Application of VSC Reliable Design to Spacecraft Attitude Tracking

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Abstract— **This study investigates variable structure reliable control issues of nonlinear systems and its applications to spacecraft attitude tracking problems. The proposed passive reliable control laws must know in advance which group of actuators is allowed to fail. These reliable controllers need not the solution of Hamilton-Jacobi (HJ) equation or inequality that are essentials in optimal approaches such as LQR and H∞ reliable designs. As a matter of fact, this approach is able to relax the computational load in computing the solution of HJ equation. The proposed reliable designs are also applied to spacecraft attitude tracking problems to explain their effectiveness and benefits. Finally, simulation results and comparisons between LQR and Variable Structure Control (VSC) reliable designs are presented to illustrate the merits of the proposed scheme. Although the proposed design is a passive one, it may provide a guideline for active design when a Fault Detection and Diagnosis (FDD) scheme is available.**

I. INTRODUCTION

Due to the growing demands of system reliability in aerospace and industrial process, the study of reliable control has recently attracted lots of attention (see e.g., [1]-[9]). The objective of this study is to design an appropriate controller and apply the effective scheme to handling spacecraft attitude tracking problems such that the closed-loop spacecraft system can tolerate the abnormal operation of some specific control components and retain an overall system stability with acceptable system performance. In general, reliable control systems can be classified (see e.g., [6]) as active [1], [2] and passive ones [3]-[9]. In an active reliable control system, faults are to be detected and identified by a fault detection and diagnosis (FDD) mechanism. Then the controllers are reconfigured according to the on-line detection results in real time. On the other hand, the passive approach exploits system inherent redundancy to design a fixed controller so that the closed-loop system can achieve an acceptable performance not only during normal operations but also under various components fail without the need of FDD and controllers reconfiguration. Although the performance of the active reliable control is in general superior to those of passive one under various faulty situations because of controllers reconfiguration, the active approach needs a reliable FDD but those of passive ones donot. This is important when the available reaction time is short after the occurrence of faults. In this paper, we only consider passive reliable control issues.

Among the existing passive reliable control studies, several approaches have been proposed, for example, ARE-based approach [4], LMI-based approach [8] and HJ-based approach [7], [8]. Although the HJ-based approach is mainly for nonlinear systems, an inevitable difficulty comes from its reliable controllers depending on the solutions of the Hamilton-Jacobi equation or inequality, which are known not easy to solve. A power series method [10] may alleviate the difficulty through computer calculation, while the obtained solution is only an approximated one and the calculating load grows fast when system is complicated. Due to these potential drawbacks of HJ-based approach, this paper investigates the reliable issues from Variable Structure Control (VSC) viewpoint.

It is known that VSC schemes have the advantages of fast response and small sensitivity to system parameter uncertainties and disturbances (see e.g., [11] and the references therein). It is then widely applied to control a variety of systems [12]-[15]. For example, Shtessel et al. [14], [15] applied multiple time scale reconfigurable sliding modes to an F-16 aircraft and a tailless aircraft. Therein, a reconfigurable two-loop continuous Sliding Mode Controller (SMC) is developed and robust tracking of angle of roll, attack and slideslip angles is achieved through an outer-loop SMC by providing angular rate commands to an inner-loop SMC. In this paper, we propose passive VSC reliable design, which are shown to be able to tolerate the outage of actuators within a prespecified subset of actuators. These controllers are easily implemented and need not to solve a Hamilton-Jacobi equation or inequality, which is known not easy to solve but is an essential part in the design of LQR and H_∞ reliable controllers. Thus, the VSC approach can alleviate the computational burden for solving the Hamilton-Jacobi equation or inequality. In order to illustrate the use of the proposed scheme, in this paper we employ the ROCSAT-II satellite [17], which contains a redundant control element, as an example to demonstrate its effectiveness. Moreover, comparison with the conventional LQR reliable control designs will show the benefits of the proposed method.

The rest of this paper is organized as follows. In Section II, reliable control designs via VSC are proposed. The issues about reliable controllers applied to spacecraft attitude tracking problems are described in Section III. Section IV gives the simulation results and the comparisons with LQR reliable control. Finally, conclusions are drawn in Section V.

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II. RELIABLE CONTROLLERS DESIGN

Consider a n-dimensional nonlinear control system as given by

$$\dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2) + G_{\Omega 1}(\mathbf{x}_1, \mathbf{x}_2)\mathbf{u}_\Omega, \quad (1)$$

$$\dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + G_{\Omega 2}(\mathbf{x}_1, \mathbf{x}_2)\mathbf{u}_\Omega + G_{\Omega'}(\mathbf{x}_1, \mathbf{x}_2)\mathbf{u}_{\Omega'}. \quad (2)$$

Here, $\mathbf{x}_1$, $\mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2) \in \mathbb{R}^{\mathrm{n-m_1}}$, $\mathbf{x}_2$, $\mathbf{u}_{\Omega'}$, $\mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) \in \mathbb{R}^{\mathrm{m_1}}$ and $\mathbf{u}_\Omega \in \mathbb{R}^{\mathrm{m-m_1}}$. $G_{\Omega 1}(\mathbf{x}_1, \mathbf{x}_2)$ and $G_{\Omega 2}(\mathbf{x}_1, \mathbf{x}_2)$ are matrices with appropriate dimensions, and $G_{\Omega'}(\mathbf{x}_1, \mathbf{x}_2)$ is a nonsingular matrix. In addition, for the interest of study, we assume that $\mathbf{f}_1(\mathbf{0}, \mathbf{0}) = \mathbf{0}$ and $\mathbf{f}_2(\mathbf{0}, \mathbf{0}) = \mathbf{0}$. In the description of (1)-(2), we have divided the actuators into two classes $\mathbf{u}_\Omega$ and $\mathbf{u}_{\Omega'}$, within which the outage of $\mathbf{u}_\Omega$ must be tolerated, while the outage within those of $\mathbf{u}_{\Omega'}$ are not taken into account.

If all actuators of $\mathbf{u}_\Omega$ fail, then system (1)-(2) becomes

$$\dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2), \quad (3)$$

$$\text{and} \quad \dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + G_{\Omega'}(\mathbf{x}_1, \mathbf{x}_2)\mathbf{u}_{\Omega'}. \quad (4)$$

Since the outage of actuators in $\mathbf{u}_\Omega$ should be tolerated, we impose the following assumption:

Assumption 1: The origin $(\mathbf{x}_1, \mathbf{x}_2) = (\mathbf{0}, \mathbf{0})$ of (3)-(4) is stabilizable.

From Assumption 1, it implies that there exists $\mathbf{x}_2(t)$ such that the origin of $\dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2)$ is asymptotically stable. In order to construct an appropriate sliding surface, we impose the next assumption:

Assumption 2: Suppose there exists a function $\mathbf{x}_2 = \phi(\mathbf{x}_1)$ such that the reduced order system $\dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \phi(\mathbf{x}_1))$ has an asymptotically stable equilibrium point at the origin $\mathbf{x}_1 = \mathbf{0}$.

According to Assumption 2, we choose a sliding surface as

$$\mathbf{s} = \mathbf{x}_2 - \phi(\mathbf{x}_1) = 0. \quad (5)$$

It follows from (3)-(4) that

$$\begin{aligned} \dot{\mathbf{s}} &= \dot{\mathbf{x}}_2 - \frac{\partial \phi}{\partial \mathbf{x}_1} \cdot \dot{\mathbf{x}}_1 \\ &= \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) - \frac{\partial \phi}{\partial \mathbf{x}_1} \cdot \mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2) + G_{\Omega'}(\mathbf{x}_1, \mathbf{x}_2)\mathbf{u}_{\Omega'}. \quad (6) \end{aligned}$$

Following the VSC design procedure [11], the VSC law is designed to be

$$\begin{aligned} \mathbf{u}_{\Omega'} = {} & {G_{\Omega'}}^{-1}(\mathbf{x}_1, \mathbf{x}_2)[-\mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + \frac{\partial \phi}{\partial \mathbf{x}_1} \cdot \mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2)] \\ & -{G_{\Omega'}}^{-1}(\mathbf{x}_1, \mathbf{x}_2)[\Lambda_{\Omega'} \cdot \mathrm{sgn}(\mathbf{s})] \quad (7) \end{aligned}$$

where $\Lambda_{\Omega'} = \mathrm{diag}(\eta_1, \cdots, \eta_{m_1})^T$ with $\eta_i > 0$ for all $i = 1, \cdots, m_1$, sgn$(\cdot)$ denotes the sign function and $\mathrm{sgn}(\mathbf{s}) = (\mathrm{sgn}(s_1), \cdots, \mathrm{sgn}(s_{m_1}))^T$. We then have

$$\begin{aligned} \mathbf{s}^T \dot{\mathbf{s}} &= -\sum_{i=1}^{m_1} \eta_i \cdot |s_i| \\ &\leq -(\min_{1 \leq i \leq m_1} \eta_i) \cdot ||\mathbf{s}|| \quad (8) \end{aligned}$$

where s_i denotes the ith entry of the sliding vector $\mathbf{s}$ and $|| \cdot ||$ is the Euclidean norm. Equation (8) implies that the system state will reach sliding surface in a finite time with time depending on the choice of η [11]. Under the designed control law, we have the next result for the case of which $\mathbf{u}_\Omega = \mathbf{0}$:

Theorem 1: Suppose Assumptions 1-2 hold. Then the origin of system (3)-(4) is locally asymptotically stable under the control given by (7).

In addition to the design of $\mathbf{u}_{\Omega'}$ as discussed above, we discuss the design of $\mathbf{u}_\Omega$ when they are available. In this case, the governing equation is given by (1)-(2). From Eqs. (1)-(2), (5) and (7), we have

$$\begin{aligned} \mathbf{s}^T \dot{\mathbf{s}} &= \mathbf{s}^T \left[G_{\Omega 2}(\mathbf{x}_1, \mathbf{x}_2) - \frac{\partial \phi}{\partial \mathbf{x}_1} \cdot G_{\Omega 1}(\mathbf{x}_1, \mathbf{x}_2) \right] \mathbf{u}_\Omega \\ & \quad -\sum_{i=1}^{m_1} \eta_i \cdot |s_i|. \quad (9) \end{aligned}$$

Clearly, one of the choices of $\mathbf{u}_\Omega$ to make $\mathbf{s}^T\dot{\mathbf{s}}$ more negative than that of $\mathbf{u}_\Omega = 0$ is

$$\mathbf{u}_\Omega = -\Lambda_\Omega \cdot$$

$$\mathrm{sgn} \left[\left(G_{\Omega 2}(\mathbf{x}_1, \mathbf{x}_2) - \frac{\partial \phi}{\partial \mathbf{x}_1} \cdot G_{\Omega 1}(\mathbf{x}_1, \mathbf{x}_2) \right)^T \mathbf{s} \right] \quad (10)$$

where $\Lambda_\Omega = (\eta_{m_1+1}, \cdots, \eta_m)^T$ and $\eta_i \geq 0$ for all $i = m_1 + 1, \cdots, m$. This means that system states moving to the sliding surface is faster than that of worst case when $\mathbf{u}_\Omega$, given by (10), is available. Moreover, the reduced order system becomes

$$\dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \phi(\mathbf{x}_1)). \quad (11)$$

These lead to the next result:

Theorem 2: Suppose that Assumptions 1-2 hold. Then the origin of system (3)-(4) is locally asymptotically stable under the control given by (7) and (10) even when some or all of control in $\mathbf{u}_\Omega$ fail.

III. APPLICATION TO SPACECRAFT ATTITUDE TRACKING

A. Spacecraft Dynamics Model

In order to explore the application of VSC reliable design to spacecraft attitude tracking, in this subsection we first consider the spacecraft dynamics. According to the definition of Euler's angle [17], the spacecraft system dynamics, in terms of angular momentum conservation law has the form

$$T + \mathbf{g} = \frac{dh}{dt} = [\frac{dh}{dt}]_b + \omega \times h \quad (12)$$

where T denotes external disturbances (including solar pressure torque, magnetic disturbance), G is gravity gradient torque, h is the total angular momentum of spacecraft and ω is the angular rate of principal axis. The symbol $[\cdot]_b$ means that it is with respect to the body coordinate frame of spacecraft. Define $\hat{i}, \hat{j}, \hat{k}$ the three standard basis vectors in the body coordinate frame. Thus, the total angular momentum can be expressed as:

$$h = (I_x\omega_x + h_{wx})\hat{i} + (I_y\omega_y + h_{wy})\hat{j} + (I_z\omega_z + h_{wz})\hat{k} \quad (13)$$

where I_x, I_y, I_z are the inertia with respect to x, y, z axis, $\omega_x, \omega_y, \omega_z$ define the angular rates with respect to x, y, z axis, and h_{wx}, h_{wy}, h_{wz} are input torques of wheels. Substituting Eq. (13) into (12), we have

$$T + \mathbf{g} = \begin{pmatrix} I_x\dot{\omega}_x + \dot{h}_{\omega x} + (I_z - I_y)\omega_y\omega_z + \omega_y h_{\omega z} - \omega_z h_{\omega y} \\ I_y\dot{\omega}_y + \dot{h}_{\omega y} + (I_x - I_z)\omega_x\omega_z + \omega_z h_{\omega x} - \omega_x h_{\omega z} \\ I_z\dot{\omega}_z + \dot{h}_{\omega z} + (I_y - I_x)\omega_x\omega_y + \omega_x h_{\omega y} - \omega_y h_{\omega x} \end{pmatrix}. \quad (14)$$

According to [17], the angular rate and Euler angular rate have the following relation:

$$\omega_\alpha = \dot{\theta}_\alpha + \omega_0 E_2 \cdot e_\alpha, \quad \alpha = x, y, z. \quad (15)$$

Define $\theta_x = \phi$, $\theta_y = \theta$, $\theta_z = \psi$ the rotational angle with respect to x , y and z axis, respectively, E_2 is the unit vector of orbit coordinate frame, e_α is unit vector of body axis frame, ω_0 is orbit rate. Then, Eqs. in (15) can be rewritten as the vector form:

$$w = \begin{pmatrix} \dot{\phi} - \omega_0 \sin\psi\cos\theta \\ \dot{\theta} + \omega_0(\cos\psi\cos\phi - \sin\psi\sin\theta\sin\phi) \\ \dot{\psi} + \omega_0(\cos\psi\sin\phi + \cos\phi\sin\psi\sin\theta) \end{pmatrix}, \quad (16)$$

and the gravity gradient torque can be described as:

$$\mathbf{g} = \begin{pmatrix} -3/2\omega_0^2(I_y - I_z)\cos^2\theta\sin 2\phi \\ 3/2\omega_0^2(I_z - I_x)\sin 2\theta\cos\phi \\ -3/2\omega_0^2(I_x - I_y)\sin 2\theta\sin\phi \end{pmatrix}. \quad (17)$$

For simlicity, we assume that thrust is the only applied control force. This implies that $h_{\omega i} = 0$ and $\dot{h_{\omega i}} = 0$ for $i = x, y, z$. After combining Eq. (16) and (17) with (15) and defining the state variables as $x_1 = \phi, x_2 = \dot{\phi}, x_3 = \theta, x_4 = \dot{\theta}, x_5 = \psi$ and $x_6 = \dot{\psi}$, the spacecraft dynamics can be represented in the form of

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + G(\mathbf{x})\mathbf{u}, \quad (18)$$

with $\mathbf{x} = (x_1\ x_2\ x_3\ x_4\ x_5\ x_6)^T$, $u = (T_x\ T_y\ T_z)^T$, $\mathbf{f} = (f_1\ f_2\ f_3\ f_4\ f_5\ f_6)^T$ and

$$G = \begin{pmatrix} 0 & 0 & 0 \\ 1/I_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/I_y & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/I_z \end{pmatrix} \quad (19)$$

where details about f_1 to f_6 are described in Appendix.

Note that, the above spacecraft dynamics is primarily depicted for three control input, however, in many practical applications, a system often equips with redundancy to allow safe operation when the outage of some of the actuators happen. An example can be found in the design of ROCSAT-II satellite which is equipped with four actuators. The connection between the three and four actuators in ROCSAT-II is through the transformation matrix

$$S = \begin{pmatrix} 0.67 & 0.67 & 0.67 & 0.67 \\ 0.69 & -0.69 & -0.69 & 0.69 \\ 0.28 & 0.28 & -0.28 & -0.28 \end{pmatrix}_{3\times 4}. \quad (20)$$

Therefore, the dynamics can be rewritten as the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + G'(\mathbf{x})\mathbf{u}' \quad (21)$$

where the function $\mathbf{f}(\mathbf{x})$ is the same with $\mathbf{f}(\mathbf{x})$ in Eq. (26). However, $G'(\mathbf{x})$ and $\mathbf{u}'$ are

$$G'(\mathbf{x}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{0.67}{I_x} & \frac{0.67}{I_x} & \frac{0.67}{I_x} & \frac{0.67}{I_x} \\ 0 & 0 & 0 & 0 \\ \frac{0.69}{I_y} & \frac{-0.69}{I_y} & \frac{-0.69}{I_y} & \frac{0.69}{I_y} \\ 0 & 0 & 0 & 0 \\ \frac{0.28}{I_z} & \frac{0.28}{I_z} & \frac{-0.28}{I_z} & \frac{-0.28}{I_z} \end{pmatrix},$$

$$\text{and} \quad \mathbf{u}' = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}. \quad (22)$$

Here, u_1, u_2, u_3 and u_4 are the control torques in four directions of ROCSAT-II. .

B. Controllability Analysis

Here, we investigate the linear controllability of the spacecraft dynamics different faulty situations. To attain this end, we linearize the nonlinear state equation (21) with equilibrium point being the origin. It yields

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -2a_1 & 0 & 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2a_2 & 0 & 0 & a_1 & 0 \end{pmatrix},$$

$$\text{and} \quad B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.67 & 0.67 & 0.67 & 0.67 \\ 0 & 0 & 0 & 0 \\ 0.69 & -0.69 & -0.69 & 0.69 \\ 0 & 0 & 0 & 0 \\ 0.28 & 0.28 & -0.28 & -0.28 \end{pmatrix} \quad (23)$$

where $a_1 = \omega_0^2(I_y - I_x)/I_z$, $a_2 = \omega_0 I_y/I_x$. After calculating, the characteristic polynomial of A has the form

$$\lambda^2 \cdot [\lambda^4 + (a_1 + a_2^2)\lambda^2 - 2a_1^2] = 0.$$

Thus, the eigenvalues of A matrix are $(0, 0, \pm\mu_1, \pm j\mu_2)$, where $\mu_1 = \sqrt{\frac{\sqrt{(a_1+a_2^2)^2+8a_1^2}-(a_1+a_2^2)}{2}}$ and $\mu_2 = \sqrt{\frac{\sqrt{(a_1+a_2^2)^2+8a_1^2}+(a_1+a_2^2)}{2}}$. Because μ_1 and μ_2 are greater than zero, it is clear that the uncontrolled version of the system is unstable. To analyze the linear controllability of the spacecraft, we select the system parameters as $\omega_0 = 1.0312 \times 10^{-3}$, $I_x = I_z = 2000$ and $I_y = 400$, which means that the inertia of system is symmetric with respect to y axis, we have $a_1 = -8.507 \times 10^{-7}$ and $a_2 = 2.6024 \times 10^{-4}$. By direct calculation, the controllability matrices associated with the number of healthy thrusters being greater than or equal to two have full rank. This means that the system can be stabilized by any two or more thrusters. However, when system has two actuators outage, the controllability matrix is nearly lost rank which implies that the faulty system with two outages is weakly linear controllable.

C. VSC Reliable Design

In this section, we employ the VSC scheme to design the reliable controller for ROCSAT-II spacecraft. According to the proposed design procedure as described in Section II, we choose the sliding surface as

$$\mathbf{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} \dot{e}_1 + m_{11}e_1 \\ \dot{e}_2 + m_{21}e_2 \\ \dot{e}_3 + m_{31}e_3 \end{pmatrix} \tag{24}$$

where $e_1 = \phi - \phi_d$, $e_2 = \theta - \theta_d$, $e_3 = \psi - \psi_d$, ϕ_d, θ_d and ψ_d are the desired attitude, m_{11}, m_{21} and m_{31} are positive constants. The passive reliable controller then has the form as below. For simplicity, we only present the case for u_2 fail.

$$\begin{aligned} \begin{pmatrix} {u_1}^{eq} \\ {u_3}^{eq} \\ {u_4}^{eq} \end{pmatrix} &= (\mathbf{g_1}\ \mathbf{g_3}\ \mathbf{g_4})^{-1} \cdot \\ &\quad \begin{pmatrix} -f_2(x) + \ddot{\phi}_d - m_{11}\dot{e}_1 \\ -f_4(x) + \ddot{\theta}_d - m_{21}\dot{e}_2 \\ -f_6(x) + \ddot{\psi}_d - m_{31}\dot{e}_3 \end{pmatrix}, \\ \begin{pmatrix} {u_1}^{re} \\ {u_3}^{re} \\ {u_4}^{re} \end{pmatrix} &= (\mathbf{g_1}\ \mathbf{g_3}\ \mathbf{g_4})^{-1} \begin{pmatrix} -(\rho_1 + \eta_1)\mathrm{sgn}(s_1) \\ -(\rho_2 + \eta_2)\mathrm{sgn}(s_2) \\ -(\rho_3 + \eta_3)\mathrm{sgn}(s_3) \end{pmatrix}, \\ {u_2}^{eq} &= 0 \quad \text{and} \quad {u_2}^{re} = -\eta_4 \mathrm{sgn}(s^T \mathbf{g}_2) \end{aligned} \tag{25}$$

where $\mathbf{g_i}$ denotes the ith column of G, i= 1, 2, 3, 4.

IV. SIMULATION RESULTS

This section presents simulation results of VSC reliable designs and compares its performances with those by nonlinear LQR reliable designs. It is known that the controllers of LQR reliable design depend on the solution of the associative Hamilton-Jacobi equation, which is known difficult to solve [8]. A power series method [10] may alleviate the difficulty through computer calculation. However, the obtained solution is only an approximated one and, when system is complicated, the computational load grows fast as the order of the approximated solution increases. In this paper, we adopt the numerical scheme of [10] to estimate the LQR reliable controller up to order 3 with quadratic performance being chosen as $\int (\mathbf{x}^T Q\mathbf{x} + \mathbf{u}^T R\mathbf{u})dt$, $Q = I$, $R = I$. In these simulations, the parameters of spacecraft are chosen as $I_x = 2000$, $I_y = 400$, $I_z = 2000$ (spacecraft is symmetric with respect to y-axis) and orbit rate $\omega_0 = 1.0312 \times 10^{-3}$. The initial state $x(0) = (-0.7\ \ -0.07\ \ 1.5\ \ 0.3\ \ 1.3\ \ -0.2)^T$. The parameters of VSC scheme are chosen to be $m_{11} = m_{21} = m_{31} = 2$ and the sign function is replaced by the saturation function with boundary layer width $\epsilon_i = 0.1$ for $i = 1, 2, 3$.

Simulation results are summarized in Tables 1-2 where both controllers are designed regarding u_2 as susceptible actuator. The controllers for LQR reliable design include linear (denoted by 1st), linear+second order (denoted by 2nd) and linear+second+third order (denoted by 3rd) controllers. Tables 1 and 2 assume that the system is in normal operation and experienced u_2 outage, respectively. From these results, the quadratic performances $\int_0^\infty (\mathbf{x}^T Q\mathbf{x} + \mathbf{u}^T R\mathbf{u})dt$ of LQR are observed better than those by VSC design. However, all the other performances of VSC including convergent time, energy assumption $\int_0^\infty (\mathbf{u}^T R\mathbf{u})dt$ and the required maximum control magnitude $||\mathbf{u}||_\infty$ are seen better than those by LQR. It is also noted that the convergent time and the maximum control magnitude can easily be accommodated by adjusting the control parameters η_i in VSC design. As a matter a fact, the control parameters of VSC can be tuned as large as possible in practical applications to promote the responding performances while fulfill the maximum control magnitude constraint. This example demonstrates the benefits of the VSC reliable design.

V. CONCLUSIONS

In this paper, a class of passive VSC reliable laws are proposed and applied to the spacecraft attitude control. These reliable laws are shown to be able to tolerate the outage of actuators within a prespecified subset of actuators

TABLE I

PERFORMANCE INDICES BY LQR AND VSC RELIABLE DESIGNS (REGARDING u_2 AS SUSCEPTIBLE ACTUATOR) WHEN ALL THRUSTERS ARE IN NORMAL OPERATION.

Performance index	Controller				
	LQR (1st)	LQR (2nd)	LQR (3rd)	VSC (η_i=1)	VSC (η_i=0.4)
Time when $\max_i\lvert x_i\rvert < 0.01$	9.1338	9.6064	9.5696	4.101	6.630
$\int \mathbf{x}^T Q\mathbf{x} + \mathbf{u}^T R\mathbf{u}$	10.5168	9.8355	9.6689	14.666	12.631
$\int \mathbf{u}^T R\mathbf{u}$	4.7975	4.0835	3.8446	8.4222	3.328
$\lvert\lvert\mathbf{u}\rvert\rvert_\infty$	5.39	4.66	4.66	2.974	1.492

TABLE II

PERFORMANCE INDICES BY LQR AND VSC RELIABLE DESIGNS (REGARDING u_2 AS SUSCEPTIBLE ACTUATOR) WHEN u_2 FAILS.

Performance index	Controller				
	LQR (1st)	LQR (2nd)	LQR (3rd)	VSC (η_i=1)	VSC (η_i=0.4)
Time when $\max_i\lvert x_i\rvert < 0.01$	8.8736	9.4136	9.7187	4.354	8.370
$\int \mathbf{x}^T Q\mathbf{x} + \mathbf{u}^T R\mathbf{u}$	10.9406	10.0168	9.9871	11.880	11.891
$\int \mathbf{u}^T R\mathbf{u}$	4.6418	3.6109	3.5037	5.677	1.114
$\lvert\lvert\mathbf{u}\rvert\rvert_\infty$	5.39	4.66	4.66	2.974	1.492

as those by LQR and H_∞ reliable designs. However, the controllers of VSC reliable design do not depend on the solution of a Hamilton-Jacobi (HJ) equation, which is known difficult to solve but is a necessary part in the LQR and H_∞ reliable controllers. Thus, the VSC approach can alleviate computational burden for solving the HJ equation. Simulation results for spacecraft attitude stabilization are given to demonstrate the benefits of the VSC approach. It is shown that the performance indices by VSC reliable design, including convergent time, quadratic performance, energy consumption and maximum torque requirement, are all better than those by LQR reliable design. When any actuator other than the dedicated ones fails, the proposed design may be cooperated with a Fault Detection and Diagnosis (FDD) scheme to proceed an active reliable control.

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APPENDIX

$$
\begin{aligned}
f_i &= x_i+1, \;\; i=1, \;\; 3 \;\text{ and } 5,\\
f_2 &= \omega_0 x_6 cx_5 cx_3 - \omega_0 x_4 sx_5 sx_3 + (I_y - I_z)/I_x[x_4 x_6\\
&\quad +\omega_0 x_4 cx_1 sx_5 sx_3 + \omega_0 x_4 cx_5 sx_1\\
&\quad +\omega_0 x_6 cx_5 cx_1 + 1/2\omega_0^2 s(2x_5)c^2 x_1 sx_3\\
&\quad +1/2 w_0^2 c_2 x_5 s(2x_1) - \omega_0^2 c^2 x_5 s(2x_1)\\
&\quad -\omega_0 x_6 sx_5 sx_3 sx_1 - 1/2\omega_0^2 s_2 x_3 s^2 x_5 s(2x_1)\\
&\quad -1/2\omega_0^2 s(2x_5) sx_3 s^2 x_1 - 3/2\omega_0^2 c^2 x_3 s(2x_1)]\\
&\quad -\dot{h}_{\omega x} - h_{\omega z}(x_4 + \omega_0 cx_5 cx_1 - \omega_0 sx_5 sx_3 sx_1)\\
&\quad +h_{\omega x}(x_6 + \omega_0 cx_1 sx_5 sx_3 + \omega_0 cx_5 sx_1),\\
f_4 &= \omega_0 x_6 sx_5 cx_1 + \omega_0 x_2 cx_5 sx_1 + \omega_0 x_6 cx_5 sx_3 sx_1\\
&\quad +\omega_0 x_4 sx_5 cx_3 sx_1 + \omega_0 x_2 sx_5 sx_3 cx_1\\
&\quad +(I_z - I_x)/I_y[x_2 x_6 + \omega_0 x_2 cx_1 sx_5 sx_3\\
&\quad +\omega_0 x_2 cx_5 sx_1 - \omega_0 x_6 cx_5 cx_3\\
&\quad -1/2\omega_0^2 s(2x_3) s^2 x_5 cx_1 - 1/2 w_0^2 c_3 sx_1 s(2x_5)\\
&\quad +3/2\omega_0^2 s(2x_3) cx_1] - \dot{h}_{\omega y}\\
&\quad -h_{\omega x}(x_6 + \omega_0 cx_1 sx_5 sx_3 + \omega_0 cx_5 sx_1)\\
&\quad +h_{\omega z}(x_2 - \omega_0 sx_5 cx_3),\\
f_6 &= \omega_0 x_2 sx_1 sx_5 sx_3 - \omega_0 x_6 cx_1 cx_5 sx_3\\
&\quad -\omega_0 x_4 cx_1 sx_5 cx_3 + \omega_0 x_6 sx_5 sx_1\\
&\quad -\omega_0 x_2 cx_5 cx_1 + (I_x - I_y)/I_z[x_2 x_4 + \omega_0 x_2 cx_5 cx_1\\
&\quad -\omega_0 x_2 sx_5 sx_3 sx_1 - \omega_0 x_4 sx_5 cx_3\\
&\quad -1/2\omega_0^2 s(2x_5) cx_3 cx_1 + 1/2 w_0^2 s^2 x_5 sx_1 s(2x_3)\\
&\quad -3/2\omega_0^2 s(2x_3) sx_1] - \dot{h}_{\omega z} - h_{\omega y}(x_2 - \omega_0 sx_5 cx_3)\\
&\quad -h_{\omega x}(x_4 + \omega_0 cx_5 cx_1 - \omega_0 sx_5 sx_3 sx_3). \qquad (26)
\end{aligned}
$$

Here, c and s denote cos and sin function, respectively.

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