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Stabilization of NCSs: Asynchronous Partial Transfer Approach

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***Abstract*— In this paper, a framework for stabilization of networked control systems with limited data rates is presented. The key technology is that instead of sending the complete state vector(for an n-dimensional system, which means a data array with n items) to the network, we only send one item of a nonsingular transform of the state vector at each step. The main advantage of our mechanism is dramatically reducing the amount of the data sent to the network. In order to implement such a technology, in our framework, two auxiliary instruments are included into the overall closed-loop state feedback system over the network. One instrument is used as an encoder, which runs a transform of the system state and sends the items of the transformed state vector in turn. The other takes the role as a decoder, which receives items of the transformed state vector from the network and reconstructs the state vector for the controller. It is shown that if the plant is stabilizable via direct state feedback control, then it can be stabilized with the same controller under our framework over the network.**

I. INTRODUCTION

In distributed control systems, a feedback control loop is closed through a network. Distributed control systems with networks are called *networked control systems(NCSs)*. The use of a data network in a control loop has gained increasing attention in recent years due to its cost effective and flexible applications. The advantages of NCSs include drastic decreasing the number and the total length of cables, easiness of system diagnosis and maintenance, and increasing system agility. Thus, many industrial applications, such as robotic systems, jacking systems for train cars, networks on automobiles, and process control systems can be reconstructed and modelled as NCSs.

The analysis and design of NCSs are much more difficult due to the insertion of the communication network in the feedback control loop where the conventional control theories with ideal assumptions can not be applied directly.

One main drawback of NCSs is the network-induced delay. Various methodologies have been formulated to treat the network-induced delays. A survey of recent control methodologies for a closed-loop control system over a data network has been presented in [1]. That paper provided the overview of NCSs including system configuration, network delay characteristics, the effects of networked delays, and corresponding control methodologies.

The other main drawback of NCSs is the limitation on bandwidth. A communication network has limited bandwidth which is shared by different devices connected to the network. To overcome the bandwidth constraint quite a lot of approaches have been proposed in recent years.

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[2] introduces the notion of minimum attention control. The basic idea is a tradeoff between open loop and closed loop control. Open loop control means no utilization of network but with poor performance, while closed loop control means utilization of network and with good performance. The former requires no bandwidth, but the later requires sufficient bandwidth. Then the problem is posed as an optimization problem.

[3] introduces the notion of maximal allowable transfer interval, MATI. In this framework, the controller is designed without taking the network into account. And if the time between transfers of information from the sensor to the controller is smaller than MATI, then the networked control system still keeps stable. In [4], this framework is extended to deal with nonlinear plants. In [5], based on the results in [3], a state predictor for the plant to estimate the state between updates is introduced. Two kinds of predictors are presented. Some sufficient conditions are given for stability of the NCS setup.

Similar to the framework in [5], [6] introduces a compensated model of the plant, the state of which is updated with the plant's state. The compensated model is asked to be stable. Necessary and sufficient conditions for stability via state and output feedback control are derived.

Quantization theory is another primary approach used to achieve lower bit-rates. The leading work is referred to [7], though it was not motivated from an NCS point of view. There are abundant results from stability analysis to stabilization synthesis via different quantization algorithms(see [8][9] [10][11][12][13] and the references therein).

The common feature of the above approaches is being minimizing the transfer of information between the sensor and the controller/actuator. Different from the above mechanisms, in this paper, a new framework is presented for stabilization of NCSs which aims to reduce the size of the data transferred at each transaction. The key idea of this approach is that instead of sending the entire state vector(for an n-dimensional system, this means a data array with n items) to the network, only one item of a nonsingular transform of the state vector is sent at each time.

In order to realize this approach, two auxiliary instruments are inserted into the overall closed-loop state feedback system over the network. One instrument is used as an encoder, which runs a transform of the state vector and sends the items of the transformed state vector in turn. The other takes the role as a decoder, which receives the items of the transformed state vector from the network and reconstructs the state vector which is being used by the

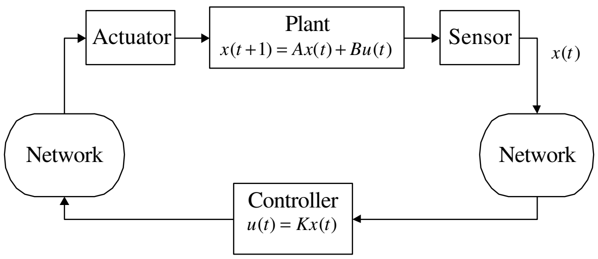

Fig.1. Standard NCS model.

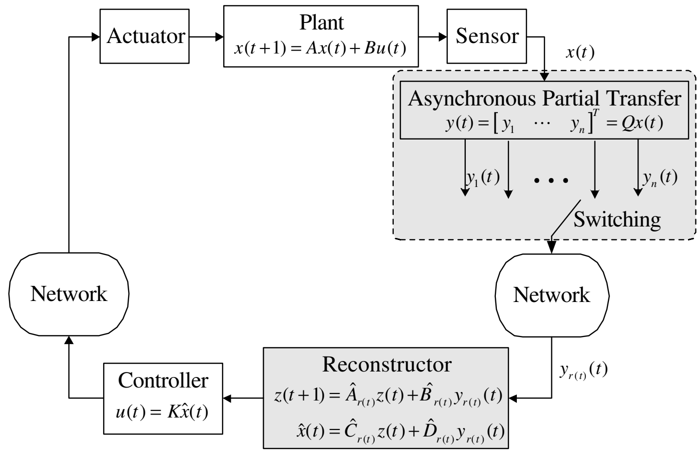

Fig.2. NCS model with asynchronous partial transfer.

controller. It is shown that if the plant is stabilizable via direct state feedback control, then it can be stabilized with the same controller under our framework over the network.

The main advantage of our mechanism is dramatically reducing the amount of the data sent to the network and decreasing the load of the network.

This paper is organized as follows. Section II formulates the problem and presents the preliminary results. Section III is the main results. Section IV contains a numerical example. Finally, the conclusion is provided in Section V.

Notations: We use standard notations throughout this note. $\mathbb{R}$ is the set of real number, $\mathbb{C}$ is the of complex number. I_n is the unit matrix of dimension n.

II. PROBLEM FORMULATION

In this paper, we consider a standard discrete-time linear-time-invariant system described as follows:

$$x(t+1) = Ax(t) + Bu(t), t = 0, 1, 2, \cdots \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^p$ is the control input. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$ are known constant matrices.

Different from general framework of NCSs(shown in Fig.1), two auxiliary instruments are added into the overall closed-loop feedback system over the network(shown in Fig.2):

• After the sensor, an auxiliary instrument, named as *asynchronous partial transfer*, is inserted to upload signals to the communication network, where a state transformation is made as

$$y(t) = [y_1(t), \cdots, y_n(t)]^T = Qx(t),$$

where Q is a constant matrix to be designed. At each time instant, one and only one item of $y(t)$ is uploaded to the network. A switching signal

$$r(t) : \{0, 1, 2, \cdots\} \to \{1, 2, \cdots, n\}$$

is introduced to describe when and which item of $y(t)$ is uploaded. In detail, $r(t) = i$ means that the i-th item of $y(t)$, $y_i(t)$, is sent to the network. The switching signal $r(t)$ is also a control factor to be designed.

• Before the controller, another auxiliary instrument, named as *reconstructor*, is interposed to received the signal from the network. In the reconstructor, a dynamic estimator of the system state is given as follows:

$$\begin{aligned} z(t+1) &= \hat{A}_{r(t)}z(t) + \hat{B}_{r(t)}y_{r(t)}(t) \\ \hat{x}(t) &= \hat{C}_{r(t)}z(t) + \hat{D}_{r(t)}y_{r(t)}(t) \end{aligned} \quad (2)$$

where $z(t)$ is the state of the estimator, $y_{r(t)}(t)$ is the input and $\hat{x}(t)$ is the output. The constant matrices $\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i$ are given as follows:

$$\begin{aligned} &\hat{A}_i = Q(A+BK)Q^{-1}F_i, \quad \hat{B}_i = Q(A+BK)Q^{-1}e_i, \\ &\hat{C}_i = Q^{-1}F_i, \qquad\qquad\quad \hat{D}_i = Q^{-1}e_i, \\ &\qquad\qquad\qquad\qquad\qquad\qquad i = 1, \cdots, n. \end{aligned} \quad (3)$$

Moreover, the vectors $e_1, \cdots, e_n \in \mathbb{R}^n$ and matrices $F_1, \cdots, F_n \in \mathbb{R}^{n\times n}$ are defined as follows:

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \cdots, e_n = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad (4)$$

$$E_i = e_i e_i^T, \quad F_i = I_n - E_i, \quad i = 1, \cdots, n.$$

We take the initial state of the estimator to be zero, i.e., $z(0) = 0$ and the matrix K is the feedback gain matrix to be designed.

To avoid unnecessary complexity, we assume that there is no time delay in our framework. Here, we are interested in the synthesis problem for the NCS in Fig.2 which is formulated as follows:

Problem. *To design a transform matrix Q, a feedback gain K and a switching signal $r(t)$ such that the overall closed-loop system is asymptotically stable.*

Before given the main result, we first give some basic lemmas.

Lemma 1: [14] Given a matrix $G \in \mathbb{C}^{n\times n}$ (or $\mathbb{R}^{n\times n}$), there exists a nonsingular matrix $H \in \mathbb{C}^{n\times n}$(or $\mathbb{R}^{n\times n}$) such that the matrix product HGH^{-1} to be a triangular matrix.

Lemma 2: Given a triangular matrix $G \in \mathbb{C}^{n\times n}$ or $\mathbb{R}^{n\times n}$, all the eigenvalues of the matrix product $GF_1 \cdots GF_n$ are zero, where $F_1, \cdots, F_n$ are defined by (4).

Proof: See Appendix A. ■

III. Stabilization Design

In this section, we consider the stabilization design problem formulated in the previous section.

By the definition of $E_1, \cdots, E_n$, it is easy to verify that for any switching signal $r(t)$,

$$e_{r(t)}y_{r(t)}(t) \equiv E_{r(t)}y(t).$$

It follows that

$$\hat{B}_{r(t)}y_{r(t)}(t) \equiv \hat{B}_{r(t)}e^T_{r(t)}y(t),$$
$$\hat{D}_{r(t)}y_{r(t)}(t) \equiv \hat{D}_{r(t)}e^T_{r(t)}y(t).$$

Thus, we can rewrite (2) as follows:

$$\begin{array}{rl} z(t+1) & = \hat{A}_{r(t)}z(t) + \bar{B}_{r(t)}y(t) \\ \hat{x}(t) & = \hat{C}_{r(t)}z(t) + \bar{D}_{r(t)}y(t) \end{array} \tag{5}$$

where $\hat{A}_i$, $\hat{C}_i$ is given by (3) and

$$\bar{B}_i = Q(A+BK)Q^{-1}E_i,\ \bar{D}_i = Q^{-1}E_i,\ i = 1, \cdots, n. \tag{6}$$

It is easy to see that the estimator (5) is a standard switched linear system(See [15] for details on switched systems). It follows that

$$\hat{x}(t) = Q^{-1}(F_{r(t)}z(t) + E_{r(t)}y(t)) \tag{7}$$

Applying the state feedback $u(t) = K\hat{x}(t)$ to the plant, we get the closed-loop system as follows:

$$\begin{aligned} x(t+1) &= Ax(t) + BKQ^{-1}(F_{r(t)}z(t) + E_{r(t)}y(t)) \\ z(t+1) &= Q(A+BK)Q^{-1}(F_{r(t)}z(t) + E_{r(t)}y(t)) \end{aligned} \tag{8}$$

Since $y(t) = Qx(t)$, we get

$$\begin{aligned} y(t+1) = {} & QAQ^{-1}y(t) + QBKQ^{-1} \\ & (F_{r(t)}z(t) + E_{r(t)}y(t)) \\ z(t+1) = {} & Q(A+BK)Q^{-1}(F_{r(t)}z(t) + E_{r(t)}y(t)) \end{aligned} \tag{9}$$

Denote

$$\xi(t) = \begin{bmatrix} y(t) \\ y(t) - z(t) \end{bmatrix} \tag{10}$$

It is easy to prove that augment variable ξ satisfies the following dynamic equation:

$$\xi(t+1) = \Gamma_{r(t)}\xi(t) \tag{11}$$

where

$$\Gamma_i = \begin{bmatrix} Q(A+BK)Q^{-1} & -QBKQ^{-1}F_i \\ 0 & QAQ^{-1}F_i \end{bmatrix}, \quad i = 1, \cdots, n. \tag{12}$$

The system (11) is also a standard switched linear system and it is obvious that such a system is asymptotically stable implies that the closed-loop NCS in Fig.2 is asymptotically stable.

Theorem 1: Assume that the system matrix A has only real eigenvalues, if the pair (A, B) is stabilizable, then there exist real matrices Q, K and a periodically switching signal $r(t)$ such that the NCS in Fig.2 is asymptotically stable.

Proof: First, by Lemma 1, we select suitable nonsingular matrix Q such that QAQ^{-1} is in triangular form. Next, we select suitable matrix K such that $A + BK$ is Schur stable. Finally, we select the periodically switching signal $r(t)$ given by

$$r(kn + i) = n - i + 1, i = 1, \cdots, n, k = 0, 1, \cdots. \tag{13}$$

It is obvious that the period of $r(t)$ is n. Thus, we get

$$\xi((k+1)n) = \Gamma\xi(kn), k = 0, 1, \cdots. \tag{14}$$

where

$$\Gamma = \begin{bmatrix} Q(A+BK)^nQ^{-1} & * \\ 0 & QAQ^{-1}F_1 \cdots QAQ^{-1}F_n \end{bmatrix} \tag{15}$$

Since QAQ^{-1} is in triangular form, by Lemma 2, we know that the eigenvalues of $QAQ^{-1}F_1 \cdots QAQ^{-1}F_n$ are all zero. It follows that the matrix Γ is Schur stable. Thus, the system (11) is asymptotically stable. This means that the NCS in Fig.2 is asymptotically stable. ■

Corollary 1: Assume that the system matrix A has complex eigenvalue, and the pair (A, B) is stabilizable, then there exist a complex matrix Q, a real matrix K and a periodically switching signal $r(t)$ such that the NCS in Fig.2 is asymptotically stable.

Proof: The proof is similar to that of Theorem 1. ■

Remark 1: In Theorem 1, since all eigenvalues of the matrix $QAQ^{-1}F_1 \cdots QAQ^{-1}F_n$ are zero, it follows that

$$(QAQ^{-1}F_1 \cdots QAQ^{-1}F_n)^n = 0_{n\times n}$$

This implies that

$$Qx(t) - z(t) = y(t) - z(t) \equiv 0,\ t > n^2.$$

It means that the state of the estimator can converge to the exact value of the state in finite steps. This fact shows that the estimator is a *deadbeat observer*.

Remark 2: In Theorem 1, if (A, B) is reachable, we can select suitable K such that all eigenvalues of $A + BK$ are zero. It follows that all the eigenvalues of the matrix Γ are zero. Thus, we get

$$\Gamma^{2n} = 0_{2n\times 2n}.$$

This implies that the plant can be stabilized in finite steps. This fact shows that the NCS in Fig.2 can realize *deadbeat control* when the system is reachable.

IV. Example

To illustrate the efficiency of the proposed approach, a numerical example is presented.

Example 1: Consider the plant model with

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 2 & 3 & 4 \\ 1.5 & -0.5 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} \tag{16}$$

By simple calculation, we know that the eigenvalues of A are 1, 3 and -2, and (A, B) is reachable. Thus, by Theorem 1, the system can be stabilized. In fact, we take

$$Q = \begin{bmatrix} 7 & 1 & 0 \\ 29 & 3 & 4 \\ -16.5 & -0.5 & 1 \end{bmatrix}$$

such that

$$QAQ^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

is a triangular matrix. Next, we take

$$K = \begin{bmatrix} -6.2347 & -1.4840 & -5.3853 \end{bmatrix}$$

such that $A + BK$ is Schur stable. Finally, applying the periodic switching signal (12) with initial state $x(0) = [100, 10, -50]^T$, the simulation results show that the closed-loop system is stable indeed(See Fig. 3). The numerical simulation is finished with Matlab 6.5.

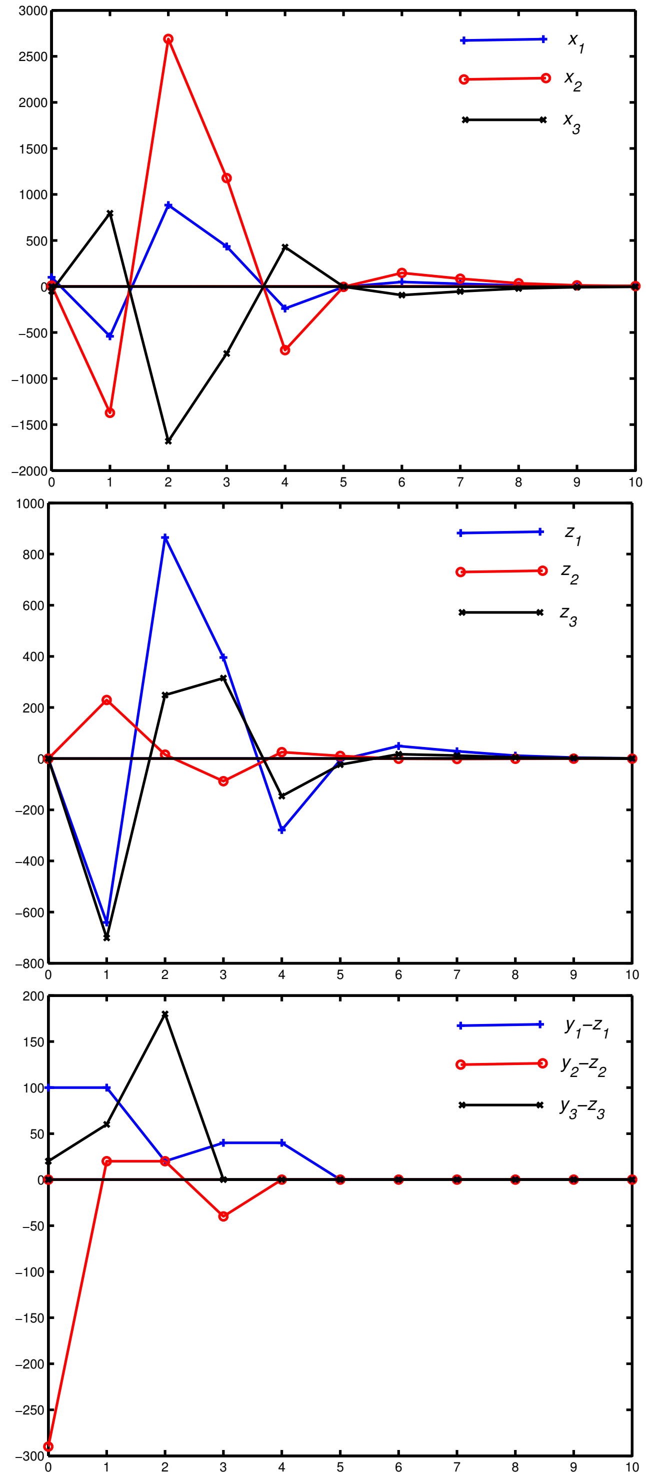

Fig.3. Simulation results of Example 1.

V. Conclusion

In this paper, a new framework for stabilization of networked control systems with limited data rates has been presented. The approach is based on reducing the size of the data transferred at each transaction. In our framework, two auxiliary instruments are included into the overall closed-loop state feedback system over the network. One instrument is used as an encoder, which sends only a scalar value each step. The other takes the role as a decoder, which receives signals from the network and reconstructs the state vector for the controller. By adding these two instruments, it has been shown that if the plant is stabilizable via direct state feedback control, then it can be stabilized with the same controller under our framework over the network. The future work includes considering network-induced delay into the framework.

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Appendix A

Proof: [Proof of Lemma 2] Supposing

$$G = \begin{bmatrix} g_1 & * & \cdots & * \\ 0 & g_2 & \cdots & * \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & g_n \end{bmatrix},$$

we have

$$
\begin{aligned}
&GF_1 \cdots GF_n \\
= &\begin{bmatrix} 0 & * & \cdots & * \\ 0 & g_2 & \cdots & * \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & g_n \end{bmatrix} GF_2 \cdots GF_n \\
= &\begin{bmatrix} 0 & * & \cdots & * \\ 0 & 0 & \cdots & * \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & g_n \end{bmatrix} GF_3 \cdots GF_n \\
= &\cdots \\
= &\begin{bmatrix} 0 & * & \cdots & * \\ 0 & 0 & \cdots & * \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}
\end{aligned}
$$

■

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