected number of $\lambda_1 = 20 \times 10^{-6}$/m-mrad for sensor 1 and $\lambda_2 = 2 \times 10^{-4}/\text{mrad}^2$ for sensor 2. These statistics were used for generating the clutter in all simulations. However, a nonparametric clutter model was used for implementing all the algorithms for target tracking.

Other Parameters: The gates for setting up the validation regions for both the sensors were based on the threshold γ=16 corresponding to a gate probability P_G=0.9997.

Simulation Results: The results were obtained from 1000 Monte Carlo runs. Fig. 1 shows the true trajectory of the two targets as a function of time. The two targets start out far apart, move close to each other from 38 to 42 sec., cross at 52 sec. and then move apart again. Fig. 2(a) shows the results of proposed IMM/JPDAMCF based on 953 successful runs (target swap occurred in 3 runs with 44 track failures). Fig. 2(b) shows the standard IMM/JPDACF (the merged target case is not accounted for in this approach) based on 902 successful runs (target swap occurred in 4 runs with 94 track failures) in terms of the RMSE in position. Table 1 shows the number of successful runs (including target swappings) for the two approaches IMM/JPDAMCF and IMM/JPDACF. It is seen from Table 1 and Figs. 1 and 2 that the proposed IMM/JPDAMCF has better performance than IMM/JPDACF especially in terms of the track estimation accuracy and the loss of tracks.

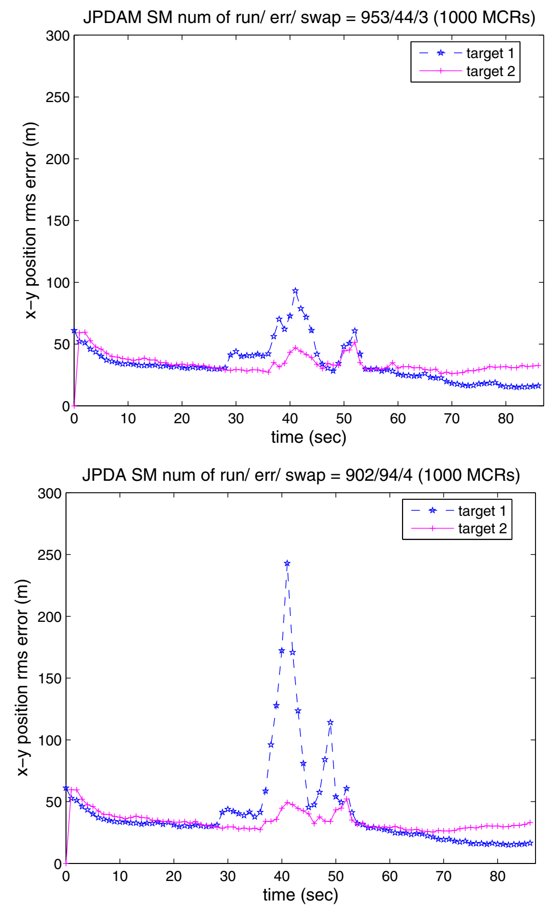

Figure 2. *Performance (RMSE in position) of the proposed IMM/JPDAMCF and the IMM/JPDACF of [5] based on successful runs (read top to bottom): (a) IMM/JPDAMCF (proposed), (b) IMM/JPDACF [5].*

REFERENCES

[1] A. Houles and Y. Bar-Shalom, "Multisensor tracking of a maneuvering target in clutter", *IEEE Trans. Aerospace & Electronic Systems*, vol. AES-25, pp. 176-188, March 1989.

[2] G.V. Trunk, "Range resolution of targets using automatic detectors", *IEEE Trans. Aerospace & Electronic Systems*, vol. AES-14, pp. 750-755, Sept. 1978.

[3] G.V. Trunk and J.D. Wilson, "Track initiation of occasionally unresolved radar targets", *IEEE Trans. Aerospace & Electronic Systems*, vol. AES-17, pp. 122-130, Jan. 1981.

[4] Y. Bar-Shalom and X.R. Li, *Multitarget-Multisensor Tracking: Principles and Techniques*. Storrs, CT: YBS Publishing, 1995.

[5] J.K. Tugnait, "Tracking of multiple maneuvering targets in clutter using multiple sensors, IMM and JPDA coupled filtering," *IEEE Trans. Aerospace & Electronic Systems*, vol. AES-40, pp. 320-330, Jan. 2004.

[6] B. Chen and J.K. Tugnait, "Tracking of multiple maneuvering targets in clutter using IMM/JPDA filtering and fixed-lag smoothing", *Automatica*, vol. 37, pp. 239-249, Feb. 2001.

[7] K.-C. Chang and Y. Bar-Shalom, "Joint Probabilistic Data Association for Multitarget Tracking with Possibly Unresolved Measurements and Maneuvers", *IEEE Trans. Automatic Control*, Vol. AC-29, pp. 585-594, July 1984.

[8] W. Koch and G.V. Keuk, "Multiple hypothesis Track Maintenance with Possibly Unresolved Measurements", *IEEE Trans. Aerospace & Electronic Systems*, Vol. AES-33, pp. 883-892, July 1997.