# Adaptive Accommodation of Failures in Second-Order Flight Control Actuators with Measurable Rates 

Jovan D. Bošković, Sarah E. Bergstrom, and Raman K. Mehra


#### Abstract

In this paper effective Failure Detection, Identification and Reconfiguration (FDIR) algorithms are developed for a class of linearized aircraft models and second-order actuator dynamics. Assuming that the actuator dynamics are fast, a baseline controller is designed and, using the singular perturbation arguments, shown to achieve the control objective. Typical failures in flight control actuators described by first and second order dynamics are considered next, and the FDI algorithms are derived for the latter case. This is followed by the design of a corresponding adaptive reconfigurable controller, and the main theorem is proved stating that all the signals in the system are bounded and that the tracking error converges to zero asymptotically despite multiple simultaneous actuator failures. The properties of the proposed FDIR algorithms are evaluated through numerical simulations of the F-18 aircraft.


## I. Introduction

In the past several years there has been significant progress in the area of on-line Failure Detection, Identification and Reconfiguration (FDIR) in flight control applications [1], [2], [3], [4], [6], [7], [8]. Several of the proposed approaches have been demonstrated as efficient tools for reconfigurable control design, and some have even been flight tested [3]. However, while the case of firstorder actuator dynamics was considered in [5], the available techniques do not appear to be well suited for the case of higher-order actuator dynamics. On the other hand, it is well established that the dynamics of most of the flight control actuators are of (at least) second order, and can be sufficiently accurately described by linear second-order relative degree two models. In addition, in many situations only the output of the actuator is available for measurement, i.e. its rate is not measurable, which makes the related FDIR problem highly challenging.

In this paper new Failure Detection, Identification and Reconfiguration (FDIR) algorithms that are well suited for the case of second-order actuator dynamics with measurable rates are proposed. This is the first step toward solving a more general problem when the actuator rates are not available. The algorithms are a part of the Integrated FDIR scheme shown in Figure 1 that is seen to be based on the local FDI observers. While the local observers estimate failure-related parameters on-line for each of the actuators,

[^0]the FDI information is used globally by the adaptive reconfigurable controller to accommodate the failure. The main advantage of such a scheme, described in detail in [5], is that it is well suited for multiple simultaneous actuator failures.


Fig. 1. The Integrated FDIR Scheme
The flight control actuator failures can be broadly divided into two categories: (i) Failures that result in a total loss of effectiveness of the control effector; and (ii) Failures that cause partial loss of effectiveness. The former category includes Lock-In-Place (LIP), float, and Hard-Over Failure (HOF), while the latter is referred to as the Loss-OfEffectiveness (LOE) type of failure. Distinguishing between these types of failures is not an easy task. When it needs to be solved in the presence of second-order actuator dynamics, the problem becomes highly difficult.

Another issue arising in on-line FDIR in flight control is that of failure recovery. In the case of failure accommodation, immediately following the control reconfiguration the system achieves a new operating regime characterized by the accommodated failure. For instance, if there is a lock-in-place of an actuator in an over-actuated flight control system, the corresponding control channel is disconnected, and the remaining control inputs are reconfigured to accommodate the failure and achieve the control objective. If, in such a system, there is a failure recovery, the recovered actuator may cause a disturbance that may perturb the system even more than the original failure, and the system can become unstable. Hence techniques that effectively deal both with failures and failure recoveries are of importance in practice.

In this paper a FDIR scheme is developed for multiple simultaneous failures and failure recoveries of control actuators whose behavior is described by second-order dynamic models. It is shown that the stability of the overall closedloop system can be guaranteed, and that the convergence of the tracking error to zero is assured despite the presence of multiple simultaneous failures. The properties of the
proposed scheme are illustrated through simulations of the F/A-18C/D aircraft.

## II. Problem Statement

In this paper our focus is on linearized aircraft models with state variables accessible. The model is of the form:

$$
\begin{align*}
\dot{x}_{1} & =x_{2}  \tag{1}\\
\dot{x}_{2} & =A x+B u_{1}  \tag{2}\\
\dot{u}_{1} & =u_{2}  \tag{3}\\
\dot{u}_{2} & =-\lambda_{1} u_{1}-\lambda_{2} u_{2}+\lambda_{1} u_{c} \tag{4}
\end{align*}
$$

where $x \in \mathbb{R}^{n}$ and $u \in \mathbb{R}^{m}(m>n)$ denote respectively the state and control input vectors, $u_{c} \in \mathbb{R}^{m}$ is the signal generated by the controller, and $\lambda_{1} \gg \lambda_{2}$ and $\lambda_{1} \gg 1$. It is interesting to note that, in many cases, the dynamics of the flight control actuators is described by the above model where $\lambda_{1}=\omega_{n}^{2}, \lambda_{2} \in\left[0.7 \omega_{n}, 1.4 \omega_{n}\right], \omega_{n}$ denotes the natural frequency, and $\omega_{n} \geq 30$, so that $\lambda_{1} / \lambda_{2} \geq 20$. Hence, in practice, the above assumptions are justified in most of the cases.

In this paper the focus will be on a class of failure scenarios that satisfy the following assumption:

## Assumption 1:

(a) Up to $m-n$ effectors can undergo total LOE failure
(b) All effectors can undergo partial LOE failure.

Reference Model: The reference model is chosen in the form:

$$
\begin{align*}
\dot{x}_{1}^{*} & =x_{2}^{*},  \tag{5}\\
\dot{x}_{2}^{*} & =A_{m} x^{*}+B_{m} r, \tag{6}
\end{align*}
$$

where $x^{*}$ is the state of the reference model, the reference model is asymptotically stable, and $r$ is a vector of bounded piece-wise continuous reference inputs.
Control Objective: The objective is to design a control law $u_{c}(t)$ such that the error $x(t)-x^{*}(t)$ tends to zero asymptotically even in the presence of different control effector failures.

Baseline Control Strategy: To achieve the control objective in the case without control effector failures, the Inverse Dynamics Control Law (IDCL) is chosen:

$$
u_{c}=W B^{T}\left(B W B^{T}\right)^{-1} \eta
$$

where

$$
\begin{equation*}
\eta=-A x+A_{m} x+B_{m} r . \tag{7}
\end{equation*}
$$

To demonstrate that the above control law achieves the objective, the expression (1) is rewritten as:

$$
\dot{x}_{2}=A_{1} x_{1}+A_{2} x_{2}+B u_{1},
$$

and multiplied by $s^{2}+\lambda_{2} I s+\lambda_{1} I$ to obtain:

$$
\begin{aligned}
x_{2}^{(3)}+\lambda_{2} \ddot{x}_{2}+\lambda_{1} \dot{x}_{2}= & A_{1} \ddot{x}_{1}+\lambda_{2} A_{1} \dot{x}_{1}+\lambda_{1} A_{1} x_{1}+A_{2} \ddot{x}_{2} \\
& +\lambda_{2} A_{2} \dot{x}_{2}+\lambda_{1} A_{2} x_{2}+\lambda_{1} B u_{c},
\end{aligned}
$$

from where one has:

$$
\begin{aligned}
& \frac{1}{\lambda_{1}}\left(x_{2}^{(3)}-A_{1} \ddot{x}_{1}-A_{2} \ddot{x}_{2}\right) \\
& +\frac{\lambda_{2}}{\lambda_{1}}\left(\ddot{x}_{2}-A_{2} \dot{x}_{2}-A_{1} \dot{x}_{1}\right)+\dot{x}_{2}=A x+B u_{c} .
\end{aligned}
$$

Since $\lambda_{1} \gg 1$ and $\lambda_{1} \gg \lambda_{2}$, it follows that $1 / \lambda_{1} \cong 0$ and $\lambda_{2} / \lambda_{1} \cong 0$. Hence, using the singular perturbation arguments, it can be concluded that the approximate lower order dynamics of the plant is of the form:

$$
\begin{aligned}
\dot{x}_{1} & =x_{2} \\
\dot{x}_{2} & =A x+B u_{c} .
\end{aligned}
$$

Upon substituting the control law (7) into the above equation, one obtains $\ddot{x}_{1}=A_{m} x+B_{m} r$. Hence the reduced-order dynamics of the closed-loop system coincides with that of the reference model (modulo initial conditions), and the IDCL achieves the objective in an approximate sense.

## III. Failure Modeling

In [4] actuator failure models were derived for the case of insignificant actuator dynamics, while in [5] the case of first-order actuator dynamics was considered. This case will be also described. However, the focus of this paper is on second-order actuator dynamics. In both cases, the failures are modeled in terms of some minimum number of uncertain failure-related parameters. Uncertainty associated with each of the actuator models is due to: (i) Unknown time of failure $t_{F i}$, (ii) Unknown LOE coefficient $k_{i}$, and (iii) Unknown value at which the control effector locks.

## A. First-Order Actuator Dynamics

Nominal Model: In this case the nominal (no-failure) model is of the form:

$$
\begin{equation*}
\dot{u}_{i}=-\lambda\left(u_{i}-u_{c i}\right), i=1,2, \ldots, m \tag{8}
\end{equation*}
$$

where $u_{i}$ denotes the output of $i$ th the actuator, $u_{c i}$ is the signal generated by the controller, and $\lambda \gg 1$.
Total LOE: The case of total Loss-Of-Effectiveness (LOE) includes Lock-In-Place (LIP), float, and Hard-Over-Failure (HOF). For this case the following model is proposed [5]:

$$
\begin{equation*}
\dot{u}_{i}=-\sigma_{i} \lambda\left(u_{i}-u_{c i}\right), \tag{9}
\end{equation*}
$$

where $\sigma_{i}(t)=1$ in the case of no failure, and $\sigma_{i}(t)=$ $0, u\left(t_{F i}\right)={ }^{-}{ }_{i t}$ when the failure occurs at $t=t_{F i}$, where $t_{F i}$ denotes the time of failure of the $i$ th effector. Hence in the case of failure at $t_{F i}$, one has that $\dot{u}_{i}(t)=0$ for $t \geq t_{F i}$, and $u(t)=u\left(t_{F i}\right)$ for all $t \geq t_{F i}$. In the case of LIP, $u\left(t_{F i}\right)$ has the value of $u\left(t_{F i}^{-}\right)$, while in the case of HOF, it jumps to the upper or lower position limit.
Partial LOE: In this case the actuator gain $k_{i}$, whose nominal value is one, decreases to a value from the interval $\left[\varepsilon_{i}, 1\right)$, where $\varepsilon_{i} \ll 1$. The corresponding model that covers both the nominal and the failure cases is of the form:

$$
\begin{equation*}
\dot{u}_{i}=-\lambda\left(u_{i}-k_{i} u_{c i}\right), \tag{10}
\end{equation*}
$$

where $k_{i} \in\left[\varepsilon_{i}, 1\right]$.
Total and Partial LOE: Both types of failures can be described by a single model of the form:

$$
\begin{equation*}
\dot{u}_{i}=-\sigma_{i} \lambda\left(u_{i}-k_{i} u_{c i}\right) . \tag{11}
\end{equation*}
$$

## B. Second-Order Actuator Dynamics

In this section the index $i$ will be omitted to simplify the notation. It is still assumed that there are $m$ actuators.
Nominal Model: In this case the actuator dynamics is described by a stable second-order model of the form:

$$
\begin{align*}
\dot{u}_{1} & =u_{2}  \tag{12}\\
\dot{u}_{2} & =-\lambda_{1} u_{1}-\lambda_{2} u_{2}+\lambda_{1} u_{c} \tag{13}
\end{align*}
$$

$\lambda_{j i} \gg 1$ and $\lambda_{2} \gg \lambda_{1}, j=1,2$.
Total LOE: In this paper the total LOE failure model is proposed in the form:
$\dot{u}_{1}(t)=u_{2}(t)$
$u_{2}(t)=\left\{\begin{array}{cc}-\int_{0}^{t}\left[\lambda_{1} u_{1}(\tau)+\lambda_{2} u_{2}(\tau)-\lambda_{1} u_{c}(\tau)\right] d \tau, & t<t_{F}, \\ 0, & t \geq t_{F}\end{array}\right.$
The above model describes the fact that $u_{2}(t)$ becomes zero instantaneously when there is a total LOE failure. However, the model is not in a convenient form for observer design since the dynamics of $u_{2}$ is described by an integral equation. For this reason an approximate total LOE model is proposed in the form:

$$
\begin{align*}
& \dot{u}_{1}=\sigma u_{2}  \tag{14}\\
& \dot{u}_{2}=-\left[\lambda_{2}+(1-\sigma) \beta\right] u_{2}+\sigma \lambda_{1}\left(u_{c}-u_{1}\right), \tag{15}
\end{align*}
$$

where $\lambda_{2}+\beta \gg 1$, and

$$
\sigma= \begin{cases}1, & \text { if } t<t_{F} \\ 0, & \text { if } t \geq t_{F}\end{cases}
$$

It is seen that, when $\sigma=0, u_{2}$ tends to zero asymptotically with the rate of convergence dominated by $\lambda_{2}+\beta$. By properly choosing $\beta$, arbitrarily fast convergence can be obtained to emulate the situation in the case of total LOE failures when, at the time instant of the failure, all derivatives are instantaneously set to zero.
Total and Partial LOE: The model that describes both partial and total LOE is proposed in the form:

$$
\begin{align*}
& \dot{u}_{1}=\sigma u_{2}  \tag{16}\\
& \dot{u}_{2}=-\left[\lambda_{2}+(1-\sigma) \beta\right] u_{2}+\sigma \lambda_{1}\left(k u_{c}-u_{1}\right) \tag{17}
\end{align*}
$$

where $k \in[\varepsilon, 1]$. It is seen that, for $\sigma=k=1$, the above model reduces to the form (12)-(13).

## IV. On-Line FDI for Second-Order Actuator Dynamics

In this section the local FDI algorithms are designed for second-order actuator dynamics. The following assumption is introduced:

## Assumption 2:

1. $u_{1}$ and $\dot{u}_{1}$ are measurable.
2. $\lambda_{2}$ is sufficiently large to assure fast convergence of $u_{2}(t)$ to zero in the case of failure. Hence $\beta$ in (17) can be set to zero.

Observer: To design an observer for the model (16), (17), a derivative of the expression (16) is taken for $\beta=0$ to obtain:

$$
\begin{aligned}
\ddot{u}_{1} & =\sigma \dot{u}_{2} \\
& =-\lambda_{2} \dot{u}_{1}+\sigma^{2} \lambda_{1}\left(k u_{c}-u_{1}\right) \\
& =-\lambda_{1} u_{1}-\lambda_{2} \dot{u}_{1}+\sigma \lambda_{1} k u_{c}+(1-\sigma) \lambda_{1} u_{1},
\end{aligned}
$$

where the fact is used that, since $\sigma \in\{0,1\}$, one has that $\sigma^{2}=\sigma$. It is important to note that the above system can be expressed in the form:

$$
\begin{equation*}
u_{1}=\frac{\lambda_{1}}{s^{2}+\lambda_{2} s+\lambda_{1}}\left[\sigma k u_{c}-(1-\sigma) u_{1}\right] \tag{18}
\end{equation*}
$$

modulo exponentially decaying initial conditions.
By letting $\eta_{1}=u_{1}$ and $\eta_{2}=\dot{u}_{1}$, one now has:

$$
\begin{align*}
\dot{\eta}_{1} & =\eta_{2} \\
\dot{\eta}_{2} & =-\lambda_{2} \eta_{2}+\sigma \lambda_{1}\left(k u_{c}-\eta_{1}\right) \tag{19}
\end{align*}
$$

Since both unknown parameters are in the second equation of the transformed system, the observer is now chosen in the form:

$$
\begin{equation*}
\dot{\hat{\eta}}_{2}=-\lambda_{2} \eta_{2}+\hat{\sigma} \lambda_{1}\left(\hat{k} u_{c}-\eta_{1}\right)-\tau \hat{e} \tag{20}
\end{equation*}
$$

where $\tau>0$, and $\hat{e}=\hat{\eta}_{2}-\eta_{2}$.
Error Model: After subtracting (19) from (20), one obtains:

$$
\begin{equation*}
\dot{\hat{e}}=-\tau \hat{e}+\phi_{\sigma} \lambda_{1}\left(\hat{k} u_{c}-\eta_{1}\right)+\sigma \lambda_{1} \phi_{k} u_{c} \tag{21}
\end{equation*}
$$

where $\phi_{\sigma}=\hat{\sigma}-\sigma$ and $\phi_{k}=\hat{k}-k$.
Let $\omega_{\sigma}=\hat{k} u_{c}-\eta_{1}$ and $\omega_{k}=u_{c}$. The following theorem is considered next:
Theorem 1: The following adaptive laws assure that $\hat{e} \in$ $\mathscr{L}^{\infty} \cap \mathscr{L}^{2}$ :

$$
\begin{align*}
\dot{\hat{\sigma}} & =\operatorname{Proj}_{[0,1]}\left\{-\gamma_{\sigma} \hat{e} \omega_{\sigma}\right\},  \tag{22}\\
\dot{\hat{k}} & =\operatorname{Proj}_{[\varepsilon, 1]}\left\{-\gamma_{k} \hat{e} \omega_{k}\right\}, \tag{23}
\end{align*}
$$

where the projection operator is used to keep the parameter estimates within the parameter bounds.
Proof: Let a tentative Lyapunov function be of the form:

$$
\begin{equation*}
V\left(\hat{e}, \phi_{\sigma}, \phi_{k}\right)=\frac{1}{2}\left[\hat{e}^{2}+\frac{\phi_{\sigma}}{\gamma_{\sigma}}+\sigma \frac{\phi_{k}}{\gamma_{k}}\right] . \tag{24}
\end{equation*}
$$

The following property of the adaptive algorithms with projection is used next (see e.g.[4]): if the adaptive law is of the form $\dot{\theta}=\operatorname{Proj}_{[-\bar{\theta}, \bar{\theta}]}\{-e \omega\}$, then:

$$
\theta \dot{\theta} \leq-e \theta \omega
$$

With this fact the first derivative of $V$ along the solutions of the system is:

$$
\begin{equation*}
\dot{V} \leq-\tau \hat{e}^{2} \leq 0 \tag{25}
\end{equation*}
$$

Hence $\hat{e}$ is bounded ( $\phi_{\sigma}$ and $\phi_{k}$ are bounded due to the use of the projection algorithm). Upon integrating $\dot{V}$ from 0 to $\infty$, one obtains:

$$
V(0)-V(\infty) \geq \tau \int_{0}^{\infty} \hat{e}^{2}(\tau) d \tau
$$

Since the term on the left hand side is bounded, it follows that $\hat{e} \in \mathscr{L}^{2}$.

## V. Adaptive Reconfigurable Controller

To design a reconfigurable controller that effectively compensates for the effect of both total and partial LOE, the expression (1) is first rewritten as:

$$
\begin{equation*}
\dot{x}_{2}=A_{1} x_{1}+A_{2} x_{2}+B u, \tag{26}
\end{equation*}
$$

and multiplied by $s^{2}+\lambda_{2} I s+\lambda_{1} I$ to obtain:

$$
\begin{aligned}
x_{2}^{(3)}+\lambda_{2} \ddot{x}_{2}+\lambda_{1} \dot{x}_{2}= & A_{1} \ddot{x}_{1}+\lambda_{2} A_{1} \dot{x}_{1}+\lambda_{1} A_{1} x_{1}+A_{2} \ddot{x}_{2} \\
& +\lambda_{2} A_{2} \dot{x}_{2}+\lambda_{1} A_{2} x_{2} \\
& +\lambda_{1} \sum_{i=1}^{m} b_{i}\left[\sigma_{i} k_{i} u_{c i}+\left(1-\sigma_{i}\right) u_{1 i}\right]
\end{aligned}
$$

where the expression (18) was used.
Upon dividing the above expression by $\lambda_{1}$ and neglecting the terms containing the ratio $1 / \lambda_{1}$ or $\lambda_{2} / \lambda_{1}$, one obtains:

$$
\begin{equation*}
\dot{x}_{2}=A x+\sum_{i=1}^{m}\left[\sigma_{i} k_{i} u_{c i}+\left(1-\sigma_{i}\right) u_{1 i}\right] . \tag{27}
\end{equation*}
$$

Ideal Reconfigurable Controller: To design the reconfigurable controller, let:

$$
\begin{align*}
B_{o}(\sigma) & =\left[\begin{array}{lll}
\sigma_{1} b_{1} & \sigma_{2} b_{2} \ldots \sigma_{m} b_{m}
\end{array}\right]  \tag{28}\\
K & =\operatorname{diag}\left[k_{1}, k_{2}, \ldots k_{m}\right] . \tag{29}
\end{align*}
$$

The ideal reconfigurable controller is now chosen in the following form:

$$
\begin{equation*}
u_{c}=W K B_{o}^{T}\left(B_{o} K W K B_{o}^{T}\right)^{-1}\left(\eta-\sum_{i=1}^{n}\left(1-\sigma_{i}\right) u_{1 i}\right) \tag{30}
\end{equation*}
$$

where

$$
\begin{equation*}
\eta=-A x+A_{m} x+B_{m} r . \tag{31}
\end{equation*}
$$

Hence, if $\sigma_{i}$ and $k_{i}$ are known, the above controller achieves the objective. However, since the time and type of failure are generally unknown, the adaptive reconfigurable controller is implemented by replacing the failure-related parameters $\sigma_{i}$ and $k_{i}$ with their estimates, as discussed below.

Adaptive Reconfigurable Controller: Let

$$
\begin{align*}
\hat{B}_{o} & =\left[\begin{array}{llll}
\hat{\sigma}_{1} b_{1} & \hat{\sigma}_{2} b_{2} & \ldots & \hat{\sigma}_{m} b_{m}
\end{array}\right]  \tag{32}\\
\hat{K} & =\operatorname{diag}\left[\hat{k}_{1}, \hat{k}_{2}, \ldots . . \hat{k}_{m}\right] \tag{33}
\end{align*}
$$

The adaptive reconfigurable controller is now chosen in the form:

$$
\begin{equation*}
u_{c}=W \hat{K} \hat{B}_{o}^{T}\left(\hat{B}_{o} \hat{K} W \hat{K} \hat{B}_{o}^{T}\right)^{-1}\left(\eta-\sum_{i=1}^{n}\left(1-\hat{\sigma}_{i}\right) u_{i}\right) \tag{34}
\end{equation*}
$$

A question that arises in this context is whether the above controller, with the true values of parameters replaced by their estimates, achieves the control objective. This is discussed in the following section.

## VI. Main Result

The properties of the overall FDIR system are summarized in the following theorem:

Theorem 2: The closed-loop system (1)-(4), (34), where $\eta$ is given by (31), and where the adaptive parameters are adjusted using (22), (23), is stable and, even in the presence of total or partial LOE failures, $\lim _{t \rightarrow \infty}\left[x(t)-x^{*}(t)\right]=0$.
Proof: It is first recalled that, using the results of Theorem


The main objective is to show that the tracking error $e_{c}=x-x^{*}$ is bounded. The expression (27) is first rewritten as:

$$
\begin{aligned}
\dot{x}_{1}= & x_{2} \\
\dot{x}_{2}= & \left.A x+\sum_{i=1}^{m} b_{i}\left[\sigma_{i} k_{i} u_{c i}+\left(1-\sigma_{i}\right) u_{i}\right)\right] \\
= & A x+\sum_{i=1}^{m} b_{i}\left[-\sigma_{i} \phi_{k i} u_{c i}\right. \\
& \left.+\hat{\sigma}_{i} \hat{k}_{i} u_{c i}+\phi_{\sigma i}\left(u_{i}-\hat{k}_{i} u_{c i}\right)+\left(1-\hat{\sigma}_{i}\right) u_{i}\right] \\
= & A x+\sum_{i=1}^{m} b_{i}\left[\hat{\sigma}_{i} \hat{k}_{i} u_{c i}+\left(1-\hat{\sigma}_{i}\right) u_{i}+\phi_{i}^{T} \omega_{i}\right] .
\end{aligned}
$$

The controller equation (34) is substituted next, and the expressions (5), (6) are used to obtain:

$$
\dot{e}_{c}=A_{m} e_{c}-\sum_{i=1}^{m} b_{i} \phi_{i}^{T} \omega_{i}
$$

The above expression can be rewritten as:

$$
e_{c}=-W(s) \sum_{i=1}^{m} b_{i} \frac{1}{s+\tau} \phi_{i}^{T} \omega_{i},
$$

where exponentially decaying terms due to initial conditions are neglected, and where $W(s)=\left(s I-A_{m}\right)^{-1}(s+\tau)$. It is noted that the transfer function matrix $\left(s I-A_{m}\right)^{-1}$ is asymptotically stable and minimum phase, and the minimum relative degree of the individual transfer functions is one. Adding a stable zero will not change these properties, and the elements of the resulting transfer function matrix $W(s)$ will be at most proper.

Since from (21) it follows that:

$$
\hat{e}_{i}=\frac{1}{s+\tau} \phi_{i}^{T} \omega_{i},
$$

where exponentially decaying terms due to initial conditions are again neglected, one has that

$$
\begin{equation*}
e_{c}=-W(s) \sum_{i=1}^{m} b_{i} \hat{e}_{i} \tag{35}
\end{equation*}
$$

Since it has already been shown that each $\hat{e}_{i}$ is bounded and belongs to $\mathscr{L}^{2}$, and since $W(s)$ is asymptotically stable and minimum phase transfer function matrix with proper or strictly proper elements, it follows that $e_{c}$ is also bounded and belongs to $\mathscr{L}^{2}$. Since $x^{*}$ is bounded, it follows that $x$ is bounded as well. Boundedness of $x$ implies the boundedness of $u$, which in turn implies boundedness of each $\omega_{i}$. Hence each $\dot{\hat{e}}_{i} \in \mathscr{L}^{\infty}$, which now, from the Barbalat's lemma [9], implies that $\lim _{t \rightarrow \infty} \hat{e}_{i}(t)=0$. From (35) it can now be concluded that $\lim _{t \rightarrow \infty} e_{c}(t)=0$.

## VII. Simulations

As a simulation example, a linearized dynamics of a F-18 aircraft is chosen during carrier landing. The most critical actuator during landing is that for the stabilator that controls the pitch rate. Ailerons are fully deflected, while the leading-edge flaps and rudder toe-ins are not used during normal operation. The dynamics of the stabilators and rudders is described by the model (3), (4), where $\lambda_{1}=$ $1325, \lambda_{2}=30$ for the stabilator, and $\lambda_{1}=5200, \lambda_{2}=99.5$ for the rudders. A typical response in the nominal (no-failure) case is shown in Figure 2.

The FDIR scheme is designed for stabilators and rudders only, since the other control surfaces are not critical during landing. Also, in this case it turns out that the loss-ofeffectiveness can be effectively handled by the baseline controller. Hence the focus will be on multiple lock-in-place failures. The FDIR algorithms presented in the paper are used, and their performance is evaluated in both the failure and failure recovery cases. Failure recovery capability is important since a standard approach is to stop adjusting the failure-related parameters after the failure, so that, even if there is a recovery, the system continues working without that actuator. This may be beneficial for avoiding the disturbance caused by a potential recovery. However, if there is a failure of another actuator, the system may become under-actuated even though there is a sufficient number of healthy actuators.

The failure scenario chosen is listed in Table 1.

| Actuator | Failure | Recovery | Time |
| :---: | :---: | :---: | :---: |
| Left Stabilator | + |  | 2 sec |
| Left Rudder | + |  | 2 sec |
| Left Stabilator |  | + | 6 sec |
| Right Rudder | + |  | 7 sec |
| Right Stabilator | + |  | 8 sec |
| Left Rudder |  | + | 9 sec |

Table 1: The failure scenario used in simulations
It is noted that the system quickly becomes unstable if the failures are not accommodated. Also, if, after the first failure (left stabilator and left rudder at $t=2$ seconds),
the parameter adjustment is stopped, the system becomes unstable. The response in the case when the recovery is allowed is shown in Figure 3. It is seen that all failures and recoveries are accurately detected and identified, and that the overall response is excellent despite multiple failures and recoveries.

## VIII. Conclusions

In this paper effective Failure Detection, Identification and Reconfiguration (FDIR) algorithms are developed for a class of linearized aircraft models and second-order actuator dynamics. Assuming that the actuator dynamics are fast, a baseline controller is designed and, using the singular perturbation arguments, shown to achieve the control objective. Typical failures in flight control actuators described by first and second order dynamics are considered next, and the FDI algorithm is derived for the latter case. This is followed by the design of a corresponding adaptive reconfigurable controller, and the main theorem is proved stating that all the signals in the system are bounded and that the tracking error converges to zero asymptotically despite multiple simultaneous actuator failures. The properties of the proposed FDIR algorithm are evaluated through numerical simulations of the F-18 aircraft.
The proposed FDIR algorithms assume that the actuator velocity is measurable. A challenging related problem is to design an efficient FDIR scheme for the case when only the actuator output is available. Such a case is the focus of our current research.

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Fig. 2. Response with the baseline controller in the no-failure case


Fig. 3. Response with the adaptive reconfigurable controller for the scenario from Table 1


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    The authors are with Scientific Systems Company, Inc., 500 W. Cummings Park, Suite 3000, Woburn, MA 01801, jovan@ssci.com, seb@ssci.com, rkm@ssci.com

