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Design of Linear Phase Lead Repetitive Control for CVCF PWM DC-AC Converters

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***Abstract*— A linear phase lead repetitive controller is introduced for the constant-voltage constant-frequency (CVCF) pulse-width modulated (PWM) DC-AC converters. The design of the repetitive control (RC) is discussed. Since phase lead can compensate the phase lag of feedback control system, more harmonics can be suppressed which leads to a low total harmonic distortion (THD) of the output voltage in the presence of nonlinear load disturbances and parameter uncertainties. Simulation results show that this method has a fast response and good tracking accuracy.**

I. INTRODUCTION

In many AC power-conditioning systems, such as uninterruptable power supply and other industrial facilities, constant-voltage constant-frequency (CVCF) pulse-width modulated (PWM) DC-AC converters are widely used. Because of the nonlinearity of the loads and the parameter uncertainties, the output voltage often suffers periodic tracking error, which are major sources of total harmonic distortion (THD) in AC power systems. THD is one important index to evaluate the performance of the converters.

To minimize THD, some high precision control method for the CVCF PWM DC-AC converters are proposed. Kawamura, Kawabata proposed a deadbeat controller [1], [2], [3], respectively. Sliding mode controllers are also used to overcome uncertainties and disturbances [4], [5]. However, the deadbeat controller is highly dependent on the accuracy of the parameters while the switching pattern of sliding model controller imposes excessive stress on power devices and cause difficulty in lowpass filtering. In addition, these feedback control schemes do not have memory and any imperfection in performance will be repeated in following cycles.

The repetitive control (RC) method [6], [7], [8], originated from internal model principle [9], is employed to eliminate periodic errors in a nonlinear dynamic system to achieve high accuracy in the presence of uncertainties. Cosner *et al.* proposed a plug-in structure of repetitive control [10], which is widely used now [11], [12], [13], [14], [15]. There are also many reports about the applications of repetitive control in DC-AC converter systems [16], [17], [18], [19], [20], [21].

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In this paper, the design of discrete time linear phase lead repetitive controller is presented systematically. The repetitive controller has a plug-in structure and is developed for the one-step-ahead-preview (OSAP) [16] controlled CVCF PWM converters. The lead-step provides a linear phase compensation so that more harmonics can be suppressed. Simulations show that a repetitive controller with a well designed lead-step can minimize THD and improve the tracking accuracy substantially in the presence of parameter uncertainties and a nonlinear load.

II. LINEAR PHASE LEAD RC

According to internal model principle [9], if a generator of the reference input is included in the stable closed-loop system, the zero tracking error of the reference in the steady state can be achieved. For a periodic reference input, the repetitive controller can be plugged into the closed-loop system [11] as shown in Figure 1. In this figure, $Y_d(z)$ is the reference signal, $Y(z)$ is the output, $D(z)$ is the disturbance, $E(z)$ is the tracking error, $G_r(z)$ is the plug-in repetitive controller, $G_s(z)$ is the plant and $G_c(z)$ is the feedback controller. $G_c(z)$ is chosen so that the closed-loop transfer function $G(z) = \frac{G_c(z)G_s(z)}{1+G_c(z)G_s(z)}$ is asymptotically stable.

According to Figure 1, the repetitive controller $G_r(z)$ has transfer function of:

$$G_r(z) = \frac{k_r z^{-(N-m)} Q(z)}{1 - z^{-N} Q(z)} = \frac{k_r z^m Q(z)}{z^N - Q(z)} \tag{1}$$

in which $N = f_c/f$ is the period with f being the reference signal frequency and f_c being the sampling frequency; $Q(z)$ is a low pass filter [19]; k_r is a *repetitive control gain*; m is lead-step for linear phase lead compensation [22]. Consider in frequency domain, increase lead-step m by 1 will produce a linear phase lead, which reaches $180°$ at Nyquist frequency. Hence, by introduction of m, the phase lag of system can be compensated so that $Q(z)$ can have a high cutoff frequency value to suppress more harmonics.

The linear phase lead RC update law is:

$$U_r(z) = Q(z) z^{-N} \left[U_r(z) + k_r z^m E(z) \right] \tag{2}$$

From Figure 1, the transfer function from $y_d(z)$ to $y(z)$ is:

$$\frac{Y(z)}{Y_d(z)} = \frac{\left[1 - Q(z) z^{-N} + k_r Q(z) z^{-(N-m)} \right] G(z)}{1 - Q(z) z^{-N} \left[1 - k_r z^m G(z) \right]} \tag{3}$$

and the overall transfer function from $d(z)$ to $y(z)$ can be

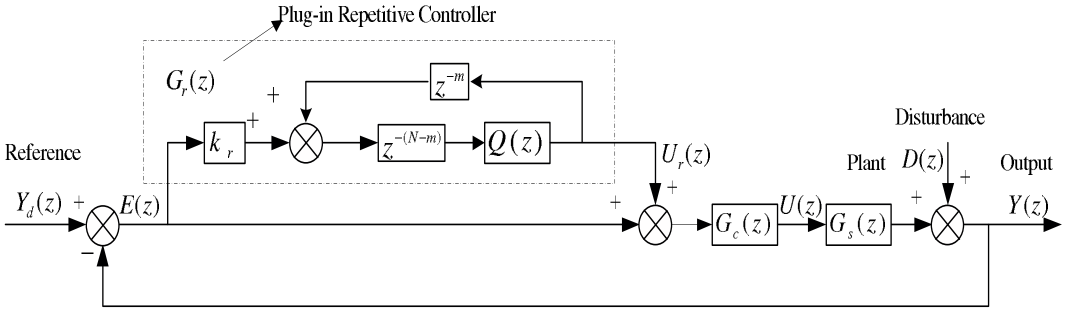


Fig. 1. The linear phase lead repetitive control system

derived as:

$$\frac{Y(z)}{D(z)} = \frac{1-Q(z)z^{-N}}{1+G_c(z)G_s(z)} \cdot \frac{1}{1-Q(z)z^{-N}\left[1-k_r z^m G(z)\right]} \quad (4)$$

From these equations, a conclusion can be drawn that the stability of the overall repetitive control system requires 1) The roots of $[1+G_c(z)G_s(z)]=0$ are inside the unit circle; and 2)

$$|Q(z)(1-k_r z^m G(z))|<1, \quad \forall\ z=e^{j\omega},\ 0<\omega<\pi \quad (5)$$

where ω is a normalized frequency with π being the Nyquist frequency [23].

From Figure 1, we get the error transfer function for the overall system as

$$G_e(z) = \frac{E(z)}{Y_d(z)-D(z)} = \frac{1-Q(z)z^{-N}}{1+G_cG_s} \cdot \frac{1}{1-Q(z)z^{-N}\left[1-k_r z^m G(z)\right]} \quad (6)$$

By realizing $z=e^{j\omega}$, it is clear that when ω approaches $\omega_l = 2\pi l f$ with $l=0,1,2,\cdots,L$ ($L=N/2$ for even N and $L=(N-1)/2$ for odd N), $z^{-N}=1$. Then, with assumption $|Q(z)|=1$, condition (5) becomes

$$|1-k_r z^m G(z)|<1 \quad (7)$$

We have $\|G_e(e^{j\omega})\|=0$ for all ω_l and, hence,

$$\|E(e^{j\omega_l})\|=0 \quad (8)$$

Suppose the closed-loop system $G(z)$ has frequency characteristics $G(e^{j\omega})=N_g(e^{j\omega})\exp(j\theta_g(e^{j\omega}))$ with $N_g(e^{j\omega})$ being its magnitude and $\theta_g(e^{j\omega})$ being its phase, then inequality (7) has the form of:

$$|1-k_r N_g(e^{j\omega})e^{j(\theta_g(e^{j\omega})+m\omega)}|<1$$

Taking square on both sides of this inequality and we have:

$$k_r^2 N_g(e^{j\omega}) < 2k_r\cos(\theta_g(e^{j\omega})+m\omega)$$

By realizing *repetitive control gain* $k_r>0$, this yields [24]:

$$k_r N_g(e^{j\omega}) < 2\cos(\theta_g(e^{j\omega})+m\omega) \quad (9)$$

With *repetitive control gain* $k_r>0$ and $N_g(e^{j\omega})>0$, condition (9) implies:

$$|\theta_g(e^{j\omega})+m\omega|<90^\circ \quad (10)$$

To enhance the robustness of overall system, a phase margin of ϵ is introduced such that condition (10) is written as [24]:

$$|\theta_g(e^{j\omega})+m\omega|<90^\circ-\epsilon \quad (11)$$

From this condition, a lead-step number m can be selected so that (11) can be satisfied in a broadest frequency band.

In the practice, it is often difficult even impossible to make condition (11) hold for all frequencies. If (11) is not satisfied in some high frequency range after linear phase compensation, a low-pass filter $|Q(z)|\le 1$ can be employed to relax the stable condition (7) in this high frequency range. In this case, (7) is modified as:

$$|1-k_r z^m G(z)| < \frac{1}{|Q(z)|} \quad (12)$$

This indicates that there is a tradeoff between tracking accuracy and system robustness.

In presence of model uncertainties $\Delta(z)$ bounded by $\|\Delta(z)\|\le\delta$ and assumption $Q(z)=1$, condition (5) yields a stable range for k_r as [23]:

$$0<k_r<\frac{2}{\max(|z^m G(z)|)+\delta} \quad (13)$$

From this condition, the *repetitive control gain* can be determined.

III. RC Controlled DC-AC Converters

As shown in Figure 2, a single-phase DC-AC converter with RC controller is proposed. In this figure, v_c is the output voltage; i_o is the output current; v_{in} is the input voltage with magnitude of E or $-E$; L_n, C_n, R_n and E_n are the nominal values of inductor L, capacitor C, load R, and dc voltage E, respectively. The dynamics of the CVCF PWM DC-AC converters is as follows [1]:

$$\begin{bmatrix} \dot{v}_c \\ \ddot{v}_c \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{L_nC_n} & -\frac{1}{C_nR_n} \end{bmatrix} \begin{bmatrix} v_c \\ \dot{v}_c \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_nC_n} \end{bmatrix} v_{in} \quad (14)$$

$$G(z) = \frac{(b_1 + b_2 z^{-1})}{(z + a_1 + a_2 z^{-1})(m_1 + m_2 z^{-1}) - (p_1 + p_2 z^{-1})(b_1 + b_2 z^{-1})} \tag{19}$$

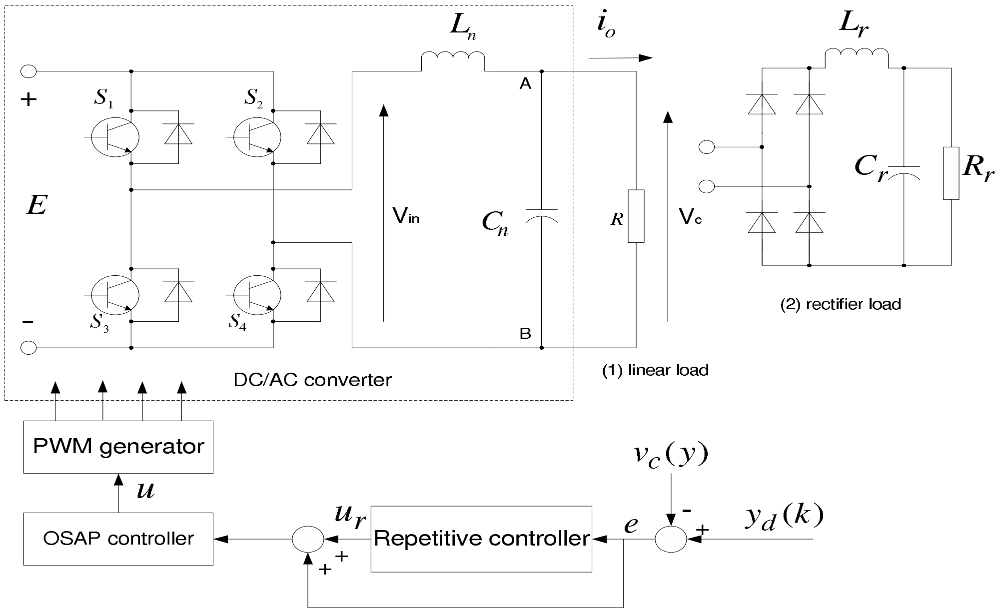


Fig. 2. RC controlled DC-AC converter system

A sampled-data form of (14) with sampling period of T can be written as:

$$\begin{bmatrix} v_c(k+1) \\ \dot{v_c}(k+1) \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix} \begin{bmatrix} v_c(k) \\ \dot{v_c}(k) \end{bmatrix} \pm \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \Delta T(k) \tag{15}$$

The coefficients $\varphi_{11} = 1 - \frac{T^2}{2L_nC_n}$, $\varphi_{21} = -\frac{T}{L_nC_n} + \frac{T^2}{2L_nC_n^2R_n}$, $\varphi_{12} = T - \frac{T^2}{2C_nR_n}$, $\varphi_{22} = 1 - \frac{T}{C_nR_n} - \frac{T^2}{2L_nC_n} + \frac{T^2}{2C_n^2R_n^2}$, $g_1 = \frac{T^2}{2L_nC_n}$, $g_2 = \frac{T}{L_nC_n}(1 - \frac{T}{2C_nR_n})$; $\Delta T(k)$ is the pulse width of input $u(k) = v_{in}(k)$.

The objective is to force the output voltage $v_c(k)$ follows a desired sinusoidal reference $y_d(k)$ with the period of $N \times T$. To design the feedback controller, the dynamics of the converter based on nominal component values can be written in discrete-time domain as [16]:

$$y(k+1) = -p_1 y(k) - p_2 y(k-1) + m_1 u(k) + m_2 u(k-1) \tag{16}$$

with $y(k) = v_c(k)$; $p_1 = -(\varphi_{11} + \varphi_{22})$, $p_2 = \varphi_{11}\varphi_{22} - \varphi_{21}\varphi_{12}$, $m_1 = g_1$, $m_2 = g_2\varphi_{12} - g_1\varphi_{22}$.

When an one-sampling-ahead-preview (OSAP) feedback controller [16]

$$u(k) = \frac{1}{m_1}[y_d(k) - m_2 u(k-1) + p_1 y(k) + p_2 y(k-1)] \tag{17}$$

is applied to the plant with nominal component values, $y(k+1) = y_d(k)$. This is a deadbeat response.

For the practical plant, the converter dynamics should be:

$$y(k+1) = -a_1 y(k) - a_2 y(k-1) + b_1 u(k) + b_2 u(k-1) \tag{18}$$

where a_1, a_2, b_1 and b_2 are calculated based on the actual parameters. If controller (17) is applied, the closed-loop transfer function $G(z)$ without repetitive controller has the form of Equation (19).

To overcome load disturbances $\Delta R = R - R_n$ and uncertainties $\Delta L = L - L_n$, $\Delta E = E - E_n$, $\Delta C = C - C_n$, a linear phase lead RC (2) is employed as:

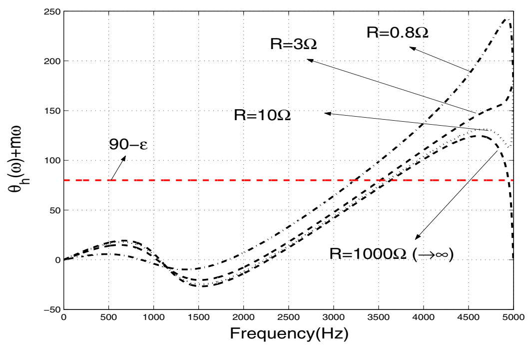
$$\begin{aligned} u_r(k) = {} & d_1(u_r(k-N-1) + k_r e(k-N+m-1)) + \\ & d_0(u_r(k-N) + k_r e(k-N+m)) + \\ & d_1(u_r(k-N+1) + k_r e(k-N+m+1)) \end{aligned} \tag{20}$$

Notice that in this update law $Q(z) = d_1 z^{-1} + d_0 + d_1 z$ with $d_0 + 2d_1 = 1$ [21].

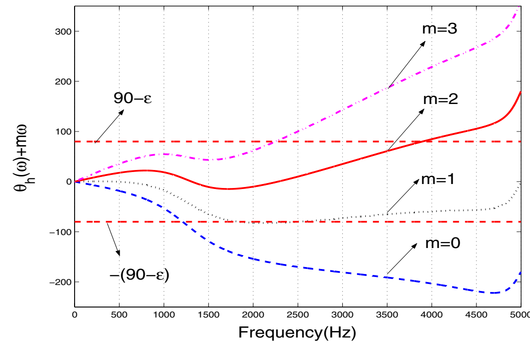
IV. Simulations

For simulation, the parameters of the converter is chosen as follows: $E_n = 200$V; $C_n = 300\mu$F; $L_n = 500\mu$H; $R_n = 3\Omega$ and $E = 180$V; $C = 500\mu$F; $L = 700\mu$H; $R = 8\Omega$, $y_d(t)$ is a 50Hz, 100V (peak) sinusoidal signal; $f = 50$Hz; $f_c = \frac{1}{T} = 10$kHz; the rectifier load parameters are $R_r = 10\Omega$, $C_r = 2000\mu$F. Based on these parameters, if $R > 0.8\Omega$, all roots of $[1 + G_c(z)G_s(z)] = 0$ are located inside the unity circle. With load $R \in (0.8, \infty)\Omega$, $\max(|z^m G(z)|)$ is less than 8. From (13), the system is stable if $k_r \in [0, 0.25]$. We choose $k_r = 0.02$.

With several selected loads $R \in (0.8, \infty)\Omega$, Figure 3(a) illustrates the condition (11) with $m = 2$. It is clear that phase angles of $G(z)$ are similar for these loads when $\omega <$ 4500Hz, especially when load ranges from nominal load $R_n = 3\Omega$ to infinity. Therefore, we can choose $R = 8\Omega$ as

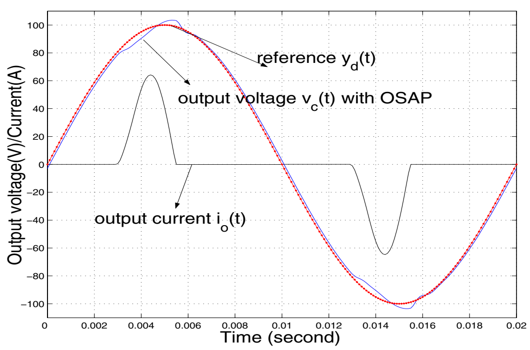


(a) (11) for different loads with $m = 2$

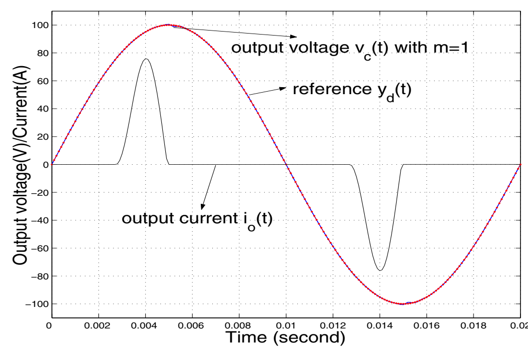


(b) Condition (11) for different m

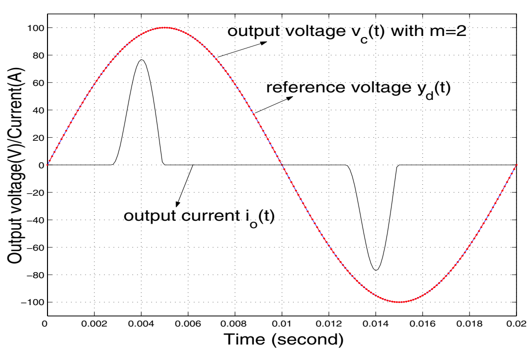
Fig. 3. The determination of linear phase lead



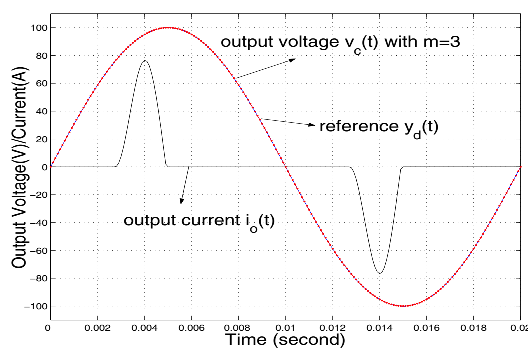
(a) OSAP



(b) OSAP plus RC (m=1)



(c) OSAP plus RC (m=2)



(d) OSAP plus RC (m=3)

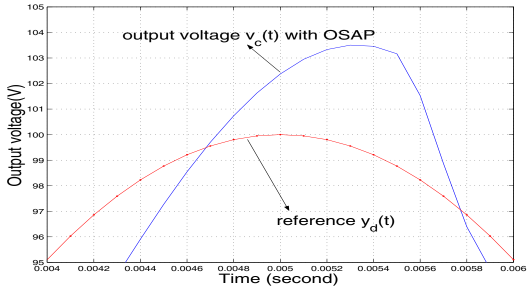
Fig. 4. Steady-state $y_d(t)$, $v_c(t)$ and $i_o(t)$ under rectifier load

a representative load to obtain the system model $G(z)$ as in (21) and determine the linear phase lead m.

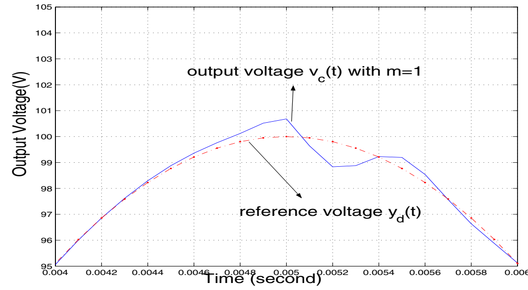
$$G(z) = \frac{0.3857z^2 + 0.3816z}{z^3 - 0.3193z^2 - 0.4667z + 0.5588} \quad (21)$$

Figure 3(b) shows that the phase lead compensation of (21) with different m and $\epsilon = 10°$. It is clear that $m = 2$ has the widest frequency band to make condition (11) hold. Hence, $m = 2$ is the best phase lead in our simulation.

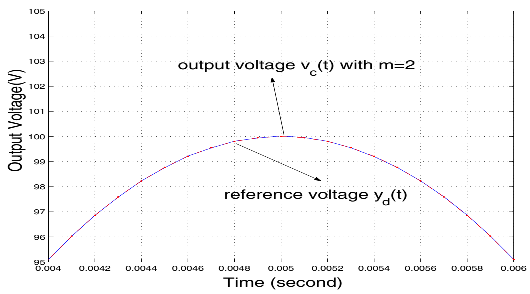
Figure 4(a)-(d) shows the steady-state response under the rectifier load with only OSAP feedback controller, OSAP plus RC with $m = 1$ ($Q(z) = 0.15z + 0.7 + 0.15z^{-1}$), with $m = 2$ ($Q(z) = 1$), and with $m = 3$ ($Q(z) = 0.05z + 0.9 + 0.05z^{-1}$), respectively, where $Q(z)$ is used to stabilize the system. Figure 5 zoomed the simulation results. When



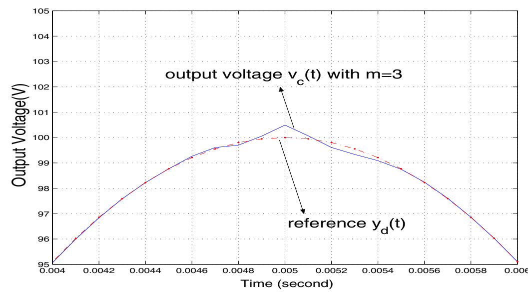
(a) Zoomed OSAP



(b) Zoomed OSAP plus RC (m=1)

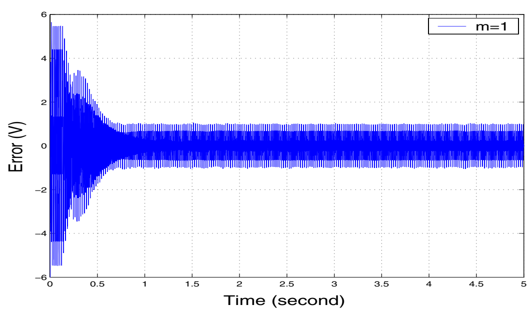


(c) Zoomed OSAP plus RC (m=2)

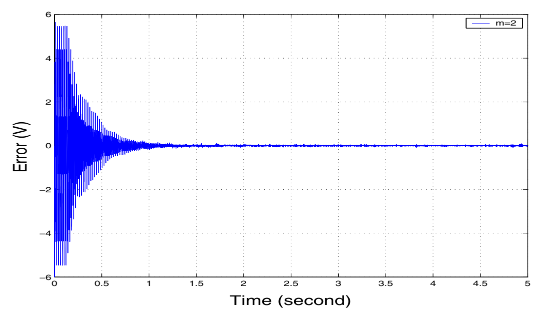


(d) Zoomed OSAP plus RC (m=3)

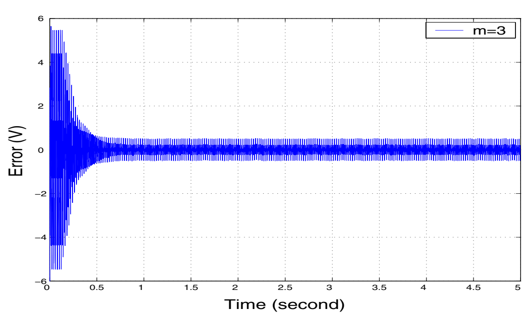
Fig. 5. Zoomed steady-state $y_d(t)$, $v_c(t)$ under rectifier load



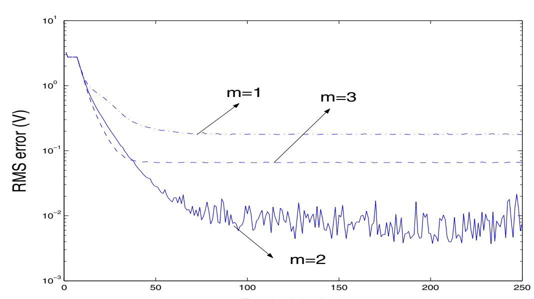
(a) OSAP plus RC (m=1) tracking error



(b) OSAP plus RC (m=2) tracking error



(c) OSAP plus RC (m=3) tracking error



(d) RMS error

Fig. 6. Tracking error history and RMS of $e(t) = y_d(t) - y(t)$

TABLE I

SIMULATION RESULTS

Controller	$Q(z)$	*peak* $e(t)$ (V)	RMS (V)	THD
OSAP	-	± 5.5	2.756	2.36%
RC(m=1)	$0.15z^{-1} + 0.7 + 0.15z$	± 1	0.179	0.977%
RC(m=2)	**1**	$\pm\mathbf{0.08}$	**0.005**	**0.945%**
RC(m=3)	$0.05z^{-1} + 0.9 + 0.05z$	± 0.6	0.066	0.950%

$m = 1$, the repetitive controller is the same with that in [21]. It is clear that the plug-in RC with $m = 2$ offers the best tracking accuracy.

Figure 6 shows the tracking error of plug-in RC with $m = 1$, $m = 2$, and $m = 3$ and their root mean square (RMS) errors along period index. The RC takes effect from $t = 0.12$ second. For OSAP controlled $v_c(t)$, its THD is 2.36%, peak of $e(t)$ is $\pm 5.5V$, and RMS error is 2.756V. For plug-in RC with $m = 1$, THD of $v_c(t)$ drops to 0.977% within 50 cycles, peak of $e(t)$ reduces to $\pm 1V$ within 0.68 second and its RMS reduces to 0.179V. For RC with $m = 2$, THD of $v_c(t)$ drops to 0.945% within 40 cycles, peak of $e(t)$ reduces to $\pm 0.08V$ within 0.88 second and its RMS drops to 0.005V. For RC with $m = 3$, THD of $v_c(t)$ drops to 0.950% within 40 cycles, peak of $e(t)$ reduces to $\pm 0.6V$ within 0.5 second and its RMS reduces to 0.066V.

The simulation results can be summarized in Table I, in which the best performance with lead step $m = 2$ is highlighted by bold font.

V. CONCLUSION

In this paper, a linear phase lead RC is introduced into the control of CVCF PWM DC-AC converters under parameter uncertainties and nonlinear load disturbances. The design of linear phase lead is discussed. Simulation results show that with a well-designed phase lead, the periodic tracking errors caused by nonlinear load and parameter uncertainties are suppressed substantially. Minimized output voltage THD and fast response are obtained at the same time.

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