2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Control of Plants which Change using Switching Controllers

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***Abstract*— A recent problem in adaptive control has been the task of controlling a system that cannot be characterized by a single mathematical model, and instead undergoes changes within a certain family of models. One strategy for controlling such a system has been to use switching between a fixed number of controllers to compensate for changing plant dynamics. This paper considers the problem of controlling an unknown plant contained in a family of models together with the additional task of controlling a plant whose plant dynamics shift from one member of the family to another; in particular, this paper describes a new switching controller that uses observers to improve switching transients, while maintaining an acceptable level of robustness.**

I. Introduction

Switching, as it relates to adaptive control, was first introduced in [1] and has since been studied in a multitude of different contexts. A recent area of focus has been the so-called 'family of plants' problem [2], [3] wherein the system to be controlled is characterized by one member of a known family of plant models, $P \in \mathcal{P}$. In this case, it is possible to establish a corresponding set of controllers $\mathcal{K}$ such that for any moment in time, within $\mathcal{K}$ there will be a suitable controller available for the system. A switching scheme was developed in [3], using observers with each of the plant models, to select which controller, amongst the bank of candidates, to apply to the system. It appears that this was the first scheme involving observers in adaptive switching control. In [2], a revised switching controller was presented that used less a priori plant information than [3] while demonstrating certain levels of integrity and robustness. An outstanding question relates to the ability of these schemes *to control not only an unknown plant $P \in \mathcal{P}$, but also the additional situation where at certain points in time, the plant dynamics shift from one member of the family to another i.e. when $P \rightarrow P_* \in \mathcal{P}$*. It will be shown that the controller developed in [2] does indeed maintain control under such circumstances, a fact that directly motivates the development of the new switching controller presented herein.

A second area of focus is in so-called Multiple Model Adaptive Control (MMAC), where banks of models are used to render a switching decision. In [4] and [5] the system to be controlled is assumed to lie within a certain parameter-space and a fixed number of identification models are used to yield error signals which are subsequently used to weight inputs from a bank of controllers. Such a scheme may, however, suffer from high susceptibility to modeling errors and unanticipated disturbances. The controller suggested in this note uses certain ideas from both MMAC and the 'family-of-plants' scheme to yield the same level of robustness as in [2], while providing significant improvement in transient response under nominal conditions; the focus of this proposed controller is in controlling LTI plants which have significant changes in the plant's dynamics.

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II. Preliminaries

Let us assume that the plant to be controlled, P_i, exists within a finite family of plants $\mathcal{P}$, such that $P_i \in \mathcal{P}$. Then let $\mathcal{K}$ denote a set of corresponding controllers, such that $\kappa_i \in \mathcal{K}$ provides acceptable reference tracking and disturbance rejection for P_i under bounded piecewise constant reference and constant disturbance inputs, where P_i is of the form:

$$\begin{aligned} \dot{x} &= A_i x + B_i u + E_i w \\ y &= C_i x + D_i u + F_i w \\ e &:= y_{ref} - y \end{aligned} \tag{1}$$

and κ_i is of the form:

$$\begin{aligned} \dot{\eta} &= G_i \eta + H_i y + J_i y_{ref} \\ u &= K_i \eta + L_i y + M_i y_{ref} \end{aligned} \tag{2}$$

where $x \in \mathbf{R}^{n_i}$ is the state, $u \in \mathbf{R}^m$ is the control input, $w \in \mathbf{R}^q$ is an unmeasurable disturbance, $\eta \in \mathbf{R}^{g_i}$ is the controller state, $y \in \mathbf{R}^r$ is the output to be regulated, and y_{ref} is the specified reference input. Here it is, in general, assumed that E_i and F_i are unknown disturbance gain matrices.

A. Notation

The ∞-norm of a vector $x \in \mathbf{R}^n$ will be denoted by:

$$\|x\|_\infty = \|x\| := \max_{1 \le i < n} |x_i| \tag{3}$$

Additionally, a function $f : \mathbf{R}^+ \rightarrow \mathbf{R}^n$ will be said to lie in $\mathcal{L}_\infty$ if $\|f\|$ is finite, where:

$$\|f\|_\infty = \|f\| := \sup_{t > 0} |f(t)|$$

B. Assumptions

We assume herein that a solution to the robust servomechanism problem for the plant P_i exists for the class of constant reference input signals y_{ref} and for the class of constant disturbance signals w such that the following conditions [6] hold:

i) (C_i, A_i, B_i) is stabilizable and detectable; and

ii) $rank \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = n_i + r$

In particular, it is assumed that the controller κ_i is obtained to solve this robust servomechanism problem for P_i.

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C. Switching Controller 1

Controller 1, developed in [2] is given below:

$$\begin{array}{l}(G(t), H(t), ..., M(t)) := (G_i, H_i, ..., M_i) \\ t \in (t_k, t_{k+1}]\end{array} \tag{4}$$

where $\eta(t_k^+) \equiv 0$ and $i := ((k-1) \bmod^1 s) + 1$, noting that $(G_i, H_i, ..., M_i)$ corresponds to κ_i as in (2), and s is the size of $\mathcal{K}$. The switching times are defined by:

$$t_k = \begin{cases} \min t \ni & \\ \text{(i)} \quad t > t_{k-1} & \min \text{exists} \\ \text{(ii)} \ \|\eta(t) \ \ e_f(t)\| = f(k-1) & \\ \infty & \text{otherwise} \end{cases} \tag{5}$$

where e_f is the output of a filter given by:

$$\dot{e}_f(t) := -\lambda e_f(t) + \lambda e(t) \tag{6}$$

Here, $f(k)$ is a member of the class of bounding functions, denoted $\mathbf{BF}$, such that $\forall\,(c_0, c_1, c_2) \in \mathbf{R}^{3+}$, as $i \to \infty$:

$$\frac{f(i)}{c_0 + c_1(i-1) + c_2 \sum_{j=1}^{i-1} f(j)} \to \infty \tag{7}$$

Theorem 1: Let $P \in \mathcal{P}$, with Controller 1 applied at $t = 0$. Assume that $\|\eta(0)\| < f(1)$ and $\|e_f(0)\| < f(1)$. Then for all $f \in \mathbf{BF}$ and $\lambda \in \mathbf{R}^+$, and all bounded piecewise continuous reference and disturbance signals, the closed loop system has the properties that

i) there exists a finite time $t_{ss} > 0$ such that $\kappa_i = \kappa_{ss} \in \mathcal{K}$ for all $t > t_{ss}$
ii) the controller states $\eta \in \mathcal{L}_\infty$, the plant states $x \in \mathcal{L}_\infty$ and the filtered error states $e_f \in \mathcal{L}_\infty$
iii) for almost all controller parameters, the error $e(t) \to 0$ as $t \to \infty$

Proof: The details of the proof are given in [2] and are omitted here. ■

The following proposition provides the basis for the main result to follow.

Proposition 1: Controller 1 maintains the properties of Theorem 1 in the presence of plant changes within $\mathcal{P}$. That is, when $P_i \to P_j$ at some $t > 0$.

Proof: Assume, without loss of generality, that at $t = t_p$ the plant transitions from $P_2 \to P_3$ and the value of k is some $k_p > 1$. Then by definition $\left\|\eta(t_p^+)\right\| < f(k_p - 1)$ and $\left\|e_f(t_p^+)\right\| < f(k_p - 1)$. Hence, if we consider t_p^+ as a new start point, (i.e. abusing notation, let $t_p^+ \equiv 0$ and $k_p^+ \equiv 2$), then the assumptions of Theorem 1 hold, and the result follows directly. ■

To demonstrate the use of Controller 1 in this regard, we use the 3-member plant and controller families introduced in [3] (also [2]), which we denote by $\mathcal{P}_a$ and $\mathcal{K}_a$ respectively.

P_1:

$$\dot{x} = \begin{bmatrix} -0.75 & 7.25 \\ -2.25 & -8.25 \end{bmatrix} x + \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix} u + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} w$$
$$y = \begin{bmatrix} -1.2 & 4.1 \end{bmatrix} x$$

[1] $a \bmod b = a - \text{floor}(\frac{a}{b}) \times \text{b}$

P_2:

$$\dot{x} = \begin{bmatrix} 1.4 & 21.1 \\ -1.65 & -25.4 \end{bmatrix} x + \begin{bmatrix} -0.18 \\ 0.33 \end{bmatrix} u + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} w$$
$$y = \begin{bmatrix} -0.45 & 7.0 \end{bmatrix} x$$

P_3:

$$\dot{x} = \begin{bmatrix} -1.9 & 10.9 \\ -1.3 & -12.2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} w$$
$$y = \begin{bmatrix} 2 & -5 \end{bmatrix} x$$

The set of servomechanism controllers $\mathcal{K}_a$ is given by:

$$\begin{array}{l} \kappa_1 : \dot{\eta} = e, u = -2.75\eta \\ \kappa_2 : \dot{\eta} = e, u = -2\eta + 7y \\ \kappa_3 : \dot{\eta} = e, u = 25\eta - 6y \end{array}$$

It should be noted that in this example, all plant-controller mismatches result in a closed loop unstable system. We are interested in studying the case when the system changes within the family $\mathcal{P}_a$. As such, we examine the case when $P_1 \to P_2 \to P_3$. Controller 1 is applied to the system, with plant shifts at $t = 15s$ and $t = 35s$. For simplicity, the unfiltered $e(t)$ is used directly [2], with the bounding function f defined by:

$$f(k) := \begin{cases} 15k & 1 \le k < 10 \\ \left(\frac{k}{6}\right)^2 \exp\left(\frac{k}{6}\right)^3 & k \ge 10 \end{cases}$$

Here, the bounding function f grows linearly over the first ten iterations to provide a higher sensitivity and speed of switching response. It then assumes the more conservative exponential form shown, which fulfills the mathematical requirement provided in (7).

The control objective is to track a constant reference of $y_{ref} = 10$, with a constant disturbance of $w = 2$. As is shown in Figure 1, the controller successfully provides tracking for the system through the different operating zones. In particular, the controller switches as required, at $t = 20.41$ ($\kappa_1 \to \kappa_2$) and $t = 36.30$ ($\kappa_2 \to \kappa_3$). Clearly, however, there are large and potentially undesirable transients associated with this switching controller. This can

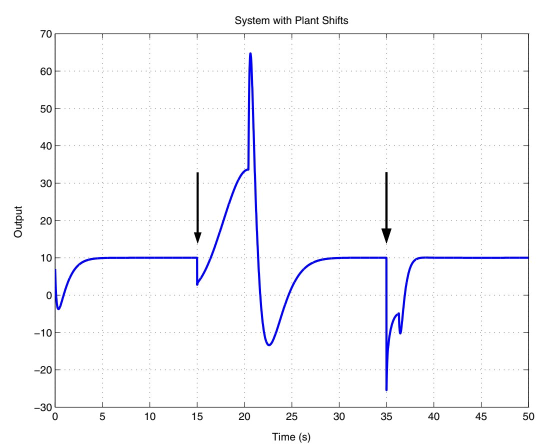

Fig. 1. Controller 1 applied to system with 2 plant shifts: $P_1 \to P_2$ ($t = 15s$), $P_2 \to P_3$ ($t = 35s$). $y_{ref} = 10$

be attributed to the fairly conservative bounding function used in the switching decision. In the main result, a method will be proposed that uses observers to render a less conservative switching trigger, and hence improve transient response under nominal conditions.

III. Main Result

Note that in developing $\mathcal{K}_a$, no mathematical knowledge of the plants in $\mathcal{P}_a$ is required. Rather, we only need to demonstrate the aforementioned property that there exists a $\kappa_i \in \mathcal{K}$ that solves the robust servomechanism problem for $P_i \in \mathcal{P}_a$. This is important, because it implies that existing controllers for a plant, which have been found using tuning procedures or other heuristic methods can be used. If however, in the process of designing these controllers, one can obtain some mathematical characterization of the various P_i, then it is possible to make dramatic gains in performance of the switching controller.

For our revised switching scheme, assume that we can establish a set of state observers $\mathcal{O}_a$ such that for each $P_i \in \mathcal{P}_a$ there exists an $O_i \in \mathcal{O}_a$ to provide adequate state observation. Such a set may be obtained by using any of the common strategies of observer design. For our example we define $\mathcal{O}_a$ to be the following.

O_1:

$$\dot{\hat{x}} = \begin{bmatrix} -18.84 & 69.07 \\ -0.064 & -16.16 \end{bmatrix} \hat{x} + \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix} u + \begin{bmatrix} -15.08 \\ 1.93 \end{bmatrix} y$$

$$\hat{y} = \begin{bmatrix} -1.2 & 4.1 \end{bmatrix} \hat{x}$$

O_2:

$$\dot{\hat{x}} = \begin{bmatrix} -4.75 & 116.8 \\ -1.34 & -30.25 \end{bmatrix} \hat{x} + \begin{bmatrix} -0.18 \\ 0.33 \end{bmatrix} u + \begin{bmatrix} -13.7 \\ 0.69 \end{bmatrix} y$$

$$\hat{y} = \begin{bmatrix} -0.45 & 7.0 \end{bmatrix} \hat{x}$$

O_3:

$$\dot{\hat{x}} = \begin{bmatrix} -19.44 & 54.74 \\ 0.046 & -15.56 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u + \begin{bmatrix} 8.77 \\ -0.67 \end{bmatrix} y$$

$$\hat{y} = \begin{bmatrix} 2 & -5 \end{bmatrix} \hat{x}$$

Let $\hat{y}_i$ denote the i^{th} observer output, and $\hat{e}_i = y - \hat{y}_i$ the corresponding observer error, where y is the actual system output. Let $f \in \mathbf{BF}$ and $t \in [nT_1, (n+1)t)$. Finally, define the accumulated error:

$$J_i(t) = \int_{nT_1}^{t} e_i^2(\tau) d\tau \, , i = 1, 2, ..., s \tag{8}$$

Here, T_1 is a predefined interval defining the rate at which the error signals $J_i(t)$ are reset, and $n = max(\mathbf{N}^+)$ such that $nT_1 < t$. We may now opt to define a second set of controllers Γ, such that $\gamma_i \in \Gamma$ offers additional performance under nominal conditions, and where γ_i is the controller to be applied when $\hat{e}_i$ is minimal. We thus amend the switching criteria (5) as follows, with $t_1 = 0$:

A. Revised Switching Criterion

$$t_k = \begin{cases} \min t > t_{k-1}, t - t_{k-1} > T_2 \ni & \\ \min\limits_{i\in[1,s]} (J_i(t)) \neq \min\limits_{i\in[1,s]} (J_i(t^-)) & k < N \\ \|\eta(t) \ \ e_f(t)\| \geq f(k-N) & k > N \\ \text{else } \infty \text{ if no such min exists} & \end{cases} \tag{9}$$

where $T_2 > 0$ and N is defined by:

$$t_N = max(t_k) \ni \|\eta(t_k) \ \ e_f(t_k)\| < f(1) \tag{10}$$

The control signal is then given by:

$$u = g_i(x, y, y_{ref}) \, , t \in (t_k, t_{k+1}] \tag{11}$$

where for $k < N$, $g_i(x, y, y_{ref})$ corresponds to γ_i, the i^{th} high performance controller such that $J_i(t_k)$ is minimal over $i = 1, 2, ..., s$. For $k > N$, $g_i(x, y, y_{ref})$ corresponds to the servomechanism controller κ_i defined in (2), with $i = (k - N) \bmod s$ and $\eta(t_k^+) \equiv 0$. A schematic block diagram of the new switching controller is given in Figure 2.

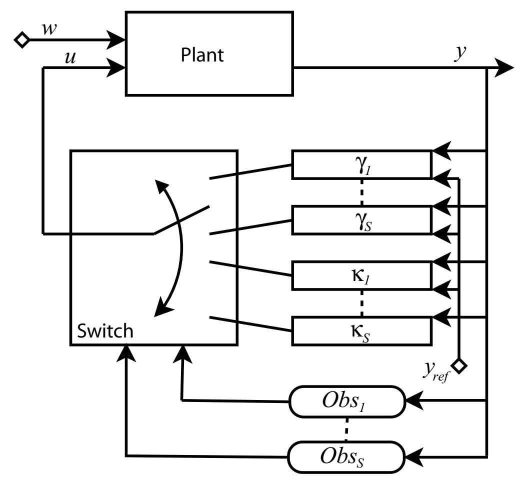

Fig. 2. Schematic block diagram of the proposed 'observer-based' switching controller

There are s state observers operating in parallel, with a maximum of $2s$ controllers (sets Γ and $\mathcal{K}$). Note that at the time of a switch, the state of the servomechanism controllers are reset to 0, leading to smaller transients [2]. The switch itself is comprised of two levels of conditioning. The first sorts the accumulated observer error signals $J_i(t)$ in order to decide which γ_i to apply. The second monitors $\eta_i(t)$ and $e_f(t)$ with respect to the bounding function f in order to detect possible errors or disturbances. Once the second level is triggered, the result in the first level is irrelevant as the switch only applies controllers from $\mathcal{K}$ via the cyclic action of Controller 1.

Remark 1: The new scheme uses a bank of observers to perform identification and controller application under nominal conditions. For all time $t < t_N$ the switching controller applies the control signal corresponding to the controller

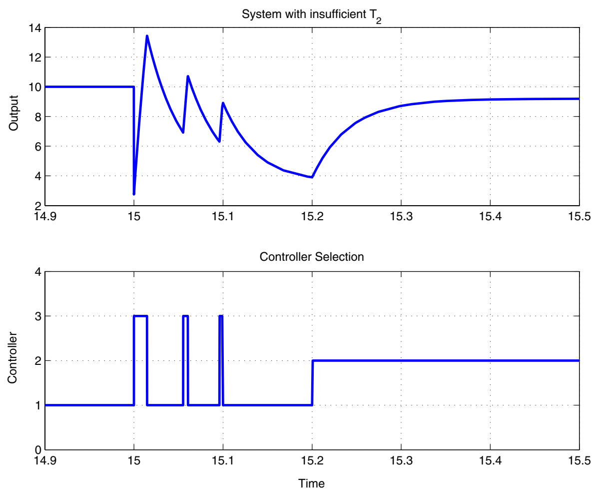

Fig. 3. System $P_1 \to P_2$ at $t = 15s$. Rapid switching transient, as a result of insufficient T_2.

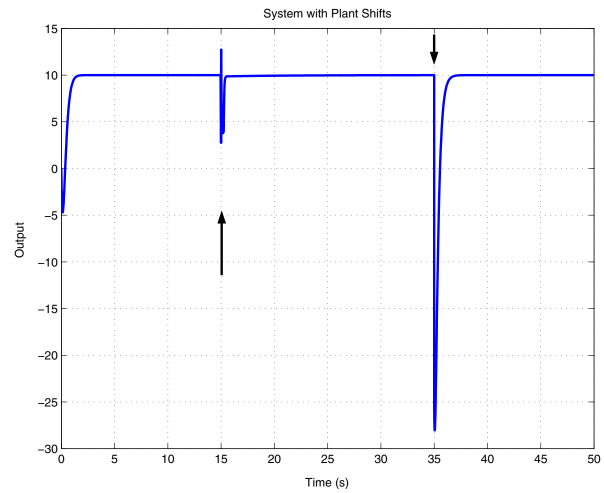

Fig. 4. New controller applied to system with 2 plant shifts: $P_1 \to P_2$ ($t = 15s$), $P_2 \to P_3$ ($t = 35s$). $y_{ref} = 10$

γ_i associated with the smallest accumulated observer error $J_i(t)$, with the constraint that switches may only occur with a maximum frequency dictated by T_2. In the event of an unmodeled disturbance or unanticipated perturbation, the system reverts to the more conservative and robust Controller 1. In this case, the servomechanism controller κ_i is applied as governed by the bounding function f, defined in (7).

Remark 2: It is not necessary to establish the second family of controllers Γ. Indeed there may be many cases where it is useful for $\gamma_i = \kappa_i$. However, there may be situations where the nominal operating conditions of a system offer certain exploitable control characteristics. In these cases, establishing a second bank of controllers may provide performance improvements.

Remark 3: The choice of intervals T_1 and T_2 may be dictated by hardware or other implementation constraints. Generically, shortening these intervals will result in a faster switching response, however this would also yield increased susceptibility to small disturbances. In particular, a choice of T_1 that is too small may lead to a series of signals $J_i(t)$ that are numerically insignificant in making a worthwhile switching decision. This is particularly true in systems involving limited precision or systems that have a certain amount of inherent uncertainty. The parameter T_2 ensures that some time is given after a switch before the system may render another switching decision. As is illustrated in Fig. 3, an insufficient T_2 may lead to switching transients that rapidly transition between candidates before settling on the correct controller. It is important to note also, that an overly conservative choice of T_2 will lead to potential delays in the switching system. In the context of our example system, T_1 and T_2 are chosen to allow ample time for $J(t)$ to accumulate, yet are sufficiently small so as to marginalize the effect of any induced delays.

Figure 4 illustrates the use of the new controller, where γ_i are feedforward tracking controllers with state feedback, of the form illustrated below.

$$u = Kx + Ny_{ref}$$

Here K is chosen to meet a quadratic performance index and N is a scaling factor to eliminate error in the steady state. Note that these γ_i are designed simply to illustrate the situation described in Remark 2, where a second set of controllers is used in addition to κ_i the servomechanism controllers of $\mathcal{K}_a$. The same simulation parameters as before are applied. The switching parameters T_1, T_2 are set as $T_1 = 0.25s$ and $T_2 = 0.1s$.

As is shown, the revised controller yields an improved transient response between plant shifts. In fact, and perhaps not surprisingly, the controller switches after just one sampling interval in each case, at $t = 15.25s$ ($\gamma_1 \to \gamma_2$) and $t = 35.25s$ ($\gamma_2 \to \gamma_3$). The following proposition shows that in addition to the improved transient response, the new controller preserves the robustness of Controller 1.

Proposition 2: Consider a system characterized by a plant P_i in a family $\mathcal{P}$, with dynamics that shift within this family. Assume that such a shift occurs from $i : m \to n$ at $t = t_k$, and assume that Controller 1 with the Revised Switching Criterion (9) is applied at time $t = 0$ with $f \in \mathbf{BF}$, $\lambda \in \mathbf{R}^+$, $\|\eta(0)\| < f(1)$ and $\|e_f(0)\| < f(1)$. Then for every bounded piecewise continuous reference and disturbance signal, the closed loop system has the following properties:

i) the controller states $\eta \in \mathcal{L}_\infty$, the plant states $x \in \mathcal{L}_\infty$ and the filtered error states $e_f \in \mathcal{L}_\infty$
ii) there exists a finite time $t_{ss} \geq t_k$ such that if $P_i = P_m$ for $t > t_k$ then $g_i(x, y, y_{ref}) = g_{ss}(x, y, y_{ref}) \in \Gamma \cup \mathcal{K}$ for all $t \geq t_{ss}$.
iii) if $P_i = P_m$ for $t > t_k$, then for constant reference and disturbance inputs, and almost all controller parameters, asymptotic error regulation occurs such that

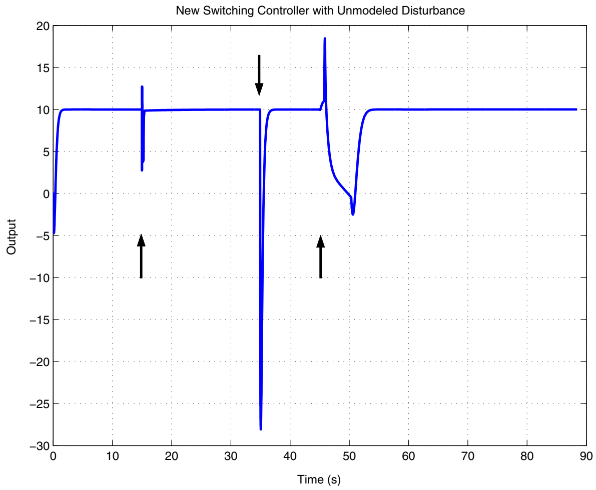

Fig. 5. New controller applied to system with 2 plant shifts: $P_1 \rightarrow P_2$ ($t = 15s$), $P_2 \rightarrow P_3$ ($t = 35s$). Unmodeled disturbance $w = 10$ applied at $t = 45s$. $y_{ref} = 10$

$e(t) \rightarrow 0$ as $t \rightarrow \infty$.

Proof: Property i) follows directly from Proposition 1, noting that $\|\eta(t)\ e_f(t)\|$ may never exceed the bounds dictated by f and the switching criterion (5). Property ii) is true by Proposition 1 and Theorem 1, with the assumed caveat that the plant must remain constant after a certain time t_k. From properties i) and ii) and Theorem 1, it is implied that iii) must hold, noting that in the worst case, the controller must switch to some servomechanism controller $\kappa_{ss} \in \mathcal{K}$ at $t = t_{ss}$ ∎

Figure 5 illustrates the robustness of the new controller, in particular demonstrating disturbance rejection properties. The system is simulated as before, with plants shifts at $t = 15s$ and $t = 35s$. At $t = 45s$ an unmodeled disturbance of $w = 10$ is applied such that γ_3 will no longer provide adequate tracking. As designed, the controller successfully reverts to the servomechanism set $\mathcal{K}_a$ and assumes tracking of $y_{ref} = 10$. The switching times are $t = 45.77$ ($\gamma_3 \rightarrow \kappa_2$) and $t = 50.32$ ($\kappa_2 \rightarrow \kappa_3$), where both switches are induced by the bound on $\eta(t)$.

IV. Conclusions and Future Work

In this paper, a new switching controller is presented that uses ideas from Multiple Model Adaptive Control to improve the transient response in a family of plants problem, where the system in question experiences shifts in dynamics within the noted family. Specifically, the controller uses observers for error-based identification and controller application. Importantly, the controller maintains an acceptable level of robustness to modeling errors and disturbances by using a bank of servomechanism controllers and a conservative bounding function trigger. This has been successfully demonstrated by means of a simulation study on a family of 3 LTI MIMO systems.

This work suggests a few avenues for further increasing switching performance and robustness. Additionally, it would be very desirable to further reduce the amount of a priori plant information required to design the controller. To this end, one potential strategy might include the use of unmixing sets [1], [7], [8] and self-tuning universal-type controllers.

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