2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Robust H_∞ Control for Uncertain Takagi-Sugeno Fuzzy Systems with Interval Time-Varying Delay

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***Abstract*— This paper is concerned with the problem of delay-dependent robust H_∞ control for uncertain time-delay fuzzy systems with norm-bounded uncertainty. The time-delay is assumed to be a time-varying continuous function belonging to a given interval, which means that the lower and upper bounds for the delay are available. No restriction on the derivative of the time-varying delay is needed, which allows the time-delay to be a fast time-varying function. The state-space Takagi-Sugeno (T-S) uncertain fuzzy model with interval time-varying delay is adopted. Delay-dependent conditions for the existence of robust H_∞ controller are presented in the form of linear matrix inequalities (LMIs). A numerical example is given to demonstrate the effectiveness of the proposed method.**

I. INTRODUCTION

Fuzzy systems in the form of the Takagi-Sugeno model [1] have attracted great interests in the past decade. It has shown that the T-S model method can give an effective way to represent complex nonlinear systems by some simple local linear dynamic systems with their linguistic description. And some nonlinear dynamic systems can be approximated by the overall fuzzy linear T-S models for the purposes of stability analysis and controller design [2], [3], [4].

Delayed fuzzy systems were introduced and studied in [5] by developing the T-S fuzzy model based on the Lyapunov-Krasovskii approach. After then, the T-S model fuzzy systems with time-delay have been widely studied, and many relevant results have been reported ([6], [7] and references therein). It is well known that delays appear in many dynamic systems. Fuzzy delayed systems of T-S models provide a method of using local linear delayed systems combined with fuzzy linguistic descriptions to achieve nonlinearity.

It is clear that the stability analysis and stabilization are important issues in analysis and design of control systems. In the time-domain, the direct Lyapunov method is a powerful tool. There are two different ideas to pursue this method: the Lyapunov-Krasovskii approach and the Lyapunov-Razumikhin approach. Both approaches can be used to handle systems with time-varying delay. The former *usually* requires both the upper bound of the time-varying delay and additional information on its derivative [5], [8], [9], while the latter has no restriction on the derivative of the time-varying delay, which allows a **fast** time-varying delay [10]. However, the obtained results using the Lyapunov-Krasovskii approach are *usually* less conservative than those using the Lyapunov-Razumikhin approach since the fromer takes advantage of the additional information of the delay.

It is well known that there exist some systems which are stable with some *nonzero* delay, but are unstable without delay [11], [12]. For such case, if there is a time-varying perturbation on the nonzero delay, it is of great significance to consider the stability analysis and controller design of the systems with *interval time-varying* delay. Other typical examples of the systems with *interval time-varying* delay are networked control systems [13], [14]. The stability of such kinds of systems was investigated in [15] using the Lyapunov-Krasovskii approach, where the derivative bound of the interval time-varying delay, i.e. $\dot{\tau}(t) \leq \tau_d < 1$, was needed. However, in most of practical applications, it is not easy to estimate the bound of the derivative of time-varying delay in advance. Sometimes, such derivative bound can not satisfy $\dot{\tau}(t) \leq \tau_d < 1$, or the time-varying delay is not even differentiable at all, such as networked control systems. To the best of our knowledge, for the case where *only* the upper and lower bounds of the interval time-varying delay are precisely known, there is no result available for the delay-dependent robust H_∞ controller design for such kinds of systems, especially for fuzzy systems by employing the Lyapunov-Krasovskii approach.

In this paper, we will consider the problem of delay-dependent robust H_∞ control design for uncertain fuzzy systems with *interval* time-varying delay. The restriction on the derivative of the interval time-varying delay is *removed*, which means that a *fast* interval time-varying delay is allowed. Based on the Lyapunov-Krasovskii functional approach, the existence condition of a delay-dependent robust H_∞ controller will be derived by introducing some relaxation matrices which can be used to reduce the conservatism of the obtained criterion which will be formulated in the form of linear matrix inequalities (LMIs). A numerical example will be given to show the effectiveness of the method.

Notation. For a symmetric matrix X, the notation $X \geq 0$ ($X > 0$) means that the matrix X is positive semi-definite

The research work was partially supported by the Central Queensland University for the 2004 Research Advancement Awards Scheme Project "Analysis and synthesis of networked control systems", the Strategic Research Project "Delay effects: analysis, synthesis and applications" (2003-2006), the Seed Grant Scheme of the Research Development and Incentives Program "Robust stability and stabilization for systems with interval time-varying delay and its application to networked control systems"(RDI(S)05/12), and the Faculty of Informatics and Communication for the Small Grant Project "On stability for systems with interval time-varying delay arising in networked control systems".

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(positive definite). I is an identity matrix of appropriate dimensions. Matrices, if not explicitly stated, are assumed to have compatible dimensions. For any real matrix A, A^T denotes the transpose of matrix A. For any nonsingular matrix A, A^{-1} denotes the inverse of matrix A. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ matrices. $\mathcal{L}_2[0,\infty)$ refers to the space of square summable infinite vector sequences. $\|\cdot\|_2$ stands for the usual $\mathcal{L}_2[0,\infty)$ norm. The symmetric terms in a symmetric matrix are denoted by *. $sgn(x)$ is a signal function, i.e.

$$sgn(x) = \begin{cases} 1, & if \quad x > 0, \\ 0, & if \quad x = 0, \\ -1, & if \quad x < 0. \end{cases}$$

II. PROBLEM STATEMENT

Consider the following T-S model

Plant Rule i :

IF $z_1(t)$ is M_{i1}, $z_2(t)$ is M_{i2}, $\cdots$, $z_g(t)$ is M_{ig}

$$\text{THEN} \begin{cases} \dot{x}(t) = [A_{i0} + \Delta A_{i0}(t)]x(t) \\ \quad +[A_{i1} + \Delta A_{i1}(t)]x(t-\tau(t)) \\ \quad +[B_i + \Delta B_i(t)]\Delta u(t) + B_{wi}w(t), \\ \tilde{z}(t) = C_i x(t) + D_i u(t), \\ x(t) = \phi(t), \ t \in [-\tau_M, 0]. \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, r$. r is the number of IF-THEN rules; $z_1(t)$, $z_2(t)$, $\cdots$, $z_g(t)$ are the premise variables of (1) and M_{ij} ($i = 1, 2, \cdots, r$; $j = 1, 2, \cdots, g$) are the fuzzy sets corresponding to $z_j(t)$ and plant rules r; $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $w(t) \in \mathcal{L}_2[0,\infty)$ is the exogenous disturbance, and $\tilde{z}(t) \in \mathbb{R}^p$ is the controlled output; A_{i0}, A_{i1}, B_i, B_{wi}, C_i and D_i ($i = 1, 2, \cdots, r$) are known parameter matrices of appropriate dimensions; $\Delta A_{i0}(t)$, $\Delta A_{i1}(t)$ and $\Delta B_i(t)(i = 1, 2, \cdots, r)$ are real-valued unknown matrices representing time-varying parameter uncertainties of (1), and are assumed to be the form

$$[\Delta A_{i0}(t), \Delta A_{i1}(t), \Delta B_i(t)] = H_i F_i(t) \left[E_{i0}, E_{i1}, E_{ib}\right] \tag{2}$$

where H_i, E_{i0}, E_{i1} and E_{ib} ($i = 1, 2, \cdots, r$) are known real constant matrices of appropriate dimensions. $F_i(t) \in \mathbb{R}^{l_1\times l_2}$ ($i = 1, 2, \cdots, r$) are unknown time-varying matrix functions with Lebesgue measurable elements satisfying

$$F_i(t)^T F_i(t) \leq I, \ i = 1, 2, \cdots, r. \tag{3}$$

$\phi(t)$ is the initial condition of system (1). $\tau(t)$ is a uniformly continuous time-varying function satisfying

$$0 < \tau_m \leq \tau(t) \leq \tau_M, \tag{4}$$

where τ_m and τ_M are two known constants. Let $h_i(z(t))$ be the normalized membership function of the inferred fuzzy set $\mu_i(z(t))$, i.e.

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^r \mu_i(z(t))},$$

where

$$\begin{aligned} z(t) &= [z_1(t), z_2(t), \cdots, z_g(t)], \\ \mu_i(z(t)) &= \prod_{j=1}^{g} M_{ij}(z_j(t)). \end{aligned}$$

$M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed that

$$\mu_i(z(t)) \geq 0, \ i = 1, 2, \cdots, r, \ \sum_{i=1}^r \mu_i(z(t)) > 0, \forall t \geq 0.$$

Then

$$h_i(z(t)) \geq 0, \ i = 1, 2, \cdots, r, \ \sum_{i=1}^r h_i(z(t)) = 1, \forall t \geq 0. \tag{5}$$

Using a center average defuzzifer, product inference, and singleton fuzzifier, the fuzzy model (1) can be expressed by the following global model

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^r h_i(z(t))[A_{i0}(t)x(t) + A_{i1}(t)x(t-\tau(t)) \\ \qquad\qquad +B_i(t)u(t) + B_{wi}w(t)], \\ \tilde{z}(t) = \sum\limits_{i=1}^r h_i(z(t))[C_i x(t) + D_i u(t)], \\ x(t) = \phi(t), \ t \in [-\tau_M, 0], \end{cases} \tag{6}$$

where $A_{i0}(t) \triangleq A_{i0} + \Delta A_{i0}(t)$, $A_{i1}(t) \triangleq A_{i1} + \Delta A_{i1}(t)$, $B_i(t) \triangleq B_i + \Delta B_i(t)$, $i = 1, 2, \cdots, r$.

Throughout this paper, delay-dependent state feedback T-S fuzzy-model-based H_∞ control laws are utilized for the robust stabilization of the T-S fuzzy system (1) as follows

$$\begin{aligned} R^i: \quad & \text{IF } z_1(t) \text{ is } M_{i1}, \ z_2(t) \text{ is } M_{i2}, \ \cdots, \ z_g(t) \text{ is } M_{ig} \\ & \text{THEN } u(t) = K_i x(t), \end{aligned} \tag{7}$$

where K_i ($i = 1, 2, \cdots, r$) are the controller gains of (7) to be determined. The defuzzified output of the controller rules is given by

$$u(t) = \sum_{i=1}^r h_i(z(t)) K_i x(t). \tag{8}$$

(6) with controller (8) can be represented as

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^r \sum\limits_{j=1}^r h_i(z(t))h_j(z(t))[(A_{i0}(t) + B_i(t)K_j)\times \\ \qquad\qquad x(t) + A_{i1}(t)x(t-\tau(t)) + B_{wi}w(t)], \\ \tilde{z}(t) = \sum\limits_{i=1}^r \sum\limits_{j=1}^r h_i(z(t))h_j(z(t))[C_i + D_i K_j]x(t), \\ x(t) = \phi(t), \ t \in [-\tau_M, 0]. \end{cases} \tag{9}$$

For a prescribed scalar $\gamma > 0$, we define the performance index

$$J(w) = \int_0^\infty [\tilde{z}^T(\theta)\tilde{z}(\theta) - \gamma^2 w^T(\theta)w(\theta)]d\theta. \tag{10}$$

The purpose of this paper is to design a delay-dependent robust H_∞ controller (8) for the T-S global model (6) such that for all admissible uncertainties satisfying (2), (3), and any $\tau(t)$ satisfying (4) for a prescribed scalar $\gamma > 0$.

(1) (9) with $w(t) = 0$ is asymptotically stable;

(2) under the zero initial condition, (9) satisfies

$$\|\tilde{z}(t)\|_2 < \gamma \|w(t)\|_2 \tag{11}$$

for all nonzero $w(t) \in \mathcal{L}_2[0,\infty)$, .

III. MAIN RESULT

In this section some delay-dependent sufficient conditions for the existence of robust H_∞ controller (8) for T-S fuzzy system (9) will be presented.

Defining $\tau_{av} = \frac{1}{2}(\tau_M + \tau_m)$ and $\delta = \frac{1}{2}(\tau_M - \tau_m)$, $\tau(t)$ satisfying (4) can be expressed as

$$\tau(t) = \tau_{av} + \delta q(t), \tag{12}$$

where

$$q(t) = \begin{cases} \frac{2\tau(t)-(\tau_M+\tau_m)}{\tau_M-\tau_m}, & \tau_M > \tau_m \\ 0. & \tau_M = \tau_m \end{cases}$$

It is clear that $|q(t)| \le 1$. For this case, $\tau(t)$ is a function belonging to the interval $[\ \tau_{av} - \delta, \quad \tau_{av} + \delta\ ]$, where δ can be taken as the range of variation of the time-varying delay $\tau(t)$.

Based on the Lyapunov-Krasovskii functional approach, the delay-dependent H_∞ fuzzy controller (8) for the following nominal closed-loop system is first investigated

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))[(A_{i0} + B_iK_j)x(t) \\ \qquad\qquad + A_{i1}x(t-\tau(t)) + B_{wi}w(t)], \\ \tilde{z}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))[C_i + D_iK_j]x(t), \\ x(t) = \phi(t),\ t \in [-\tau_M, 0]. \end{cases} \tag{13}$$

Using the fact

$$x(t-\tau_{av}) - x(t-\tau(t)) = \int_{t-\tau(t)}^{t-\tau_{av}} \dot{x}(s)ds,$$

system (13) can be rewritten as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))[(A_{i0} + B_iK_j)x(t) \\ \qquad\qquad + A_{i1}x(t-\tau_{av}) + B_{wi}w(t) \\ \qquad\qquad - A_{i1}\int_{t-\tau(t)}^{t-\tau_{av}} \dot{x}(s)ds], \\ \tilde{z}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))[C_i + D_iK_j]x(t), \\ x(t) = \phi(t), t \in [-\tau_M, 0]. \end{cases} \tag{14}$$

For the delay-dependent H_∞ controller design for fuzzy system (13), we now state and establish the following result.

Proposition 1: For a prescribed scalar $\gamma > 0$ and some given scalars $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, τ_m and τ_M, system (13) is stable and satisfies $\|\tilde{z}(t)\|_2 < \gamma \|w(t)\|_2$ for all nonzero $w(t) \in \mathcal{L}_2[0,\infty)$ and any $\tau(t)$ satisfying (4), if there exist $\tilde{P} > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\tilde{S} > 0$, X, Y_j $(j = 1, 2, \cdots, r)$, and $\tilde{M}_i$ $(i = 1, 2, 3)$ of appropriate dimensions such that the following LMIs simultaneously hold for $i, j = 1, 2, \cdots, r$,

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_1 \\ * & \Theta_{22} & \Theta_{23} & \Theta_2 \\ * & * & \Theta_{33} & \Theta_3 \\ * & * & * & \Theta_4 \end{bmatrix} < 0, \tag{15}$$

where

$$\begin{aligned} \Theta_{11} &= \tilde{Q} + A_{i0}X + B_iY_j + (A_{i0}X + B_iY_j)^T \\ &\quad + \tilde{M}_1^T + \tilde{M}_1, \\ \Theta_{12} &= A_{i1}X + \varepsilon_2(A_{i0}X + B_iY_j)^T - \tilde{M}_1^T + \tilde{M}_2, \\ \Theta_{13} &= \tilde{P} - X + \varepsilon_3(A_{i0}X + B_iY_j)^T + \tilde{M}_3, \\ \Theta_{22} &= -\tilde{Q} + \varepsilon_2(A_{i1}X + X^TA_{i1}^T) - \tilde{M}_2^T - \tilde{M}_2, \\ \Theta_{23} &= -\varepsilon_2X + \varepsilon_3X^TA_{i1}^T - \tilde{M}_3, \\ \Theta_{33} &= \tau_{av}\tilde{R} + 2\delta\tilde{S} - \varepsilon_3(X^T + X), \\ \Theta_1 &= \left[B_{wi}, -\tau_{av}\tilde{M}_1^T, \delta A_{i1}X, (C_iX + D_iY_j)^T\right], \\ \Theta_2 &= \left[\varepsilon_2B_{wi}, -\tau_{av}\tilde{M}_2^T, \delta\varepsilon_2A_{i1}X, 0\right], \\ \Theta_3 &= \left[\varepsilon_3B_{wi}, -\tau_{av}\tilde{M}_3^T, \delta\varepsilon_3A_{i1}X, 0\right], \\ \Theta_4 &= diag\{-\gamma^2I, -\tau_{av}\tilde{R}, -\delta\tilde{S}, -I\}. \end{aligned}$$

Moreover, the state feedback controller gains of (8) are given by $K_j = Y_jX^{-1}$ for $j = 1, 2, \cdots, r$.
In order to prove the above proposition, the following result is needed.

Lemma 1: [16] There exists a symmetric matrix X such that

$$\begin{bmatrix} P_1 + X & Q_1 \\ Q_1^T & R_1 \end{bmatrix} > 0, \begin{bmatrix} P_2 - X & Q_2 \\ Q_2^T & R_2 \end{bmatrix} > 0$$

if and only if

$$\begin{bmatrix} P_1 + P_2 & Q_1 & Q_2 \\ Q_1^T & R_1 & 0 \\ Q_2^T & 0 & R_2 \end{bmatrix} > 0.$$

Now, the proof of Proposition 1 is given as follows.

Proof: Choose a Lyapunov-Krasovskii functional as

$$\begin{aligned} V(x_t) &= x^T(t)Px(t) + \int_{t-\tau_{av}}^{t} x^T(s)Qx(s)ds \\ &\quad + \int_{-\tau_{av}}^{0} ds \int_{t+s}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta \\ &\quad + \int_{-\tau_{av}-\delta}^{-\tau_{av}+\delta} ds \int_{t+s}^{t} \dot{x}^T(\theta)S\dot{x}(\theta)d\theta, \end{aligned} \tag{16}$$

where $P > 0, Q > 0, R > 0, S > 0$. Taking the derivative of $V(x_t)$ with respect to t along the trajectory of (14) yields

$$\begin{aligned} \dot{V}(x_t) &= 2x^T(t)P\dot{x}(t) - x^T(t-\tau_{av})Qx(t-\tau_{av}) \\ &\quad + x^T(t)Qx(t) + \dot{x}^T(t)(\tau_{av}R + 2\delta S)\dot{x}(t) \\ &\quad - \int_{t-\tau_{av}}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\ &\quad - \int_{t-\tau_{av}-\delta}^{t-\tau_{av}+\delta} \dot{x}^T(s)S\dot{x}(s)ds. \end{aligned} \tag{17}$$

It is easy to see that

$$\begin{aligned} & - \int_{t-\tau_{av}-\delta}^{t-\tau_{av}+\delta} \dot{x}^T(s)S\dot{x}(s)ds \\ \le & -sgn(\Delta\tau(t)) \int_{t-\tau(t)}^{t-\tau_{av}} \dot{x}^T(s)S\dot{x}(s)ds. \end{aligned} \tag{18}$$

where $\Delta\tau(t) \triangleq \tau(t) - \tau_{av}$.

Note that

$$2\tilde{\xi}^T(t)N^T\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))[(A_{i0}+B_iK_j)x(t) \\ +A_{i1}x(t-\tau_{av})+B_{wi}w(t) \\ -A_{i1}\int_{t-\tau(t)}^{t-\tau_{av}}\dot{x}(s)ds]-\dot{x}(t)\}=0, \quad (19)$$

and

$$2\tilde{\xi}^T(t)M^T[x(t)-x(t-\tau_{av})-\int_{t-\tau_{av}}^{t}\dot{x}(s)ds]=0, \quad (20)$$

where $\tilde{\xi}^T(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau_{av}) & \dot{x}^T(t) \end{bmatrix}$, $N = \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix}$, $M = \begin{bmatrix} M_1 & M_2 & M_3 \end{bmatrix}$, N_i, M_i $(i = 1,2,3)$ are some matrices of appropriate dimensions. Then according to (5) and (17)-(20), we have

$$\dot{V}(x_t) \le \begin{cases} \sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\Sigma_{ij}, & \text{if } \tau(t)\neq\tau_{av}, \\ \sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\tilde{\Sigma}_{ij}, & \text{if } \tau(t)=\tau_{av}, \end{cases} \quad (21)$$

where

$$\begin{aligned}
\Sigma_{ij} &= \frac{1}{\tau_{av}}\int_{t-\tau_{av}}^{t}\xi^T(t,s)\Phi_1\xi(t,s)ds \\
&\quad +\frac{1}{\tau(t)-\tau_{av}}\int_{t-\tau(t)}^{t-\tau_{av}}\xi^T(t,s)\Phi_2\xi(t,s)ds, \\
\tilde{\Sigma}_{ij} &= \frac{1}{\tau_{av}}\int_{t-\tau_{av}}^{t}\xi^T(t,s)\tilde{\Phi}_1\xi(t,s)ds, \\
\xi^T(t,s) &= \left[x^T(t), x^T(t-\tau_{av}), \dot{x}^T(t), w^T(t), \dot{x}^T(s)\right], \\
\Phi_1 &= \tilde{\Phi}_1 + Z, \\
\Phi_2 &= \begin{bmatrix} 0 & 0 & 0 & 0 & -\Delta\tau(t)N_1^TA_{i1} \\ * & 0 & 0 & 0 & -\Delta\tau(t)N_2^TA_{i1} \\ * & * & 0 & 0 & -\Delta\tau(t))N_3^TA_{i1} \\ * & * & * & 0 & 0 \\ * & * & * & * & -sgn(\Delta\tau(t))\cdot\Delta\tau(t)S \end{bmatrix} \\
&\quad -Z, \\
\tilde{\Phi}_1 &= \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & N_1^TB_{wi} & -\tau_{av}M_1^T \\ * & \Phi_{22} & \Phi_{23} & N_2^TB_{wi} & -\tau_{av}M_2^T \\ * & * & \Phi_{33} & N_3^TB_{wi} & -\tau_{av}M_3^T \\ * & * & * & 0 & 0 \\ * & * & * & * & -\tau_{av}R \end{bmatrix}, \\
Z &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & 0 \\ * & Z_{22} & Z_{23} & Z_{24} & 0 \\ * & * & Z_{33} & Z_{34} & 0 \\ * & * & * & Z_{44} & 0 \\ * & * & * & * & 0 \end{bmatrix}, \\
\Phi_{11} &= Q + N_1^T(A_{i0}+B_iK_j) \\
&\quad +(A_{i0}+B_iK_j)^TN_1 + M_1^T + M_1, \\
\Phi_{12} &= N_1^TA_{i1} + (A_{i0}+B_iK_j)^TN_2 - M_1^T + M_2, \\
\Phi_{13} &= P - N_1^T + (A_{i0}+B_iK_j)^TN_3 + M_3, \\
\Phi_{22} &= -Q + N_2^TA_{i1} + A_{i1}^TN_2 - M_2^T - M_2, \\
\Phi_{23} &= -N_2^T + A_{i1}^TN_3 - M_3, \\
\Phi_{33} &= \tau_{av}R + 2\delta S - N_3^T - N_3,
\end{aligned}$$

$Z_{11} > 0$, $Z_{22} > 0$, $Z_{33} > 0$, $Z_{44} > 0$, Z_{12}, Z_{13}, Z_{14}, Z_{23}, Z_{24} and Z_{34} are some matrices of appropriate dimensions.

First, assuming that $w(t) = 0$, we consider the asymptotic stability of system (13). From (21) we obtain

$$\dot{V}(x_t) \le \begin{cases} \sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\{\frac{1}{\tau_{av}}\int_{t-\tau_{av}}^{t}\xi^T(t,s)\Omega_1\xi(t,s)ds \\ +\frac{1}{\tau(t)-\tau_{av}}\int_{t-\tau(t)}^{t-\tau_{av}}\xi^T(t,s)\Omega_2\xi(t,s)ds\}, \\ \qquad\qquad \text{if } \tau(t)\neq\tau_{av}, \\ \sum_{i=1}^{r}\sum_{j=1}^{r}\frac{h_ih_j}{\tau_{av}}\int_{t-\tau_{av}}^{t}\xi^T(t,s)\tilde{\Omega}_1\xi(t,s)ds, \\ \qquad\qquad \text{if } \tau(t)=\tau_{av}, \end{cases} \quad (22)$$

where

$$\begin{aligned}
\Omega_1 &= \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & -\tau_{av}M_1^T \\ * & \Phi_{22} & \Phi_{23} & -\tau_{av}M_2^T \\ * & * & \Phi_{33} & -\tau_{av}M_3^T \\ * & * & * & -\tau_{av}R \end{bmatrix} + \tilde{Z}, \\
\Omega_2 &= \begin{bmatrix} 0 & 0 & 0 & -\Delta\tau(t)N_1^TA_{i1} \\ * & 0 & 0 & -\Delta\tau(t)N_2^TA_{i1} \\ * & * & 0 & -\Delta\tau(t)N_3^TA_{i1} \\ * & * & * & -sgn(\Delta\tau(t))\cdot\Delta\tau(t)S \end{bmatrix} - \tilde{Z}, \\
\tilde{\Omega}_1 &= \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & -\tau_{av}M_1^T \\ * & \Phi_{22} & \Phi_{23} & -\tau_{av}M_2^T \\ * & * & \Phi_{33} & -\tau_{av}M_3^T \\ * & * & * & -\tau_{av}R \end{bmatrix}, \\
\tilde{Z} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & 0 \\ * & Z_{22} & Z_{23} & 0 \\ * & * & Z_{33} & 0 \\ * & * & * & 0 \end{bmatrix}.
\end{aligned}$$

From (22) we can see that if $\Omega_1 < 0$, $\Omega_2 < 0$ or $\tilde{\Omega}_1 < 0$, then $\dot{V}(t) \le -\lambda x^T(t)x(t)$ for some scalar $\lambda > 0$. Noting that $|\Delta\tau(t)| \le \delta$, $\forall t$, pre- and post-multiplying the both sides of $\Omega_2 < 0$ by $diag\{I,\ I,\ I,\ -sgn(\Delta\tau(t))I\}$, it is easy to see that matrix inequalities

$$\begin{bmatrix} -Z_{11} & -Z_{12} & -Z_{13} & \delta N_1^TA_{i1} \\ * & -Z_{22} & -Z_{23} & \delta N_2^TA_{i1} \\ * & * & -Z_{33} & \delta N_3^TA_{i1} \\ * & * & * & -\delta S \end{bmatrix} < 0 \quad (23)$$

imply $\Omega_2 < 0$, $\forall t \ge 0$ for $i = 1,2,\cdots,r$. From Lemma 1 and (23), the following matrix inequalities imply $\Omega_1 < 0$, $\Omega_2 < 0$ or $\tilde{\Omega}_1 < 0$ for $i,j = 1,2,\cdots,r$.

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & -\tau_{av}M_1^T & \delta N_1^TA_{i1} \\ * & \Phi_{22} & \Phi_{23} & -\tau_{av}M_2^T & \delta N_2^TA_{i1} \\ * & * & \Phi_{33} & -\tau_{av}M_3^T & \delta N_3^TA_{i1} \\ * & * & * & -\tau_{av}R & 0 \\ * & * & * & * & -\delta S \end{bmatrix} < 0 \quad (24)$$

Therefore, the system (13) is asymptotically stable according to Theorem 5.2.1([17], page 132) if (24) simultaneously hold for $i,j = 1,2,\cdots,r$.

Next, assuming that $\phi(t) = 0$, $t \in [-\tau_M, 0]$, we consider the performance index (10) of system (13). From (21) we have

$$
\begin{aligned}
J(w) &= \int_0^\infty [\tilde{z}^T(\theta)\tilde{z}(\theta) - \gamma^2 w^T(\theta)w(\theta)]d\theta \\
&\le \int_0^\infty [\tilde{z}^T(\theta)\tilde{z}(\theta) - \gamma^2 w^T(\theta)w(\theta) + \dot{V}(x_\theta)]d\theta \\
&= \begin{cases} \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \tilde{\Sigma}_{ij}, & \text{if } \tau(t) \ne \tau_{av}, \\ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \bar{\Sigma}_{ij}, & \text{if } \tau(t) = \tau_{av}, \end{cases} \qquad (25)
\end{aligned}
$$

where

$$
\begin{aligned}
&\tilde{\Sigma}_{ij} = \tfrac{1}{\tau_{av}} \int_0^\infty d\theta \int_{\theta-\tau_{av}}^{\theta} \xi^T(\theta,s)\Psi_1\xi(\theta,s)ds \\
&\quad + \tfrac{1}{\tau(t)-\tau_{av}} \int_0^\infty d\theta \int_{\theta-\tau(\theta)}^{\theta-\tau_{av}} \xi^T(\theta,s)\Phi_2\xi(\theta,s)ds, \\
&\bar{\Sigma}_{ij} = \tfrac{1}{\tau_{av}} \int_0^\infty d\theta \int_{\theta-\tau_{av}}^{\theta} \xi^T(\theta,s)\tilde{\Psi}_1\xi(\theta,s)ds, \\
&\Psi_1 = \Phi_1 + \Gamma, \\
&\tilde{\Psi}_1 = \tilde{\Phi}_1 + \Gamma, \\
&\Gamma = \begin{bmatrix} \tilde{C}^T\tilde{C} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & 0 \end{bmatrix}, \\
&\tilde{C} = C_i + D_i K_j.
\end{aligned}
$$

From (25) one can see that if $\Psi_1 < 0$, $\Phi_2 < 0$ or $\tilde{\Psi}_1 < 0$ simultaneously hold for $i, j = 1, 2, \cdots, r$, then $J(w) < 0$ for all nonzero $w(t) \in \mathcal{L}_2[0, \infty)$. Pre- and post-multiplying both sides of $\Phi_2 < 0$ by $diag\{I,\, I,\, I,\, I,\, -sgn(\Delta\tau(t))I\}$, it is easy to see that

$$
\begin{bmatrix} -Z_{11} & -Z_{12} & -Z_{13} & -Z_{14} & \delta N_1^T A_{i1} \\ * & -Z_{22} & -Z_{23} & -Z_{24} & \delta N_2^T A_{i1} \\ * & * & -Z_{33} & -Z_{34} & \delta N_3^T A_{i1} \\ * & * & * & -Z_{44} & \delta N_4^T A_{i1} \\ * & * & * & * & -\delta S \end{bmatrix} < 0 \quad (26)
$$

imply $\Phi_2 < 0$, $\forall t \ge 0$ for $i = 1, 2, \cdots, r$.

By Lemma 1,

$$
\begin{bmatrix} \tilde{\Phi}_1 + \Gamma & (1,2) \\ * & -\delta S \end{bmatrix} < 0, \quad (27)
$$

$$
(1,2) = \begin{bmatrix} \delta A_{i1}^T N_1 & \delta A_{i1}^T N_2 & \delta A_{i1}^T N_3 & 0 & 0 \end{bmatrix}^T,
$$

simultaneously hold for $i, j = 1, 2, \cdots, r$ imply $\Psi_1 < 0$, (26) or $\tilde{\Psi}_1 < 0$ for $i, j = 1, 2, \cdots, r$. In addition, (27) imply (24) for $i, j = 1, 2, \cdots, r$.

Noting that (27) are not LMIs, one can not solve it directly using MATLAB LMI Toolbox. In order to solve the matrix inequalities (27) efficiently, we define $N_1 = N_0$, $N_2 = \varepsilon_2 N_0$, and $N_3 = \varepsilon_3 N_0$, where $\varepsilon_i > 0$ ($i = 2, 3$). It is clear to see that (27) implies that N_0 is a nonsingular matrix. Furthermore, defining $X = N_0^{-1}$, $Y_j = K_j X$ ($j = 1, 2, \cdots, r$), $\tilde{P} = X^T P X$, $\tilde{M}_i = X^T M_i X$ ($i = 1, 2, 3$), $\tilde{Q} = X^T Q X$, $\tilde{R} = X^T R X$ and $\tilde{S} = X^T S X$. Then pre- and post-multiplying both sides of (27) with $diag\{\, X^T \;\; X^T \;\; X^T \;\; I \;\; X^T \;\; X^T \,\}$ and its transpose, respectively, we can obtain (15) by Schur complement. This completes the proof. ■

Remark 1: From the process of proof of Proposition 1, one can see that the system (14) with $w(t) \equiv 0$ is asymptotically stable if (24) hold for $i, j = 1, 2, \cdots, r$. In [15], Han and Gu studied the stability of the following system with time-varying interval delay using a generalized discretized Lyapunov functional approach.

$$
\dot{x}(t) = Ax(t) + A_1 x(t - \tau(t)).
$$

But additional information regarding the derivative of the time-varying delay, i.e. $\dot{\tau}(t) \le \tau_d < 1$, is needed. From Proposition 1, one can see that this restriction is **removed**, which means that a **fast** time-varying delay is allowed.

Concerning uncertainty appeared in system (9), by Proposition 1, one can easily obtain the following result.

Corollary 1: For a prescribed scalar $\gamma > 0$ and some given scalars $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, τ_m and τ_M, system (6) is robustly stablizable under controller (8) and satisfies $\|\tilde{z}(t)\|_2 < \gamma \|w(t)\|_2$ for all nonzero $w(t) \in \mathcal{L}_2[0, \infty)$, any $\tau(t)$ satisfying (4) and all admissible uncertainties satisfying (2) and (3), if there exist some scalars $\chi_i > 0$ ($i = 1, 2, \cdots, r$), some matrices $\tilde{P} > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\tilde{S} > 0$, X, Y_j ($j = 1, 2, \cdots, r$), $\tilde{M}_i$ ($i = 1, 2, 3$) of appropriate dimensions such that the following LMIs simultaneously hold for $i, j = 1, 2, \cdots, r$

$$
\begin{bmatrix} \Theta & \bar{\Theta}_1 & \bar{\Theta}_2 \\ * & -\chi_i I & 0 \\ * & * & -\chi_i I \end{bmatrix} < 0, \quad (28)
$$

where

$$
\begin{aligned}
\bar{\Theta}_1 &= \left[H_i^T \chi_i^T, H_i^T \varepsilon_2 \chi_i^T, H_i^T \varepsilon_3 \chi_i^T, 0, 0, 0, 0\right]^T, \\
\bar{\Theta}_2 &= \left[E_{i0}X + E_{ib}Y_j, E_{i1}X, 0, 0, 0, \delta E_{i1}X, 0\right]^T.
\end{aligned}
$$

Moreover, the state feedback controller gains of (8) are given by $K_j = Y_j X^{-1}$ for $j = 1, 2, \cdots, r$.

Proof: Replacing A_{i0}, A_{i1} and B_i with $A_{i0} + H_i F_i(t) E_{i0}$, $A_{i1} + H_i F_i(t) E_{i1}$ and $B_i + H_i F_i(t) E_{ib}$, respectively, in (15) yields

$$
\Theta + \vartheta_{i1} F_i(t) \vartheta_{i2}^T + \vartheta_{i2} F_i^T(t) \vartheta_{i1}^T < 0, \quad (29)
$$

where $i = 1, 2, \cdots, r$,

$$
\begin{aligned}
\vartheta_{i1}^T &= \left[H_i^T, \varepsilon_2 H_i^T, \varepsilon_3 H_i^T, 0, 0, 0, 0\right], \\
\vartheta_{i2}^T &= \left[E_{i0}X + E_{ib}Y_j, E_{i1}X, 0, 0, 0, \delta E_{i1}X, 0\right].
\end{aligned}
$$

It is obvious that (29) is equivalent to

$$
\Theta + \chi_i \vartheta_{i1} \vartheta_{i1}^T + \chi_i^{-1} \vartheta_{i2} \vartheta_{i2}^T < 0, \quad (30)
$$

for any $\chi_i > 0$, $i = 1, 2, \cdots, r$. By Schur complement, (30) is equivalent to (28). This completes the proof of this Corollary. ■

IV. A NUMERICAL EXAMPLE

In this section, we will apply the proposed method to design a delay-dependent robust H_∞ controller for an uncertain nonlinear delay system. The uncertain nonlinear time-delay system is decribed as follows

$$
\begin{cases}
\dot{x}_1(t) = -x_1(t)(2 + \sin^2 x_2(t)) + x_2(t) \\
\qquad +0.1x_1(t - \tau(t)) + 0.2x_2(t - \tau(t)) \cos^2 x_2(t) \\
\qquad +c(t)x_2(t) \sin^2 x_2(t) + c(t)x_1(t) \cos^2 x_2(t) \\
\qquad +u_1(t) + (1 + \sin^2 x_2(t))w(t) \\
\dot{x}_2(t) = x_1(t) - x_2(t)(1 - \cos^2 x_2(t)) \\
\qquad +0.2x_1(t - \tau(t)) \sin^2 x_2(t) - 0.5x_2(t - \tau(t)) \\
\qquad +0.5u_2(t) + 0.1c(t)x_2(t)
\end{cases} \quad (31)
$$

where $c(t)$ is an uncertain parameter satisfying

$$c(t) \in [-0.2, 0.2]. \tag{32}$$

If we select the membership functions as

$M_1(x_2(t)) = \sin^2(x_2(t))$ and $M_2(x_2(t)) = \cos^2(x_2(t))$,

then, the nonlinear time-delay system (31) can be represented by the following uncertain time-delay T-S model

Plant Rule 1:

IF $x_2(t)$ is M_1

$$\text{THEN} \begin{cases} \dot{x}(t) = [A_{10} + \Delta A_{10}(t)]x(t) \\ \quad +A_{11}x(t-\tau(t)) + B_1\Delta u(t) + B_{w1}w(t), \\ \tilde{z}(t) = C_1 x(t) + D_1 u(t), \end{cases}$$

and

Plant Rule 2:

IF $x_2(t)$ is M_2

$$\text{THEN} \begin{cases} \dot{x}(t) = [A_{20} + \Delta A_{20}(t)]x(t) \\ \quad +A_{21}x(t-\tau(t)) + B_2\Delta u(t) + B_{w2}w(t), \\ \tilde{z}(t) = C_2 x(t) + D_2 u(t), \end{cases}$$

where

$$\tau(t) = 1 + 0.2q(t), \ |q(t)| \leq 1, \tag{33}$$

$$\begin{aligned}
A_{10} &= \begin{bmatrix} -3 & 1 \\ 1 & -1 \end{bmatrix}, A_{11} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & -0.5 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, B_{w1} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, D_1 = I; \\
A_{20} &= \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.5 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, B_{w2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, D_2 = I,
\end{aligned}$$

and $\Delta A_{10}(t)$ and $\Delta A_{20}(t)$ can be represented in the from of (2) and (3) with

$$\begin{aligned}
H_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_{10} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix}; \\
H_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_{10} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix}.
\end{aligned}$$

For this example, the H_∞ performance level is chosen as $\gamma = 1$. In order to design a robust H_∞ feedback controller (8) for the given T-S model, choosing $\varepsilon_2 = 0.02$, $\varepsilon_3 = 0.54$ and using MATLAB LMI Toolbox to solve the LMIs (28), a desired robust H_∞ fuzzy feedback controller can be constructed as (8) with

$$\begin{aligned}
K_1 &= \begin{bmatrix} -1.1583 & -0.9259 \\ -0.8872 & -0.6348 \end{bmatrix}, \\
K_2 &= \begin{bmatrix} -0.7461 & -0.5782 \\ -0.5130 & -1.1010 \end{bmatrix}.
\end{aligned} \tag{34}$$

It is to say that the given T-S model is robustly stablizable under controller (8) with controller gain (34) and satisfies $\|\tilde{z}(t)\|_2 < \gamma \|w(t)\|_2$ for all nonzero $w(t) \in \mathcal{L}_2[0, \infty)$, any time-varying delay $\tau(t)$ satisfying (33) and all admissible uncertainties satisfying (32). In fact, setting $\tau_{av} = 1$, we can obtain the maximum allowed bound $\delta_{\max} = 0.2358$ to guarantee the robust H_∞ fuzzy stablizable of (31) for any $\tau(t) \in [0.7642,\ 1.2358]$.

V. CONCLUSION

The problem on the delay-dependent robust H_∞ controller design has been studied for a class of Takagi-Sugeno fuzzy-model-based systems with interval time-varying delay and norm-bounded parameter uncertainty. Based on the Lyapunov-Krasovskii Functional approach, a sufficient condition for the existence of the robust H_∞ controller, which robustly stabilizes the T-S fuzzy-model-based uncertain systems and guarantees a prescribed level on disturbance attenuation, has been obtained in an LMI form. No restriction on the derivative of the time-varying delay is needed, which has allowed a **fast** time-varying delay. The given numerical example has shown effectiveness of the proposed method.

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