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A Three-Step Design Method for Performance Improvement of Robust Repetitive Control

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***Abstract*— This paper presents a three-step design method to improve performance of repetitive control systems. The performance is enhanced for rejection of high frequency harmonic disturbances by extending the bandwidth of low-pass filter Q in repetitive module. In this method, an interim feedback controller K' is designed firstly to ensure the realizability of the Q design in repetitive module. Then Q filter is designed to satisfy robustness stability of system. Finally the feedback controller K' is redesigned as K to guarantee overall system robustness performance. This method is referred as "K'-Q-K" procedure and is applied to active vibration control of Hexapod. Simulation results demonstrate the improved performance by using the proposed approach.**

I. INTRODUCTION

Repetitive Control is an effective feed-forward solution for exogenous periodic signals tracking and/or rejection with known period. Based on the Internal Model Principle (IMP) [1], a periodic signal generator, which generates infinitely large feedback gains at periodic signal's fundamental frequency and its harmonics, is placed into a stable closed-loop feedback control system to track or reject periodic signals asymptotically. Repetitive control has been widely used in many fields, including hard disk driver [2], [3], industrial robots [4], [5], non-circular turning [6], [7], satellite control [8], PWM inverter and rectifier [9], [10] and so on.

Repetitive control is often regarded as an "outer" loop controller which is usually designed after an "inner" loop controller which provides stability and performance with respect to other disturbances. This procedure is referred as "K-Q" design procedure in this paper. Hara et al. [11] presented the stability analysis of an infinite dimensional repetitive control system and introduced a low-pass filter filter to suppress the high-gain feedback at high frequencies. Similarly, Tsao and Tomizuka [12] introduced a zero-phase low-pass filter Q in the internal model to address robust stability in discrete time domain. The tradeoff between robustness and disturbance rejection performance is made in the design of Q filter. To get the optimal Q filter, graphical design method based on freqency domain analysis of linear interval system [13] and LMI method [14], [15] are proposed.

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However, due to the waterbed effect, the repetitive controller will unavoidably amplify some non-periodic disturbances in this "K-Q" procedure, and thus degrade system performance. To guarantee overall system performance, feedback controller needs to be designed in the presence of repetitive module, which is referred as "Q-K" procedure in this paper. Li and Tsao [16] proposed a Robust Repetitive Control (RRC) scheme based on structured singular value to satisfy overall system robustness performance. In this RRC scheme, a fictitious uncertainty is introduced to replace the long delay chain in the internal model of repetitive system. Based on the same compensator structure, a modified structure of Q filter is proposed to enhance performance at periodic disturbance's low frequency harmonics with higher order repetitive controllers [17]. Both of these two schemes, however, use a fixed bandwidth Q filter, which will limit periodic disturbance rejection performance at high frequency harmonics.

In this paper, we propose a Three-Step Robust Repetitive Control (TSRRC) scheme to reject more high frequency harmonics of periodic disturbances by extending the bandwidth of low-pass filter Q. An interim feedback controller "K' " is firstly designed by μ synthesis to realize the design of Q filter. Then, based on the Q filter, the feedback controller K' is redesigned as K to guarantee the robustness stability and performance. The remainder of the paper is organized as follows: In section 2, two repetitive controller design procedures,"K-Q" and "Q-K" procedure, and their weaknesses are addressed. Section 3 proposes TSRRC design approach. Section 4 shows a design example for active vibration control of a Hexapod system, which is followed by concluding remarks in section 5.

II. TWO RC DESIGN PROCEDURES

A. *"K-Q" design procedure*

The above mentioned "K-Q" design procedure is actually a two-step design method: 1) A feedback controller K is designed to provide stability and performance against uncertainties and disturbances before the introduction of repetitive control module; 2) A repetitive control module with Q filter is then designed and plugged in. The key in repetitive control module design is to determine the cutoff frequency of Q filter, which decides the tradeoff between robustness stability and disturbance rejection performance.

Figure 1 shows the traditional plug-in repetitive controller structure. Assuming that a feedback controller K has been designed to suppress the non-periodic part of disturbance d, only the repetitive control module design is considered here. The periodic signal generator consists of Q, Z^{-N+L} and Z^{-L}, where N is the period of disturbance and L is the plant delay. Without loss of generality, assume $N \gg L$. Q is a unity gain low-pass filter. K_g is learning gain and Φ is a compensator. K_g, Φ and Q are three parameters to be determined during the design of a repetitive control system.

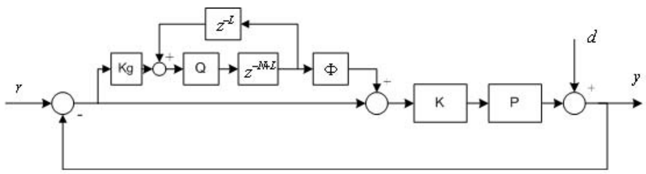

Fig. 1. Plug-in Repetitive Control Scheme

For a repetitive control system in Figure 1, the following is a basic result for stability:

Theorem 1: [11] Consider the repetitive control system in Figure 1. If

1) the roots of $1 + K(z)P(z) = 0$ are inside the unit circle;
2)

$$\|Q(z)(1 + K(z)P(z))^{-1}\|_\infty < 1,$$

$$\text{for all } z = e^{j\omega},\ \omega \in [0, \pi] \tag{1}$$

then the system is exponentially stable.

It is clear that the stability condition 2 in Theorem 1 will become milder as Q approaches zero, which implies that a Q filter with lower cutoff frequency will enhance system stability. However, lower cutoff frequency deteriorates periodic disturbances rejection performance and the repetitive control module become useless when Q=0. As a result, we have following remarks,

Remark 1: The design of low-pass filter Q represents the trade-off between periodic disturbance rejection performance and system robustness stability.

Remark 2: Theorem 1 addresses only robustness stability of repetitive control system, not robustness performance. Actually, the introduction of repetitive control module will degrade system performance since it will amplify the some non-periodic disturbances. This is the main weakness of the "K-Q" procedure.

B. "Q-K" design procedure

The basic idea of "Q-K" procedure is to consider repetitive control module as a component of augmented plant and design the feedback controller K to guarantee the overall system performance. To achieve robustness performance of a repetitive system, RRC scheme was proposed [16] in the framework of "Q-K" design procedure.

Figure 2 shows the design structure of RRC. Where P is the nominal plant model, $\Delta_r, \Delta_f, W_r, W_f \in \mathcal{RH}_\infty$ with $\|\Delta_r\|_\infty \leq 1$ and $\|\Delta_f\|_\infty \leq 1$. W_r and W_f are multiplicative and performance weighting functions, respectively. It is a simplified plug-in repetitive control structure with $\Phi = 1$, $K_g = 1$ and Q as a pre-determined zero-phase low-pass FIR filter, for example,

$$Q(z) = \frac{z + 2 + z^{-1}}{4} \tag{2}$$

Hence the feedback controller K can be designed according to this known repetitive control module.

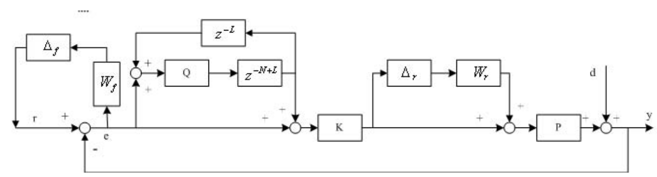

Fig. 2. Block Diagram of Robust Repetitive Control System

According to H_∞ optimal control theory [18], the order of controller is equal to the order of augmented plant. Because of the big delay N, the augmented plant including repetitive control module will have very high order. In order to reduce the order of controller, a fictitious uncertainty Δ_k is introduced to replace the big delay z^{-N+L} in the design. Suppose that Δ_k is stable and the norm of Δ_k is less than or equal to 1, i.e., $\Delta_k \in \mathcal{RH}_\infty$ and $\|\Delta_k\|_\infty \leq 1$, it is obvious that z^{-N+L} is an admissible element of Δ_k, i.e., $z^{-N+L} \in \Delta_k$.

Define an augmented block structure:

$$\Delta_{RRC} := \begin{bmatrix} \Delta_f & 0 & 0 \\ 0 & \Delta_r & 0 \\ 0 & 0 & \Delta_k \end{bmatrix}$$

By Linear Fractional Transformation (LFT), the repetitive control system in Figure 2 can be converted into the following form, where P_{aug} is the augmented plant of system.

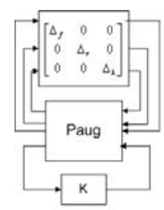

Fig. 3. LFT Form for Robust Repetitive Control Design

The robustness performance of repetitive system in Figure 3 is described by the following theorem:

Theorem 2: [16] Let $\beta > 0$ be a constant. For all $\Delta_r \in \mathcal{RH}_\infty$ with $\|\Delta_r\|_\infty < 1/\beta$, the loop in Figure 3 is well-posed, internally stable, and $\|\mathcal{F}_u(P_{aug}, \Delta_r)\|_\infty \leq \beta$ if and only if

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{RRC}}(P_{aug}(e^{j\omega})) \leq \beta \tag{3}$$

Remark 3: Since the low-pass filter Q is determined before the "Q-K" design procedure, the tradeoff between disturbance rejection and system robustness can not be made during the design. This is the main weakness of the "Q-K" procedure, including the RRC scheme.

III. Three-step Robust Repetitive Control Scheme

In this section, a Three-Step Robust Repetitive Control (TSRRC) scheme is proposed. The advantage of the proposed method lies in that Q filter can be designed according to the characteristic of system. Hence, the bandwidth of Q filter can be extended and periodic disturbance rejection performance at high frequencies can be enhanced.

The first step is to design a robust feedback controller K' (the prime here means it is an interim controller) without the repetitive control module. The role of K' is to ensure the design of Q in repetitive control module realizable. The second step is to design a Q filter to address robustness stability of system based on K'; In the third step, the feedback controller K is redesigned in the presence of the repetitive control module to replace the original K'. This way, system robustness performance can be achieved. This three-step method can be denoted as "K'-Q-K" procedure.

A. Step 1: K' design

A conventional robust optimal feedback controller is designed by μ synthesis [18] in this step. Consider system in Figure 4 before a repetitive control module is plugged-in. The LFT form of this system is shown in Figure 5, with G_p being the augmented plant. Define an augmented block structure:

$$\Delta_P := \begin{bmatrix} \Delta_f & 0 \\ 0 & \Delta_r \end{bmatrix}$$

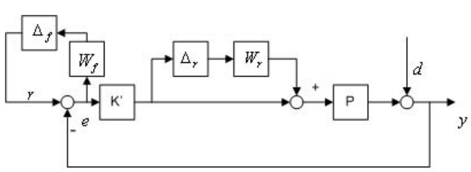

Fig. 4. K' Design in TSRRC

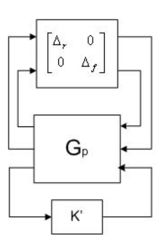

Fig. 5. LFT Form of K' Design

The following theorem gives the analysis result of robustness performance by structured singular value or μ-analysis:

Theorem 3: [18] Let $\beta > 0$. For all $\Delta_r \in \mathcal{RH}_\infty$ with $\|\Delta_r\|_\infty < 1/\beta$, the loop in Figure 5 is well-posed, internally stable, and $\|\mathcal{F}_u(G_p, \Delta_r)\|_\infty \le \beta$ if and only if

$$\sup_{\omega\in\mathbb{R}} \mu_{\Delta_P}(G_p(e^{j\omega})) \le \beta \tag{4}$$

B. Step 2: Q filter design

1) Q filter cut-off frequency determination: A repetitive control module is plugged in the system shown in Figure 4 by assuming $K_g = 1$ and $\Phi = 1$ as in RRC scheme, and the result is shown in Figure 6. The cutoff frequency of Q filter is the only parameter to be determined. Because of the existence of system uncertainty, we can not use Theorem 1 directly to design the Q filter. The following theorem gives a convenient Q design method to guarantee robustness stability of repetitive control system.

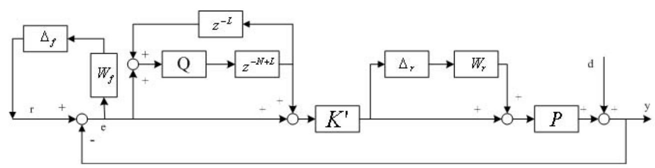

Fig. 6. Q design in TSRRC

Theorem 4: Let $\beta > 0$. For all $\Delta_r \in \mathcal{RH}_\infty$ with $\|\Delta_r\|_\infty < 1/\beta$, consider the repetitive control system in Figure 6. K' is an $\mathcal{H}_\infty$ stabilizing controller designed in step 1, which ensures that the system satisfies

$$\sup_{\omega\in\mathbb{R}} \mu_{\Delta_P}(G_p(e^{j\omega})) = \beta \le 1. \tag{5}$$

If

$$\|Q\| \le \|W_f\|/\beta \tag{6}$$

then the repetitive control system is robustly stable.

Proof: From the design of K', system robustness performance can be achieved, that is,

$$\sup_{\omega\in\mathbb{R}} \mu_{\Delta_P}(G_p(e^{j\omega})) = \beta \le 1$$

then $\|\mathcal{F}_u(G_p, \Delta_r)\|_\infty = \beta \le 1$. By the definition of LFT,

$$\|\mathcal{F}_u(G_p, \Delta_r)\|_\infty = \|\frac{W_f}{I + KP(I + W_r\Delta_r)}\|_\infty = \beta \le 1 \tag{7}$$

Combine (1) in Theorem 1 and (7), the relationship between performance weighting function W_f and Q, which is shown in (6), can be derived. ■

Remark 4: Performance weighting function W_f is actually the bound of sensitivity function S in presence of uncertainties, and thus the Q design based on this bound can guarantee robust stability of repetitive control system.

Remark 5: Since only the performance weighting function W_f is used in step 2, one can directly start from step 2 in real applications, without designing K'. In the following design example, however, we still start with the step 1 for easy understanding.

2) Zero-phase low-pass IIR filter implementation: In our proposed design, a zero-phase low-pass IIR filter is utilized, because it is easier to get sharp cutoff. IIR has nonlinear phase property which is not easy to config to zero-phase. However, using the filter backward in time can introduce a phase lead equal to its original phase lag, i.e., reverse the sequence of the signal to be filtered, pass it through the filter, then reverse the result again [19]. The backward implementation of filter is possible in repetitive control because of the N-steps delay.

C. Step 3: K design

The purpose of this step is to address the robustness performance of a system. The design method of robust controller K in this step is the same as the procedure of RRC design. The only difference lies in the different Q structures. The redesigned feedback controller K will replace K' in the system. The following theorem states that this replacement will not cause any instability of repetitive control system.

Theorem 5: Suppose an $\mathcal{H}_\infty$ stabilizing controller K can guarantee the robustness performance of the system in Figure 4, i.e.,

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{RRC}}(P_{aug}(e^{j\omega})) \leq 1$$

then the same controller can also guarantee the robustness performance of system in Figure 6, i.e.,

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_P}(G_p(e^{j\omega})) \leq 1$$

Proof: This can be proved by the main loop theorem [18] and is omitted here. ■

Remark 6: The repetitive control module in Figure 6 is considered as an uncertainty constraint during the design of $\mathcal{H}_\infty$ controller K. Hence the controller designed by the system in Figure 6 is more conservative than the one designed by Figure 4.

IV. Design Example

In this section, the proposed TSRRC design procedure will be applied to the active vibration isolation of a hexapod. Vibrations propagating in mechanical systems can cause many problems at different levels such as performance degradation.

Active vibration isolation methods can make use of a platform, such as the six-degree of freedom (6-DOF) active hexapod based on Stewart platform configuration [20]. In many applications, the vibrations contain substantial harmonics and a feedback controller is not so efficient in addressing each frequency individually. Repetitive control is very suitable for this situation and has been successfully applied to Ultra Quiet Platform [21].

A. System model and performance weighting

Since the cubic configuration [20] of Hexapod ensures the decoupled motions of six legs, we only focus on one leg for simulation study. The model of a leg can be estimated by frequency response using "swept sine" method. Figure 7 shows actual frequency response of one leg and the nominal plant model. The discrepancy between these two curves are plant uncertainty, which can be expressed as multiplicative form:

$$P_\Delta = P(I + \Delta_r W_r)$$

where P is the model and W_r is a uncertainty bound function, P_Δ is a set of plants which contain the actual plant. The uncertainty bound W_r can be selected as the upper bound of the differences between the model and the actual plant. The selection of W_r is shown in Figure 8.

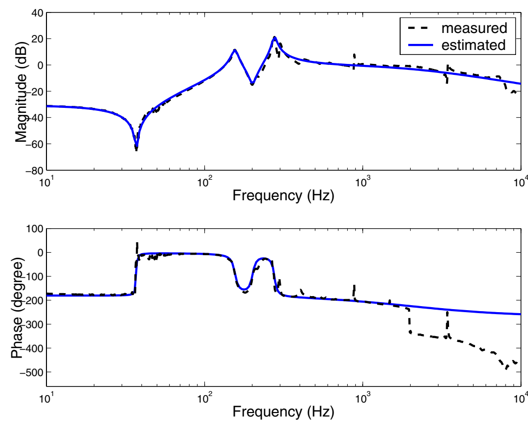

Fig. 7. Measured vs Estimated Model Frequency Response

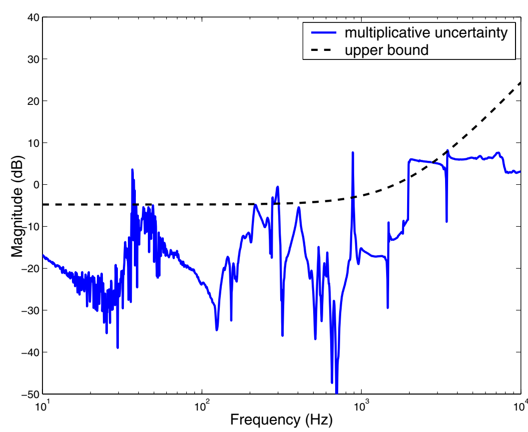

Fig. 8. Model uncertainty of plant

The design of the performance weighting function $W_f(e^{j\omega})$ depends on the performance requirement. In this simulation, the performance weighting function W_f is selected to improve the performance at low frequency. A second order low-pass filter with 4.4kHz cut-off frequency is applied.

B. Three step robust repetitive controller

1) Step 1: K' design: The interim robust feedback controller K' is designed by μ synthesis. The design result is shown in Figure 9. The order of controller has been truncated from 16 to 10 by balanced realization [18].

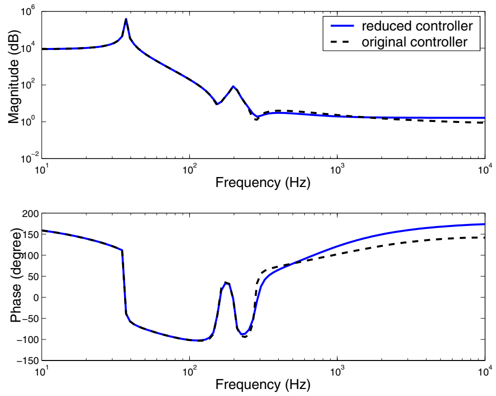

Fig. 9. Frequency Response of Robust Controller

Figure 10 shows the sensitivity function of closed-loop system. Since

$$\|W_f(e^{j\omega})S(e^{j\omega})\|_\infty < 1 \tag{8}$$

the system robust performance has been satisfied.

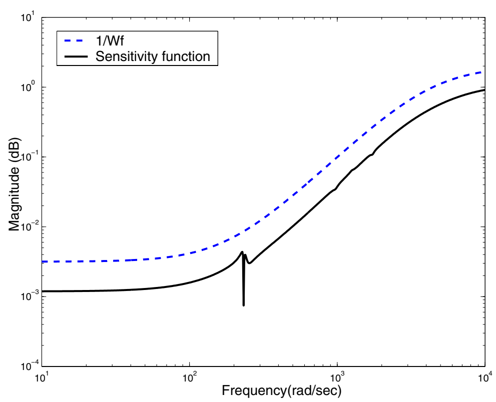

Fig. 10. Magnitude of Sensitivity and Performance Weighting Functions

2) Step 2: Q filter design: The cutoff frequency of Q filter can be determined by using Theorem 4. Since Q is a unity gain low-pass filter, only the cutoff frequency of the performance weighting function W_f, which is -3dB in bode diagram, need to be checked. From Figure 10, the cutoff frequency is determined as 4400Hz or 0.88 in normalized frequency based on the Nyquist frequency of 5000Hz. Figure 11 shows the comparison between proposed zero-phase low-pass IIR filter and fixed FIR filter of Equation (2) of RRC, in which IIR and FIR filter have the same order,

3) Step 3: K design: Redesign the feedback controller K to satisfy overall system robustness performance by μ-synthesis. The design result is shown in Figure 12. The

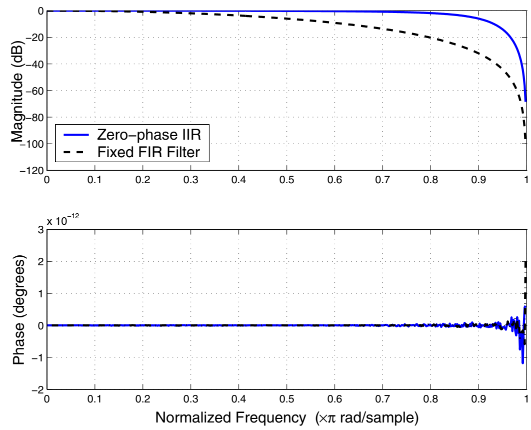

Fig. 11. Comparison of Two Filters

deep notch in sensitivity function means repetitive control module's extra suppression to periodic disturbances. Comparison between the proposed TSRRC scheme and the RRC scheme shows that our proposed method offers better periodic disturbance rejection performance at high frequency, which will be further illustrated in following simulation results.

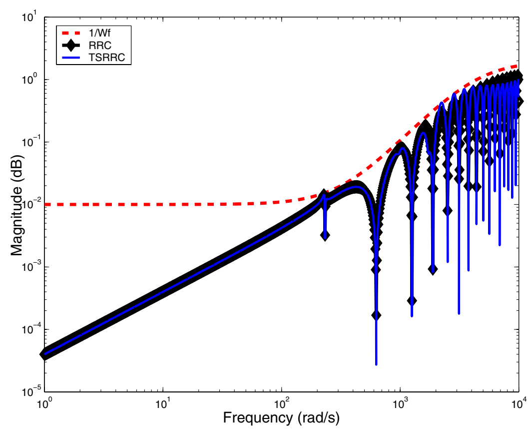

Fig. 12. Sensitivity Function Comparison of Two Schemes

C. Simulation Results

The comparison of the proposed TSRRC and the RRC is shown in Figures 13 and 14. System sampling frequency is 10kHz and the disturbance is 100Hz sinusoid signal and its 5th, 10th, 15th harmonics. Figures 13 is time response of each method. Feedback controller is switched on at time $t = 0.02s$ and the repetitive controller is added at time $t = 0.05s$.

The comparison of Root-Mean-Square (RMS) error is given in Figure 14. The cycle in this figure means period of fundamental frequency, which is 100Hz in this example. From Figure 14, we can see that the disturbances are suppressed about 5dB by robust feedback controller. After the repetitive control module being plugged in, the disturbances

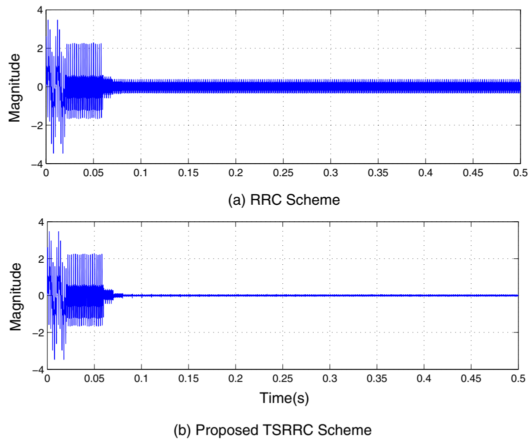

Fig. 13. Time Response of RRC and TSRRC schemes

are suppressed 20dB by RRC scheme and about 50dB by our proposed TSRRC scheme.

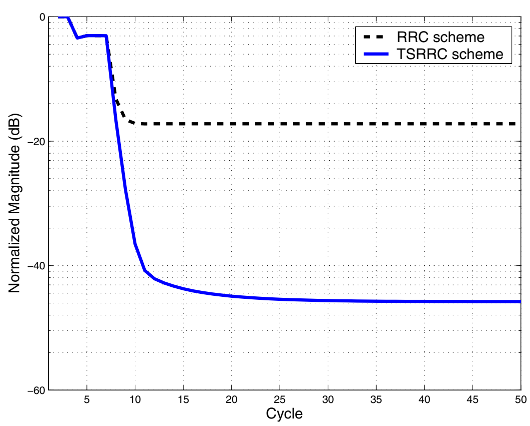

Fig. 14. RMS Comparison of Two Schemes

The different error levels of these two schemes are due to the different cutoff frequency of Q filter. The cutoff frequency of Q filter in RRC is 2kHz, which is near the highest disturbance harmonics (1.5kHz). Hence the repetitive control module with such Q filter partially loses the disturbance rejection capacity at this frequency. On the other hand, TSRRC scheme with 4.4kHz cutoff frequency rejects all the periodic disturbances.

V. Conclusion

A new three-step robust repetitive controller design method has been addressed in this paper. This method can extend the cutoff frequency of the Q filter, and hence, more periodic disturbance harmonics can be eliminated to get better performance. Both robustness stability and performance of system can be guaranteed by our proposed method. This method can be applied to either discrete or continuous time system with little changes. Simulation of active vibration control of Hexapod demonstrates that our proposed method can improve the performance greatly.

References

[1] B. A. Francis and W. M. Wonham. The internal model principle of control theory. *Automatica*, 12:457–465, 1976.

[2] K. K. Chew and M. Tomizuka. Digital control of repetitive errors in disk drive systems. *IEEE Contr. Syst. Mag.*, 10(1):16–20, 1990.

[3] Y. Onuki and H. Ishioka. Compensation for repeatable tracking errors in hard drives using discrete-time repetitive controller. *IEEE/ASME Transaction on Mechatronics*, 6(2):132–136, June 2001.

[4] H. L. Broberg and R. G. Molyet. Plug in repetitive control for industrial robotic manipulators. In *Proc. of the IEEE International conference on Robotics and Automation*, pages 1970–1975, 1990.

[5] N. Sadegh, R. Horowitz, W. W. Kao, and M. Tomizuka. A unified approach to design of adaptive and repetitive controllers for robotic manipulators. *Trans. of ASME: J. of Dynamic Systems, Measurement, and Control*, 112:618–629, 1988.

[6] T.-C. Tsao and M. Tomizuka. Adaptive and repetitive digital control algorithms for noncircular machining. In *Proceedings of American Control Conference*, pages 115–120, Atlanta, GA, 1988.

[7] T.-C. Tsao, M. Tomizuka, and K. Chew. Discrete-time domain analysis and synthesis of repetitive controllers. *ASME J. Dyn. Syst., Meas., Control*, 110:271–280, 1988.

[8] H. L. Broberg and R. G. Molyet. Correction of period errors in a weather satellite servo using repetitive control. In *1st IEEE Conference on Control Application*, pages 682–683, Dayton, OH, 1992.

[9] K. Zhou and D. Wang. Zero tracking error controller for three-phase CVCF PWM inverter. *Electronics Letters*, 36(10):864–965, 2000.

[10] K. Zhou and D. Wang. Digital repetitive learning controller for three-phase CVCF PWM inverter. *IEEE Trans. on Industrial Electronics*, 48(4):820–830, 2001.

[11] S. Hara, Y. Yamamoto, T. Omata, and M. Nakano. Repetitive control system: a new type servo system for periodic exogenous signals. *IEEE Transactions on Automatic Control*, 33(7):659–668, July 1988.

[12] T.-C. Tsao and M. Tomizuka. Robust adaptive and repetitive digital control and application to a hydraulic servo for noncircular machining. *ASME J. Dyn. Syst., Meas., Control*, 116:24–32, Mar 1994.

[13] J.-H. Moon, M.-N. Lee, and M.-J. Chung. Repetitive control for the track-following servo system of an optical disk drive. *IEEE Transaction on Control system technology*, 6(2):663–670, 1998.

[14] T.-Y. Doh, J. R. Ryoo, and M.-J. Chung. Repetitive controller design for track-following servo system of an optical disk drive. In *7th International Workshop on Advanced Motion Control*, pages 176–181, July 2002.

[15] T. Y. Doh and M. J. Chung. Repetitive control design for linear systems with time-varying uncertainties. *IEE Proc. Control Theory Appl.*, 150(4):427–432, July 2003.

[16] J. Li and T.-C. Tsao. Robust performance repetitive control system. *ASME J. Dyn. Syst., Meas., Control*, 123:330–337, Sept 2001.

[17] B.-S. Kim and T.-C. Tsao. Robust repetitive controller design with improved performance. In *Proceedings of the American Control Conference*, Arlington, VA, June 2001.

[18] K. Zhou, J. C. Doyle, and K. Glover. *Robust and optimal control*. Prentice-Hall, NY, 1996.

[19] R. W. Longman. Real time IIR zero-phase low pass filtering for robust repetitive control. In *Proceedings of the 6th International Conference on Control, Automation, Robotics and Vision*, Singapore, December 2000.

[20] Z. J. Geng and L. S. Haynes. Six degree of freedom active vibration control using the stewart platforms. *IEEE Transactions on Control Systems Technology*, 2(1):45–53, March 1994.

[21] S. G. Edwards, B. N. Agrawal, M. Phan, and R. W. Longman. Disturbance identification and rejection experiments on an ultra quiet platform. *Advances in the Astronautic Sciences*, 103:633–651, 1999.