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Unknown Input Observer of A Class of Switched Control Systems

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Abstract— **For a class of linear switched control systems(LSCSs) with unknown inputs, we study in this paper the unknown input observer(UIO) design problem. To design a full(reduced) order UIO, we first design for each subsystem a full(reduced) order UIO, and then we construct a full(reduced) order UIO for the whole system via picking the corresponding full(reduced) order UIO of the active subsystem according to the switching rule. Sufficient conditions for the existence of both full order and reduced order UIOs are derived. The design procedures of full order UIOs and reduced order UIOs are presented. Several stability results regarding to the state estimation error dynamics are established. Examples are given to show how to design UIOs for particular switched systems, and simulation results are given to show the effect of the designed UIOs.**

I. INTRODUCTION

As a special class of hybrid systems, switched systems have received lots of attention. The stability and stabilization problems have been studied extensively and fruitful results are now available. Detailed achievements in this research field can be found in survey papers by Decarlo, Branicky, Pettersson, and Lennartson [1] and Liberzon and Morse [2].

Unlike the stability and stabilization problems, the observer design problem for LSCSs received less attention and only a few results are available.

Some researchers have designed switching observers for non-switched systems, their main idea is to use switching to solve observer design problem for more complex systems, see [4], and/or to improve estimation performance, see [5]. The full order observer design problem for switched control systems has also been studied recently. An observer for continuous-time linear switched control system is designed in [6] based on coprime factorization approach. The main idea is to construct a common observer for all subsystems. Inspired by the common Lyapunov function method for stabilization problem of switched control systems [2], observers are designed for discrete-time linear switched control systems in [7] and for both continuous- and discrete-time linear switched control systems in [8]. The design problem is reduced to solve a group of linear matrix inequalities(LMI's) for a common solution. The advantage of this observer design is that the stability of state estimation error dynamics can be guaranteed for arbitrary switching sequences. The problem is that the common Lyapunov function may not exist for some cases. To circumvent this problem, [10] proposed observer designs for continuous time linear switched control systems. By extending the conventional observer design in a straightforward way and without using the common Lyapunov function technique, the designed observers can ensure the state estimation error go to zero asymptotically under some conditions.

In the literature, UIO design problem for classical systems was received great attention. Many results are now available, see [11] and [12] and the references listed therein. UIO design in classical systems has proved to be very useful in fault detection and isolation in fault diagnosis research community. When one uses multiple linear switched systems to approximate general linear systems, the modelling error can be naturally regarded as unknown input in each linear subsystems. Besides this, for real systems described by switched linear systems, external disturbances are often unavoidable. Motivated by these observations and realized that no UIO has been designed for LSCSs, we are trying to extend the traditional UIO design techniques to LSCSs in a straightforward way. Sufficient conditions are derived for the existence of both full order and reduced order UIOs. Design methods for both full order and reduced order UIOs are then given. It has been found that UIO design for LSCSs becomes much more difficult. More restrictions on the observer gains have to be required.

The rest of paper is arranged as follows. In Section 2, we introduce the model of LSCSs. In Section 3, we present existence conditions of UIOs for LSCSs and derive some sufficient conditions under which the state estimation error dynamics is asymptotically stable. In Section 4, we present methods to design UIOs. In Section 5, we give some examples and simulation results. Conclusion remarks are made in the last section.

II. LINEAR SWITCHED CONTROL SYSTEMS

We consider a class of **switched linear control systems(SLCSs)** with M subsystems described as

$$\begin{aligned} \dot{x} &= A_{\sigma(t)}x + B_{\sigma(t)}u + D_{\sigma(t)}v, x \in R^n \\ y &= C_{\sigma(t)}x, \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^k$,$v \in R^m$, and $y \in R^p$ are the state vector, known input vector, unknown input vector and the output vector of the system, respectively. A_i, B_i, D_i, and C_i with $i \in S = \{1, 2, \cdots, M\}$ are $n \times n$, $n \times k$, $n \times m$, and $p \times n$ constant matrices, respectively. $\sigma(t) : [0, \infty) \to S$ is a piecewise constant function of time and/or outputs, called a **switching rule**. The corresponding system for $\sigma(t) = i \in S$ is called the $i-$th subsystem. In such a case, we also say that the $i-$th subsystem is "active".

Assumptions

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A1– The switching rule $\sigma(t)$ is fixed and can not be designed freely. The state in (1) is continuous, that is, the state does not jump at the switching instants.

A2– $p > m$, rank $D_i = m$ and rank $C_i = p$ for all $1 \leq i \leq M$.

A3– rank $C_i D_i = m$ for all $1 \leq i \leq M$.

Remark 1: ¿From [11], we know that Assumptions A3 is necessary for each subsystem(which is a nonswitching linear system) to admit a UIO(both full and reduced order ones). To design a UIO for an SLCS, it is natural for us to require this condition.

III. CONDITIONS FOR THE EXISTENCE OF UIO

In this section, we will explore under what conditions a full order UIO and a reduced order UIO exist for system (1), and also under what sufficient conditions the state estimation error dynamics is exponentially stable.

Definition 1: A UIO is said to be an exponential UIO(EUIO) if the state estimation error tends to zero exponentially.

A. Full Order UIO

For each i, following [11], the full order observer which could be used as a UIO is described as

$$\begin{aligned} \dot{z}_i &= N_i z_i + G_i u + L_i y \\ \hat{x}_i &= z_i - E_i y \end{aligned} \tag{2}$$

Combining Theorem 1 and Theorem 2 in [11], we have the following necessary and sufficient condition for the existence of EUIO of the $i-$th subsystem.

Theorem 1: The $i-$th subsystem admits an EUIO of the form (2) if and only if there exists a matrix E_i such that

(1) $E_i C_i D_i = -D_i$;

(2) the pair $(P_i A_i, C_i)$ is detectable.

where $P_i = I + E_i C_i$.

If (2) can be designed for all i, a full order UIO for SLCS (1) could be designed as.

$$\begin{aligned} \dot{z} &= N_{\sigma(t)} z + G_{\sigma(t)} u + L_{\sigma(t)} y \\ \hat{x} &= z - E_{\sigma(t)} y \end{aligned} \tag{3}$$

where at any switching instant t_s, we let $z(t_s) = lim_{t \to t_s^-} z(t)$ such that z is continuous.

So, the existence of the full order UIO given by (3) for (1) is guaranteed by the existence of EUIOs for all its subsystems. The problem is: Is it necessarily an EUIO? Generally, without additional conditions, the answer is no. In what follows, we will study under what conditions it can be an EUIO.

Let's define the sate estimation error as

$$e = \hat{x} - x = z - x - E_{\sigma(t)} y \tag{4}$$

Since the pair $(P_i A_i, C_i)$ is detectable, we can choose K_i such that $N_i = P_i A_i - K_i C_i$ is Hurwitz. Now, if we let $L_i = K_i - N_i E_i$ and $G_i = P_i B_i$, and noticing that $P_i = I + E_i C_i$ and $P_i D_i = 0$, it follows from (1)-(3) that

$$\dot{e} = N_{\sigma(t)} e \tag{5}$$

If $e(t)$ is continuous, this is a standard switched system with Hurwitz stable subsystems. Regarding to its stability, there are many results available, see for example, those cited in [1] and [2]. Now we need to see whether the continuity of $e(t)$ can be guaranteed by (3). From the definition of $e(t)$, we know that it is not necessarily continuous. Therefore, we need the following condition to ensure the continuity of $e(t)$.

A4– $E_1 C_1 = E_2 C_2 = \cdots = E_M C_M$.

¿From (4), we see that $A4$ ensures the continuity of $e(t)$ because of the continuity of $z(t)$ and $x(t)$.

In the sequel, we present two results regarding to the stability of (5) under assumption $A4$. The first one is based on the concept of **dwell time** [2]; the second one is based on common Lyapunov approach.

With the help of the concept of **dwell time**[2], we give a result below.

Theorem 2: Under assumptions A1-A3, assume that conditions in Theorem 1 are satisfied for all $1 \leq i \leq M$ such that A4 is true, if the **dwell time** τ is large enough, then the state estimation error dynamics (5) is globally exponentially stable.

Proof: The continuity of $e(t)$ is ensured by assumptions A1 and A4.

For each $i \in \{1, 2, \cdots, M\}$, because N_i is Hurwitz, there exist $a_i \geq 0$ and $\lambda_i > 0$ such that for all $t \geq 0$ we have

$$\|e^{N_i t}\| \leq e^{a_i - \lambda_i t} \tag{6}$$

where $\|N\| = \sqrt{\lambda_{max}(N^T N)}$ and T denotes the transpose. Let

$$\tau_0 > \max_{i=1,2,\cdots,M} \{\frac{a_i}{\lambda_i}\}$$

and

$$a = max_{i=1,2,\cdots,M}\{a_i\}, \ \lambda = min_{i=1,2,\cdots,M}\{\lambda_i - \frac{a_i}{\tau_0}\}$$

For any switching signal with $\tau \geq \tau_0$, by using the continuity of $e(t)$, it is easy to show that the state transition matrix of $N_{\sigma(t)}$ satisfies

$$\Phi(t, \mu) \leq e^{a - \lambda(t - \mu)}$$

This proves the theorem. ¶

Remark 2: Note that the requirement of z being continuous is crucial for the stability of the state estimation error dynamics. Without it, the result of the theorem is not guaranteed. This is the main reason we require assumptions A1 and A4.

Remark 3: To use Theorem 2, we have to face one difficulty. It requires the knowledge of the dwell time. It is generally not available a priori. So, the result is of more theoretical importance than of practical importance.

One way to overcome the difficulties mentioned in Remark 3 is to use common Lyapunov function approach, and a result is given in the following theorem.

Theorem 3: Under assumptions A1-A3, assume that conditions in Theorem 1 are satisfied for all $1 \leq i \leq M$ such that A4 is true, if there exist two symmetric positive definite matrices P and Q such that

$$N_i^T P + P N_i \leq -Q, \ 1 \leq i \leq M$$

then the state estimation error dynamics (5) is globally exponentially stable for arbitrary switching, that is, the observer (3) is an EUIO for arbitrary switching.

Remark 4: Under the conditions of Theorem 3, we have that $V(e) = e^T P e$ is a common Lyapunov function. From the conclusions of the theorem, we know that the difficulty mentioned in Remark 3 is avoided. Another advantage is that the common Lyapunov function can be found through solving LMIs if it exists. If $N_i + N_i^T$ is negative definite for any i, we have $V(e) = e^T e$ as a common Lyapunov function. This is practical because we can choose the gains of the subobservers to guarantee that $N_i + N_i^T$ is negative definite.

Based on the above discussions, we can derive the following result immediately.

Theorem 4: Under assumptions A1-A3 and assume that there exist E_i and K_i for all $1 \leq i \leq M$ such that

(1) $E_i C_i D_i = -D_i$;

(2) $N_i + N_i^T$ is negative definite with $N_i = P_i A_i - K_i C_i$ also negative definite;

(3) Assumption A4 is true, then the state estimation error dynamics (5) can be made globally exponentially stable for arbitrary switching, that is, the observer (3) can be made an EUIO for arbitrary switching.

To use Theorem 4, we need to design the observer gain K_i (3) such that $N_i = P_i A_i - K_i C_i$ and $N_i + N_i^T$ are Hurwitz for all $1 \leq i \leq m$. This is not always possible. We give a sufficient condition for the existence of the observer gain K_i such that N_i is negative definite in the following theorem.

Theorem 5: Let $C_i = \begin{pmatrix} 0 & I_{p\times p} \end{pmatrix}$, $K_i = \begin{pmatrix} K_1^i \\ K_2^i \end{pmatrix}$ and $P_i A_i = \begin{pmatrix} A_{11}^i & A_{12}^i \\ A_{21}^i & A_{22}^i \end{pmatrix}$, where A_{12}^i is an $n-p$ by p matrix. If A_{11}^i is negative definite, then K_i can be chosen such that $\bar{A}_i$ is negative definite.

Proof: Since $\bar{A}_i = P_i A_i - K_i C_i$, we have

$$\bar{A}_i = \begin{pmatrix} A_{11}^i & A_{12}^i + K_1^i \\ A_{21}^i & A_{22}^i + K_2^i \end{pmatrix}$$

If we choose

$$K_1^i = (A_{21}^i)' - A_{12}^i$$

and

$$K_2^i = H_{22}^i - A_{22}^i + A_{21}^i (A_{11}^i)^{-1} (A_{21}^i)'$$

where H_{22}^i can be any negative definite matrix, then it can be shown that $\bar{A}_i$ is negative definite. ¶

B. Reduced Order UIO

Similar to last subsection, in this subsection, we first give conditions of the existence of reduced order UIO for system (1). Then, we derive sufficient conditions under which the state estimation error dynamics resulted from the UIO is asymptotically or exponentially stable.

To design the reduced order UIO, we need the following assumption.

A5– $C_1 = \cdots = C_M$.

Under assumption A5, after state transformation, the design of reduced order UIO can be reduced to the case $C_1 = \cdots = C_M = [I \ 0]$. Therefore, for simplicity, we only present the reduced order UIO design for the case $C_1 = \cdots = C_M = [I \ 0]$.

For each i, we partition the state vector as

$$x = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, x_1 \in R^{n-m} \ x_2 \in R^m \qquad (7)$$

Because $C_i = [I \ 0]$, we have $y = x_1$. Therefore, we do not need to estimate x_1 because y is measured, and we only need to estimate x_2.

Corresponding to the state vector partition, we partition A_i, B_i, and D_i as

$$\begin{bmatrix} A_{11}^i & A_{12}^i \\ A_{21}^i & A_{22}^i \end{bmatrix}, \ \begin{bmatrix} B_1^i \\ B_2^i \end{bmatrix}, \ \begin{bmatrix} D_1^i \\ D_2^i \end{bmatrix}$$

Following [12], the reduced order UIO for the $i-$th subsystem can be constructed as.

$$\begin{aligned} \dot{z}_i &= N_i z_i + G_i u + L_i y \\ \hat{x}_2^i &= z_i + E_i y \end{aligned} \qquad (8)$$

where

$$N_i = A_{22}^i - E_i A_{12}^i$$

$$G_i = B_2^i - E_i B_1^i$$

$$L_i = N_i E_i + A_{21}^i - E_i A_{11}^i$$

If we define $e^i = z_i + E_i y - x_2$, we have

$$\dot{e}^i = N_i e^i + (D_2^i - E_i D_1^i) v(t)$$

¿From the above equation, we can get a straightforward condition for the existence of reduced order EUIO for the $i-$th subsystem as in following theorem.

Theorem 6: The $i-$th subsystem admits an EUIO of the form (8) if and only if there exists a matrix E_i such that

(1) $D_2^i - E_i D_1^i = 0$;

(2) N_i is Hurwitz.

By combining Theorem 1 and Theorem 2 in [12], we can give a more checkable condition for the existence of reduced order EUIO for the $i-$th subsystem.

Theorem 7: If $\mathrm{rank} C_i D_i = \mathrm{rank} D_i$(that is, $\mathrm{rank} D_1^i$=$\mathrm{rank} D_i$) and the triple (C_i, A_i, D_i) has stable invariant zeros, then the $i-$th subsystem admits an EUIO of the form (8). Furthermore, if (C_i, A_i, D_i) has no invariant zeros, the eigenvalues of the UIO of the form (8) can be arbitrarily assigned.

If we use the same idea for the full order observer design, with (8) at hand, the reduced order observer for SLCS (1) would be given as

$$\begin{aligned} \dot{z} &= N_{\sigma(t)}z + G_{\sigma(t)}u + L_{\sigma(t)}y \\ \hat{x}_2 &= z + E_{\sigma(t)}y \end{aligned} \quad (9)$$

where at any switching instant t_s, we let $z(t_s) = lim_{t \to t_s^-} z(t)$ such that z is continuous.

The existence of the reduced order UIO given by (9) for (1) is guaranteed by the existence of reduced order EUIOs for all its subsystems. The problem is: Is it necessarily an EUIO? Generally, without additional conditions, the answer is no. In what follows, we will study under what conditions it can be an EUIO.

If we define $e = \hat{x}_2 - x_2 = z - x_2 + E_{\sigma(t)}y$, to analyze the stability of the state error dynamics, we need to make $e(t)$ continuous. To this end, we need the following assumption.

A6– $E_1 = E_2 = \cdots = E_M$

Note that z, x_2, and y are all continuous, assumption A6 guarantees that $e(t)$ is continuous.

If conditions in Theorem 6 are satisfied for all $1 \le i \le M$, then it is easy to derive

$$\dot{e} = N_{\sigma(t)}e \quad (10)$$

Compare the above equation with the equation (5), we see that they have exactly the same form. Therefore, all the stability results obtained for full order UIOs can also be derived here. We give the results without proof as follows.

Theorem 8: Under assumptions A1-A3 and A5, assume that conditions in Theorem 6 are satisfied for all $1 \le i \le M$ such that A6 is true, if the **dwell time** τ is large enough, then the state estimation error dynamics (10) is globally asymptotically stable.

Theorem 9: Under assumptions A1-A3 and A5, assume that conditions in Theorem 1 are satisfied for all $1 \le i \le M$ such that A6 is true, if there exist two symmetric positive definite matrices P and Q such that

$$N_i^T P + P N_i \le -Q, \ 1 \le i \le M$$

then the state estimation error dynamics (10) is globally exponentially stable for arbitrary switching, that is, the observer (9) is an EUIO for arbitrary switching.

Theorem 10: Under assumptions A1-A3 and A5, assume that rankC_iD_i = rankD_i and also that E_i for $1 \le i \le m$ can be chosen such that $N_i + N_i^T, 1 \le i \le m$ are negative definite and A6 is true, then the state estimation error dynamics (10) can be made globally exponentially stable for arbitrary switching, that is, the observer (9) can be made an EUIO for arbitrary switching.

Remark 5: All the remarks made for full order UIOs can be made here for reduced order UIOs. The reason we give the design of reduced order UIO is that it needs stronger conditions than the full order one.

IV. THE DESIGN OF UIO

In this section, we will give the design procedure of both full order and reduced order UIOs.

A. The Design Procedure Of Full Order UIO

According to Subsection III-A, to design the full order UIOs, we have to find E_i for all $1 \le i \le M$ such that

DF1– $E_iC_iD_i = D_i$ for each $1 \le i \le M$

DF2– $E_1C_1 = \cdots = E_MC_M$

DF3– (P_iA_i, C_i), that is, $((I + E_iC_i)A_i, C_i)$ is detectable or observable for each $1 \le i \le M$

According to DF1-DF3, we give a design procedure below.

Step 1– Solve $E_iC_iD_i = D_i$ to get its general solution as

$$E_i = -D_i(C_iD_i)^+ + Y_i(I_p - (C_iD_i)(C_iD_i)^+)$$

for each $1 \le i \le M$, where

$$(C_iD_i)^+ = ((C_iD_i)^T(C_iD_i))^{-1}(C_iD_i)^T$$

and Y_i is an arbitrary matrix of suitable dimension.

Step 2– Define a set for each $1 \le i \le M$ as

$$S_i = \{E_i(Y_i)C_i | for \ all \ Y_i\},$$

then let $S = \bigcap_{i=1}^M S_i$.

Step 3– Choose any element in S denoted as X such that $((I + X)A_i, C_i)$ for all $1 \le i \le M$ are detectable or observable. Since $X \in S$, there exists a Y_i for each $1 \le i \le M$ such that $E_i(Y_i)C_i = X$.

Step 4– Choose for each $1 \le i \le M$ a matrix K_i such that the eigenvalues of $N_i = (I + X)A_i - k_iC_i$ are all stable if $((I + X)A_i, C_i)$ is only detectable or all negative and different if observable.

Step 5– Compute $L_i = k_i - N_iE_i$ with $E_iC_i = X$ and $G_i = (I + X)B_i$.

Step 6– Now, we have designed N_i, G_i, L_i, E_i for all $1 \le i \le M$, therefore the full order UIO given by (3) can be implemented.

Remark 6: In the above design procedure, the most difficult parts are Step 2 and Step 3. Generally, there is no systematic way to accomplish them, and we have to do them case by case. However, for some special cases, a systematic way is possible, which is given after this remark.

If we have $C_1 = \cdots = C_M = C$ and $D_1 = \cdots = D_M = D$, we have $S_1 = \cdots = S_M$. Step 2 can be done trivially. Under this condition, we also have

$$E = E(Y) = -D(CD)^+ + Y(I - (CD)(CD)^+)$$

For simplicity, we let $U = -D(CD)^+$ and $V = I - (CD)(CD)^+$, we have $E = U + YV$. Now, we can carry out Step 3 and Step 4 simultaneously if we can find Y, $P > 0$, and $K_i, \ 1 \le i \le M$ such that

$$\begin{aligned} & ((I + UC + YVC)A_i - K_iC)^T P \\ + \ & P((I + UC + YVC)A_i - K_iC) + Q < 0 \end{aligned} \quad (11)$$

where $Q > 0$.

The matrix inequalities in (11) are bilinear matrix inequalities, to use LMI toolbox, we reformulate (11) as an LMI of the following equivalent form.

$$\begin{array}{rl} & ((I+UC)A_i)^T P + P((I+UC)A_i) \\ + & (VCA_i)^T W^T + W(VCA_i) \\ - & \bar{K}_i C - C^T \bar{K}_i^T + Q < 0, i = 1, \cdots, M \end{array} \quad (12)$$

where $Y = P^{-1}W$ and $K_i = P^{-1}\bar{K}_i$.

B. The Design Procedure Of Reduced Order UIO

For simplicity, we design for the case that $C_1 = \cdots = C_M = [I\ 0]$. According to Subsection III-B, to design the reduced order UIOs, we have to find E_i for all $1 \le i \le M$ such that

DR1– $D_2^i - E_i D_1^i = 0$.
DR2– $E_1 = \cdots = E_M$.
DR3– $N_i = A_{22}^i - E_i A_{12}^i$ is Hurwitz for all $1 \le i \le M$.

Similar to the design of full order UIOs and according to DR1-DR3, we give a design procedure for reduced order UIOs below.

Step 1– Solve $D_2^i - E_i D_1^i = 0$ to get its general solution as

$$E_i = E_i(Y_i) = D_2^i (D_1^i)^+ + Y_i(I - D_1^i(D_1^i)^+)$$

for each $1 \le i \le M$, where

$$(D_1^i)^+ = ((D_1^i)^T D_1^i)^{-1}(D_1^i)^T$$

and Y_i is an arbitrary matrix of suitable dimension.

Step 2– Define a set for each $1 \le i \le M$ as

$$S_i = \{E_i(Y_i) | for\ all\ Y_i\}$$

then let $S = \bigcap_{i=1}^M S_i$.

Step 3– Choose any element in S denoted as X such that $N_i = A_{22}^i - XA_{12}^i$ for all $1 \le i \le M$ are Hurwitz. Since $X \in S$, there exists a Y_i for each $1 \le i \le M$ such that $E_i(Y_i) = X$.

Step 4– Compute $G_i = B_2^i - XB_1^i$ and $L_i = N_i X + A_{21}^i - XA_{11}^i$.

Step 5– Now, we have designed N_i, G_i, L_i, E_i for all $1 \le i \le M$, therefore the reduced order UIO given by (9) can be implemented.

Remark 7: For reduced order UIO, same remarks can be made as those made for full order UIO. The most difficult parts are still Step 2 and Step 3.

Similar to full order UIO, we give a systematic design method for a special case, that is, $D_1 = \cdots = D_M = D$. In this case, we have $S_1 = \cdots = S_M$. Thus Step 2 can be done trivially. Under this condition, we also have

$$E = E(Y) = -D_2(D_1)^+ + Y(I - D_1(D_1)^+)$$

where D is partitioned in the same as D_i.

For simplicity, we let $U = -D_2(D_1)^+$ and $V = I - D_1(D_1)^+$, we have $E = U + YV$. Similar to full order UIO design, we can find W and $P > 0$ via solving LMIs.

$$\begin{array}{rl} & (A_{22}^i - UA_{12}^i)^T P + P(A_{22}^i - UA_{12}^i) \\ - & (VA_{12}^i)^T W^T - W(VA_{12}^i) + Q < 0 \end{array} \quad (13)$$

where $Q > 0$ and we compute Y as $Y = P^{-1}W$.

V. DESIGN EXAMPLES AND SIMULATION RESULTS

In this section, due to lcak of space, we give two examples and the corresponding results simulation results only for full order UIOs. One example is to show that our UIOs can indeed guarantee the convergence of the state estimation error; the other is given to show this is not necessarily true without assumption $A4$.

Example 1. Consider the following switched system.

$$\begin{array}{rcl} \dot{x} & = & A_{\sigma(t)}x + B_{\sigma(t)}u + D_{\sigma(t)}d(t) \\ y & = & C_{\sigma(t)}x \end{array} \quad (14)$$

where A_i, B_i, C_i, D_i are given as

$$\begin{array}{rcl} A_1 & = & \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix},\ B_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \\ C_1 & = & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ D_1 = (\,-1 \quad 0 \quad 0\,)^T \\ A_2 & = & \begin{pmatrix} -2 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{pmatrix},\ B_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \\ C_2 & = & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},\ D_2 = (\,1 \quad 0 \quad 0\,)^T \end{array} \quad (15)$$

with $\sigma(t) \in \{1, 2\}$ defined as

$$\sigma(t) = 1\ if\ t \in [20kT\ 10(2k+1)T), k = 0, 1, 2, \cdots$$

$$\sigma(t) = 2\ if\ t \in [10(2k+1)T\ 20(k+1)T), k = 0, 1, 2, \cdots$$

where T is a constant which determines how fast the switching signal switches.

Using the design procedure of a full order UIO, for system (14), we can choose

$$E_1 = E_2 = \begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix};$$

$$K_1 = \begin{pmatrix} 2 & 0 \\ -1 & -12 \\ 0 & 6 \end{pmatrix}\ K_2 = \begin{pmatrix} 2 & 0 \\ 0 & 3 \\ 0 & -3 \end{pmatrix};$$

and compute $N_i, G_i, L_i, i = 1, 2$ accordingly. According to Theorem 2, if the dwell time is large enough, the resulting state estimation error should be globally asymptotically stable.

The next example is designed to show that the stability of the state estimation error is not guaranteed without assumption $A4$ even if each subsystem admits an EUIO.

Example 2. Consider the following switched system. Consider a system in the form of (14) with D_1 and D_2 given by

$$D_1 = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix}^T \quad D_2 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T$$

with $\sigma(t) \in \{1, 2\}$ defined the same as in Example 1. We choose

$$E_1 = \begin{pmatrix} -0.2 & 0.4 \\ 0 & 0 \\ 0.4 & -0.8 \end{pmatrix}; K_1 = \begin{pmatrix} 1.2 & 0 \\ -1 & 60 \\ -0.4 & 6.8 \end{pmatrix}$$

and we use E_2, k_2 chosen in Example 1. It is easy to check that $E_1 C_1 \neq E_2 C_2$, which implies $A4$ is not met.

The simulation results for example 1 with $T = 0.001$ are presented in Figure 1. Though the switching is pretty fast, the estimates of states converge to the states of the switched systems asymptotically.

The simulation results for example 2 with $T = 0.001$ are presented in Figure 2. We see that the sate estimation error diverges. This means without Assumption A_4, the convergence of the sate estimation error is not guaranteed.

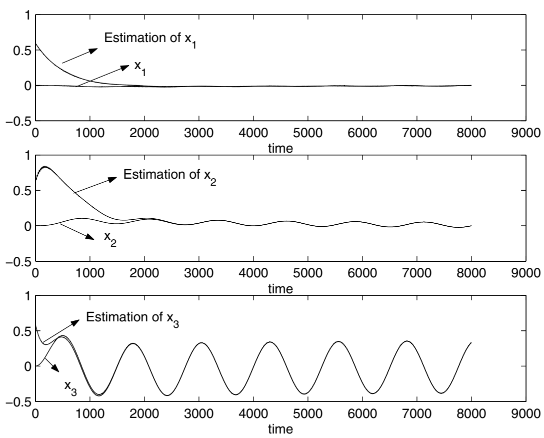

Fig. 1. Results of full order UIO

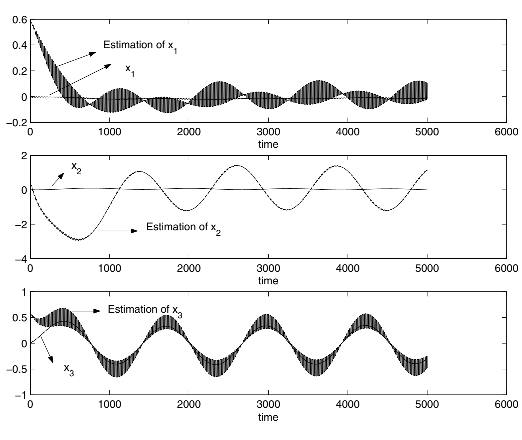

Fig. 2. Results of full order UIO with $e_1 C_1 \neq E_2 C_2$

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

We have designed both full order and reduced order UIOs for a class of switched systems. Sufficient conditions for the existence of both types of UIOs are derived. The stability of state estimation error dynamics is analyzed, and it is proved that the estimation error can converge to zero asymptotically or exponentially under certain conditions. The results for arbitrary switching are very promising because we don't need to worry about the manner of switching in our UIO design.

B. Future Works

How to design UIOs is a very challenging problem when the switching rule is unknown and how to design UIOs for other classes of hybrid systems remains to be investigated. We are also interested in studying fault detection and isolation problem of SCSs based on our UIOs.

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