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Attitude Tracking and Vibration Suppression of Flexible Spacecraft Using Implicit Adaptive Control Law

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Abstract— **Control degree of freedom is much fewer than the motion degree of freedom when a flexible system is commanded to track a desired trajectory. Many control strategies that succeed in the conventional rigid systems, can not be directly applied to control of flexible systems. In this paper, a nonlinear control scheme is proposed as a solution of these problems in attitude control of a flexible spacecraft. In particular, to cope with the existing model uncertainty, an adaptive version of the control law is proposed. The model of the spacecraft considered as a rigid central body and two elastic appendages. Stability proof of the overall closed loop system is given via Lyapunov analysis. Numerical simulations show the effectiveness of the proposed controller.**

I. INTRODUCTION

THE control of flexible systems usually requires providing the control effort for maneuvering or targeting the flexible systems/subsystems with the simultaneous vibration suppression. Attitude control of flexible spacecraft is one of the more important classes of these problems. Because of the important demands for low-energy consumption and limitation of mass, space structures are required to be light and flexible. Flexible structures are infinite dimensional systems. The equations of motion of infinite dimensional systems are usually described by partial differential equations (PDE) and the limited dimensions of practical controller design often requires discretization of original PDE model into a system of finite dimensional ordinary differential equations (ODE), and modeling error are always introduced when the Reduced Order Models (ROM) are used. In particular, for flexible space structures, the model reduction issue has been a major consideration in control law design due to the high dimensionality of the original model. Although significant progress has been made in many aspects over the last decades, the design of simple, reliable and robust controllers to accommodate modeling errors such as spill-over effects due to unmodeled dynamics and uncertain system parameters still remain an open quest.

In recent years, several studies related to the control of flexible space systems have been done, and linear and nonlinear control systems have been designed [1-12]. An excellent survey of research in this area has been published by Hyland et.al., which provides a good source of references [1]. Optimal controllers for linear and nonlinear models of flexible space structures have been designed [2,3], and bang-bang rest-to-rest maneuver of linear models has been considered [4]. Near-minimum time single axis slewing of flexible vehicle has been treated [5]. A perturbation method has been used to obtain a feedback controller [6]. Lyapunov stability theory has been used to design controllers for the maneuver and vibration control of space vehicle [7,8]. A controller based on inversion has been also derived [9]. In these studies, it is assumed that the parameters of the space structures are exactly known. In the presence of uncertainty, adaptive and sliding mode control system has been designed [10-12]. However, the derivation of these controller requires knowledge of the bounds on the uncertainty and (or) the question of stability remains open.

In the present work, an adaptive nonlinear control scheme in the presence of uncertainty has been developed. It is assumed that the parameters are completely unknown. Stability proof of the rigid and flexible dynamics is given via Lyapunov stability analysis. Attitude tracking and vibration suppression of the elastic spacecraft is accomplished.

II. MATHEMATICAL MODELING

The mathematical model of the slewing flexible spacecraft can be derived by using the Assumed Mode Method (AMM) associated with Lagrangian formulation. As shown in figure 1, a rigid hub with to elastic appendages attached is considered as the model of the spacecraft. Two clamped loaded Euler-Bernoulli beams are selected to model the elastic deflection of flexible appendages in rotational motions. Although the design approaches of the present work can be applied to multi-axis maneuver, for simplicity only single-axis maneuver is considered.

The spacecraft is controlled by a torquer on the rigid hub. When the spacecraft is maneuvered, the elastic member connecting the hub experiences structural deformation. The problem of interest here is to control the orientation of the main body of the spacecraft as well as to suppress elastic deformations. As shown in the figure, $\theta(t)$ is considered as rigid body coordinate and $y(x,t)$ as flexible body coordinate with respect to body frame. It is assumed that the rotational maneuver excite the two flexible appendages anti-symmetrically. For the configuration of the spacecraft,

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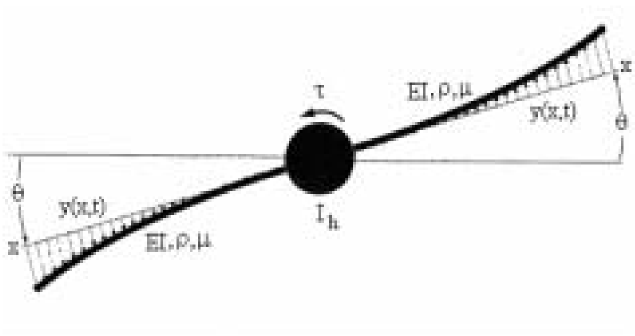

Fig. 1. Flexible spacecraft model

this assumption of anti-symmetric deformations is reasonable and has been validated [13].

The dynamic equations of the system can be obtained as follows [12]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}})\mathbf{q} + \mathbf{K}\mathbf{q} = \boldsymbol{\tau} \tag{1}$$

Equivalently, (1) can be rewritten as

$$\begin{bmatrix} M_{rr} & \mathbf{M}_{rf} \\ \mathbf{M}_{fr} & \mathbf{M}_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{q}_r \\ \ddot{\mathbf{q}}_f \end{Bmatrix} + \begin{bmatrix} h_{rr} & \mathbf{h}_{rf} \\ \mathbf{h}_{fr} & \mathbf{h}_{ff} \end{bmatrix} \begin{Bmatrix} \dot{q}_r \\ \dot{\mathbf{q}}_f \end{Bmatrix} + \begin{bmatrix} K_{rr} & \mathbf{K}_{rf} \\ \mathbf{K}_{fr} & \mathbf{K}_{ff} \end{bmatrix} \begin{Bmatrix} q_r \\ \mathbf{q}_f \end{Bmatrix} = \begin{Bmatrix} \tau \\ 0 \end{Bmatrix} \tag{2}$$

where subscripts r and f denotes rigid-mode part and flexible-modes parts, respectively. See appendix 1 and ref. [14] for more details. τ is the control torque acting on the hub. For derivation of the above equations, the elastic deformation of the appendages is considered to be:

$$y(x,t) = \boldsymbol{\varphi}^T(x)\, \mathbf{q}_f(t) \quad, 0 \le x \le l \tag{3}$$

where

$$\boldsymbol{\varphi}(x) = [\varphi_1(x) \quad \varphi_2(x) \quad \dots \quad \varphi_m(x)]^T \tag{4}$$

$$\mathbf{q}_f(x) = [q_{f1}(t) \quad q_{f2}(t) \quad \dots \quad q_{fm}(t)]^T \tag{5}$$

x is the coordinate of any point along the undeformed member. $q_{fi}(t)$ is the ith generalized displacement, or so-called modal displacement and $\varphi_i(x)$ is the ith assumed mode shape for the flexible appendage.

Generally, n rigid modes and m flexible modes may be considered. In our investigations, it is assumed that n=1 and m=5. Proposed methods and results can be expanded to more general cases.

III. ATTITUDE TRACKING

Let the desired trajectory, its first and second order derivatives be denoted as $q_{rd}(t), \dot{q}_{rd}(t)$ and $\ddot{q}_{rd}(t)$, respectively, and define the following error signals as

$$\mathbf{e} = \begin{bmatrix} e_r \\ \mathbf{e}_f \end{bmatrix} = \begin{bmatrix} q_r - q_{rd} \\ \mathbf{q}_f \end{bmatrix} \tag{6}$$

and

$$\mathbf{s} = \dot{\mathbf{e}} + \boldsymbol{\lambda}\mathbf{e} = \begin{bmatrix} \dot{e}_r + \lambda_r e_r \\ \dot{\mathbf{e}}_f + \boldsymbol{\lambda}_f \mathbf{e}_f \end{bmatrix} = \begin{bmatrix} s_r \\ \mathbf{s}_f \end{bmatrix} \tag{7}$$

where $\boldsymbol{\lambda} = \mathrm{diag}[\lambda_r \quad \boldsymbol{\lambda}_f]$ is a positive definite matrix. Note that $\mathbf{q}_{fd} = \mathbf{0}$. The dynamics of the newly defined signals s_r and $\mathbf{s}_f$ can be derived as

$$\begin{bmatrix} M_{rr} & \mathbf{M}_{rf} \\ \mathbf{M}_{fr} & \mathbf{M}_{ff} \end{bmatrix} \begin{Bmatrix} \dot{s}_r \\ \dot{\mathbf{s}}_f \end{Bmatrix} + \begin{bmatrix} h_{rr} & \mathbf{h}_{rf} \\ \mathbf{h}_{fr} & \mathbf{h}_{ff} \end{bmatrix} \begin{Bmatrix} s_r \\ \mathbf{s}_f \end{Bmatrix} + \begin{bmatrix} K_r^* & 0 \\ 0 & \mathbf{K}_f^* \end{bmatrix} \begin{Bmatrix} s_r \\ \mathbf{s}_f \end{Bmatrix} = \begin{Bmatrix} \tau + u_r + K_r^* s_r \\ \mathbf{u}_f \end{Bmatrix} \tag{8}$$

where

$$u_r = M_{rr}(-\ddot{q}_{rd} + \lambda_r \dot{e}_r) + \mathbf{M}_{rf}(\boldsymbol{\lambda}_f \dot{\mathbf{e}}_f) + h_{rr}(-\dot{q}_{rd} + \lambda_r e_r) + \mathbf{h}_{rf}\boldsymbol{\lambda}_f \mathbf{e}_f \tag{9}$$

$$\mathbf{u}_f = \mathbf{M}_{fr}(-\ddot{q}_{rd} + \lambda_r \dot{e}_r) + \mathbf{M}_{ff}(\boldsymbol{\lambda}_f \dot{\mathbf{e}}_f) + \mathbf{h}_{fr}(-\dot{q}_{rd} + \lambda_r e_r) + \mathbf{h}_{ff}\boldsymbol{\lambda}_f \mathbf{e}_f - \mathbf{K}_{ff}\mathbf{q}_f + \mathbf{K}_f^* \mathbf{s}_f \tag{10}$$

$\mathbf{K}^* = \mathrm{diag}(K_r^* \quad \mathbf{K}_f^*)$ is a positive definite arbitrary matrix. Now we propose the following control law:

$$\tau = -u_r - \tau_c - \mathbf{K}_r^* s_r \tag{11}$$

Where

$$\tau_c = \frac{(1+g)s_r}{\|s_r\|^2 + \varepsilon}(\mathbf{s}_f^T \mathbf{u}_f) \tag{12}$$

$$g = \sqrt{\gamma} \tag{13}$$

$$\dot{\gamma} = \begin{cases} 2(\gamma^{1/2}\alpha(t) + \beta(t)) & \gamma > 0 \\ 2\beta(t) & \gamma = 0, \beta(t) > 0 \\ \delta & \gamma = 0, \beta(t) \le 0 \end{cases} \tag{14}$$

$$\alpha(t) = \frac{\|s_r\|^2}{\|s_r\|^2 + \varepsilon}(\mathbf{s}_f^T \mathbf{u}_f) \tag{15}$$

$$\beta(t) = \frac{-\varepsilon}{\|s_r\|^2 + \varepsilon}(\mathbf{s}_f^T \mathbf{u}_f) \tag{16}$$

δ and ε are some small positive constants. Note that (14) is simply to define a differential equation of which its variable $\gamma(t)$ remains positive always. Then $g(t)$ can be easily checked to satisfy the following differential equation:

$$\dot{g} = \frac{1}{g}\left(\frac{g\|s_r\|^2 - \varepsilon}{\|s_r\|^2 + \varepsilon}\right)\left(\mathbf{s}_f^T \mathbf{u}_f\right) \qquad g \neq 0 \tag{17}$$

In the *theorem 1*, it is shown that the developed control law, results stable closed-loop dynamics and asymptotically tracking of the desired attitude. For better grasp of the proposed control law, we note that each term in the synthesized control torque in (11) can be associated with an interpretation as described below:

u_r denotes the computed torque to produce the desired motion trajectory, τ_c is used to cancel the effects of the elastic modes and $\mathbf{K}_r^* s_r$ represents the PD type control action.

Theorem 1: Consider a flexible spacecraft whose dynamics model is described by (1). Suppose the control objective is to rigid mode q_r track the desired trajectory while simultaneously suppression of the flexible modes. Then, the control law proposed in (11) can achieve the objective exponentially in time t.

Proof: Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\mathbf{s}^T \mathbf{M}(\mathbf{q})\,\mathbf{s} + \frac{1}{2} g^2 \tag{18}$$

Taking the time derivative of V and using (1) and (8-17) yield

$$\begin{aligned}
\dot{V} &= \mathbf{s}^T \mathbf{M}(\mathbf{q})\dot{\mathbf{s}} + \frac{1}{2}\mathbf{s}^T \dot{\mathbf{M}}(\mathbf{q})\,\mathbf{s} + g\,\dot{g} \\
&= \mathbf{s}^T\left(-\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})\mathbf{s} - \mathbf{K}^*\mathbf{s} + \begin{Bmatrix} -u_r \\ \mathbf{u}_f \end{Bmatrix}\right) \\
&\quad + \frac{1}{2}\mathbf{s}^T \dot{\mathbf{M}}(\mathbf{q})\,\mathbf{s} + \frac{g\|s_r\|^2 - \varepsilon}{\|s_r\|^2 + \varepsilon}\left(\mathbf{s}_f^T \mathbf{u}_f\right) \\
&= -\mathbf{s}^T \mathbf{K}^* \mathbf{s} - \mathbf{s}_r^T \tau_c - \mathbf{s}_f^T \mathbf{u}_f \\
&\quad + \frac{g\|s_r\|^2 - \varepsilon}{\|s_r\|^2 + \varepsilon}\left(\mathbf{s}_f^T \mathbf{u}_f\right) \\
&= -\mathbf{s}^T \mathbf{K}^* \mathbf{s} - \frac{(1+g)\|s_r\|^2 - \|s_r\|^2 - \varepsilon}{\|s_r\|^2 + \varepsilon}\left(\mathbf{s}_f^T \mathbf{u}_f\right) \\
&\quad + \frac{g\|s_r\|^2 - \varepsilon}{\|s_r\|^2 + \varepsilon}\left(\mathbf{s}_f^T \mathbf{u}_f\right) \\
&= -\mathbf{s}^T \mathbf{K}^* \mathbf{s}
\end{aligned} \tag{19}$$

Since $\mathbf{K}^*$ is positive definite, then $\dot{V} < 0$. It follows that s is bounded, and in view of (7) it follows that e and $\dot{e}$ are bounded. Using Barbalat's Lemma [15], one concludes that s tends to zero as t tends to ∞. This implies that the tracking error $\mathbf{e} = [e_r \ \mathbf{e}_f]^T$ and its time derivatives converge to zero as t tends to ∞. This concludes our proof.

In order to cope with the model uncertainty, an adaptive version of the controller is presented. Since the dynamic equations of the system possess the well-known linear-in-parameter property, it can be defined

$$\begin{aligned}
\mathbf{W}_1\mathbf{P}_1 &= M_{rr}\left(\ddot{q}_{rd} + \lambda_r \dot{e}_r\right) + \mathbf{M}_{rf}\left(\boldsymbol{\lambda}_f \dot{\mathbf{e}}_f\right) \\
&\quad + h_{rr}\left(\dot{q}_{rd} + \lambda_r e_r\right) + \mathbf{h}_{rf}\boldsymbol{\lambda}_f \mathbf{e}_f
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mathbf{W}_2\mathbf{P}_2 &= \mathbf{M}_{fr}\left(\ddot{q}_{rd} + \lambda_r \dot{e}_r\right) + \mathbf{M}_{ff}\left(\boldsymbol{\lambda}_f \dot{\mathbf{e}}_f\right) \\
&\quad + \mathbf{h}_{fr}\left(\dot{q}_{rd} + \lambda_r e_r\right) + \mathbf{h}_{ff}\boldsymbol{\lambda}_f \mathbf{e}_f \\
&\quad - \mathbf{K}_{ff}\mathbf{e}_f + \mathbf{K}_f^* \mathbf{s}_f
\end{aligned} \tag{21}$$

Where $\mathbf{W}_1$ and $\mathbf{W}_2$ are n×r1 and m×r2 regressor matrices, respectively, for some appropriate r1>0 and r2>0 and $\mathbf{P}_1$ and $\mathbf{P}_2$ are r1×1 and r2×1 vectors of unknown constant parameters, respectively. Considering the model uncertainty and denoting $\hat{\mathbf{P}}_1, \hat{\mathbf{P}}_2$ as the estimates of $\mathbf{P}_1, \mathbf{P}_2$ respectively, the control law (11) can be modified as

$$\tau = -\mathbf{W}_1\hat{\mathbf{P}}_1 - \tau_c - \mathbf{K}_r^* s_r \tag{22}$$

where

$$\tau_c = \frac{(1+g)s_r}{\|s_r\|^2 + \varepsilon}\left(\mathbf{s}_f^T \mathbf{W}_2 \hat{\mathbf{P}}_2 + \mathbf{s}_f^T \mathbf{K}_f^* \mathbf{s}_f\right) \tag{23}$$

$$g = \sqrt{\Gamma} \tag{24}$$

$$\dot{\Gamma} = \begin{cases} 2\left(\Gamma^{1/2} A(t) + B(t)\right) & \Gamma > 0 \\ 2B(t) & \Gamma = 0, B(t) > 0 \\ \delta & \Gamma = 0, B(t) \leq 0 \end{cases} \tag{25}$$

$$A(t) = \frac{\|s_r\|^2}{\|s_r\|^2 + \varepsilon}\left(\mathbf{s}_f^T \mathbf{W}_2 \hat{\mathbf{P}}_2 + \mathbf{s}_f^T \mathbf{K}_f^* \mathbf{s}_f\right) \tag{26}$$

$$B(t) = \frac{-\varepsilon}{\|s_r\|^2 + \varepsilon}\left(\mathbf{s}_f^T \mathbf{W}_2 \hat{\mathbf{P}}_2 + \mathbf{s}_f^T \mathbf{K}_f^* \mathbf{s}_f\right) \tag{27}$$

In order to estimate $\hat{\mathbf{P}}_1$ and $\hat{\mathbf{P}}_2$, the following adaptation laws can be obtained implicitly from the Lyapunov stability analysis:

$$\dot{\hat{\mathbf{P}}}_1 = -\boldsymbol{\mu}_1 \mathbf{W}_1^T s_r \tag{28}$$

$$\dot{\hat{\mathbf{P}}}_2 = -\boldsymbol{\mu}_2 \mathbf{W}_2^T \mathbf{s}_f \tag{29}$$

Where $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are adaptation gain matrices. It can be shown that the error dynamics resulting from the proposed control and adaptation laws are asymptotically stable in the sense of Lyapunov. The details will be stated in the following theorem.

Theorem 2: consider a flexible spacecraft whose dynamic model is described by Eq. (1). Suppose that the control objective is to the rigid modes track the desired trajectories while simultaneously damping out the flexible modes. Also, suppose that the system parameters are not precisely known. Then, the control law (22) and the adaptation laws (28) and (29), can achieve the objective in time t, *i.e.*, tracking errors converge to zero as *t* tends to infinity.

Proof: First, let to obtain the error dynamics by using (8) and (20-22) as

$$\begin{bmatrix} M_{rr} & \mathbf{M}_{rf} \\ \mathbf{M}_{fr} & \mathbf{M}_{ff} \end{bmatrix}\begin{Bmatrix} \dot{s}_r \\ \dot{\mathbf{s}}_f \end{Bmatrix} + \begin{bmatrix} h_{rr} & \mathbf{h}_{rf} \\ \mathbf{h}_{fr} & \mathbf{h}_{ff} \end{bmatrix}\begin{Bmatrix} s_r \\ \mathbf{s}_f \end{Bmatrix} + \begin{bmatrix} K_r^* & 0 \\ 0 & \mathbf{K}_f^* \end{bmatrix}\begin{Bmatrix} s_r \\ \mathbf{s}_f \end{Bmatrix} = \begin{Bmatrix} -\mathbf{W}_1\hat{\mathbf{P}}_1 - \tau_c \\ \mathbf{W}_2\hat{\mathbf{P}}_2 + \mathbf{K}_f^*\mathbf{s}_f \end{Bmatrix} \quad (30)$$

or more compactly as

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{s}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}})\mathbf{s} + \mathbf{K}^*\mathbf{s} = \begin{Bmatrix} \mathbf{W}_1\tilde{\mathbf{P}}_1 - \tau_c \\ \mathbf{W}_2\tilde{\mathbf{P}}_2 + \mathbf{K}_f^*\mathbf{s}_f \end{Bmatrix} \quad (31)$$

where $\tilde{\mathbf{P}}_1 = \mathbf{P}_1 - \hat{\mathbf{P}}_1$ and $\tilde{\mathbf{P}}_2 = \mathbf{P}_2 - \hat{\mathbf{P}}_2$ are the estimation errors of the parameters. Then, Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{M}(\mathbf{q})\mathbf{s} + \frac{1}{2}\tilde{\mathbf{P}}_1^T\boldsymbol{\mu}_1^{-1}\tilde{\mathbf{P}}_1 + \frac{1}{2}\tilde{\mathbf{P}}_2^T\boldsymbol{\mu}_2^{-1}\tilde{\mathbf{P}}_2 + \frac{1}{2}\Gamma \quad (32)$$

Taking the time derivative of V and using (1) and (22-29) yield

$$\begin{aligned}
\dot{V} &= \mathbf{s}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{s}} + \frac{1}{2}\mathbf{s}^T\dot{\mathbf{M}}(\mathbf{q})\mathbf{s} + \dot{\tilde{\mathbf{P}}}_1^T\boldsymbol{\mu}_1^{-1}\tilde{\mathbf{P}}_1 + \dot{\tilde{\mathbf{P}}}_2^T\boldsymbol{\mu}_2^{-1}\tilde{\mathbf{P}}_2 + g\,\dot{g} \\
&= \mathbf{s}^T\left(-\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})\mathbf{s} - \mathbf{K}^*\mathbf{s} + \begin{Bmatrix} \mathbf{W}_1\tilde{\mathbf{P}}_1 - \tau_c \\ \mathbf{W}_2\tilde{\mathbf{P}}_2 + \mathbf{K}_f^*\mathbf{s}_f \end{Bmatrix}\right) + \frac{1}{2}\mathbf{s}^T\dot{\mathbf{M}}(\mathbf{q})\mathbf{s} - \mathbf{s}_r^T\mathbf{W}_1\tilde{\mathbf{P}}_1 - \mathbf{s}_f^T\mathbf{W}_2\tilde{\mathbf{P}}_2 \\
&\quad + \left(\frac{g\|s_r\|^2 - \varepsilon}{\|s_r\|^2 + \varepsilon}\right)\left(\mathbf{s}_f^T\mathbf{W}_2\hat{\mathbf{P}}_2 + \mathbf{s}_f^T\mathbf{K}_f^*\mathbf{s}_f\right) \\
&= -\mathbf{s}^T\mathbf{K}^*\mathbf{s} - \frac{(1+g)\|s_r\|^2 - \|s_r\|^2 - \varepsilon}{\|s_r\|^2 + \varepsilon}\left(\mathbf{s}_f^T\mathbf{W}_2\hat{\mathbf{P}}_2 + \mathbf{s}_f^T\mathbf{K}_f^*\mathbf{s}_f\right) \\
&\quad + \mathbf{s}_f^T\mathbf{W}_2\hat{\mathbf{P}}_2 + \mathbf{s}_f^T\mathbf{K}_f^*\mathbf{s}_f - \mathbf{s}_f^T\mathbf{W}_2\hat{\mathbf{P}} + \left(\frac{g\|s_r\|^2 - \varepsilon}{\|s_r\|^2 + \varepsilon}\right)\left(\mathbf{s}_f^T\mathbf{W}_2\hat{\mathbf{P}}_2 + \mathbf{s}_f^T\mathbf{K}_f^*\mathbf{s}_f\right) \\
&= -\mathbf{s}^T\mathbf{K}^*\mathbf{s}
\end{aligned} \quad (33)$$

Since V is positive definite and its derivative is negative definite, it follows that s is bounded, and in view of (7) it follows that e and $\dot{e}$ are bounded. Using Barbalat's Lemma [15], one concludes that s tends to zero as t tends to infinity. This implies that the tracking error e and its time derivative converge to zero as t tends to infinity. This concludes our proof.

IV. SIMULATION RESULTS

In this section, results of the numerical simulation for the closed-loop system are presented. Numerical values of the simulation parameters are given in table 1. The first five elastic modes are retained in the model, thus m=5 . The lowest natural frequency of the system is obtained 5.12 *rad/s*. The initial conditions are assumed to be zero. Also, the initial values of the estimated parameter are arbitrarily set to zero. The controller gains are chosen to be $\lambda_r = 1.4, \lambda_f = \mathrm{diag}[1.3 \quad 1.3 \quad 1.3 \quad 1.3 \quad 1.3], K_r^* = 1.5$, $\mathbf{K}_f^* = \mathrm{diag}[1.5 \quad 1.5 \quad 1.5 \quad 1.5 \quad 1.5]$ and $\varepsilon = 0.05$. The following third order reference model with $\lambda_m = 0.07$ is chosen to generate the reference trajectory for rigid body motion:

$$\left(s^3 + 3\lambda_m s^2 + 3\lambda_m^2 s + \lambda_m^3\right)q_{rd} = 0 \quad (34)$$

Selected responses are shown in Figures 2 to 7. Smooth control of the attitude was accomplished in the closed-loop system. The response time of the rigid body mode is of the order of 50s. It is seen that the elastic modes are converging to zero as well. The estimated parameters are also converging to some constant values. It should be noted that generally, the parameters do not converge to their actual values unless persistent excitation conditions of certain signals in the closed-loop system is satisfied [14]. The control inputs also vary smoothly.

Extensive simulations showed that the control system accomplishes large angle rotational maneuvers and vibrations suppression. There are several design parameters which can be properly selected to accomplish rotational maneuver with reasonable control input magnitude and elastic deformation of the beam.

Note that the real flexible structures have infinite dimension of the state space. To investigate the robustness of the controller to the effects of unmodeled flexible dynamics, the numerical simulations have been carried out considering higher elastic modes, up to ten. Also, for obtaining the modal coordinates $q_{fi}(t)$ and $\dot{q}_{fi}(t)$ from the strain measurement, the strain dependent dynamics has been developed [14].

TABLE 1 MECHANICAL PROPERTIES OF THE SIMULATION MODEL

Parameter	Notation	Value
Appendage Length	L	5 m
Appendage stiffness	EI	15E3 Nm
Mass density of appendages	ρ	2840 kg m^{-3}
Damping coefficient	μ	0.02
Hub moment of inertia	J_h	1504 kg m^2

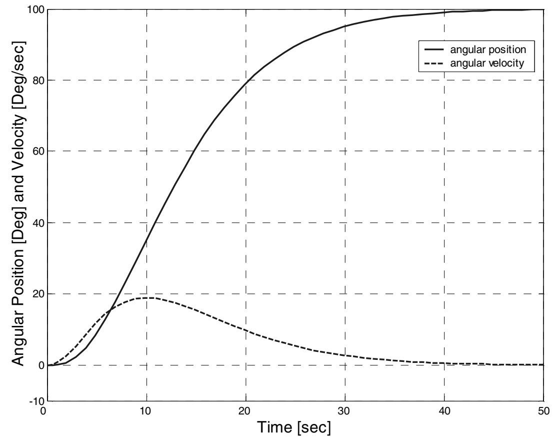

Fig. 2 Angular position and velocity

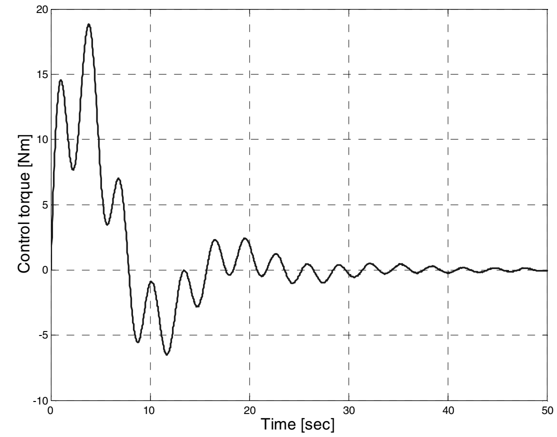

Fig. 5 Control action history

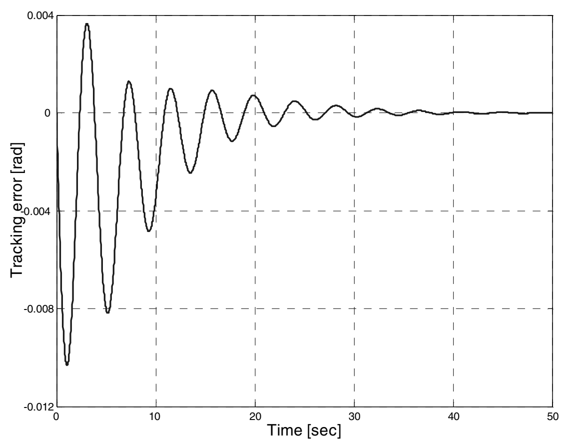

Fig. 3 Tracking error

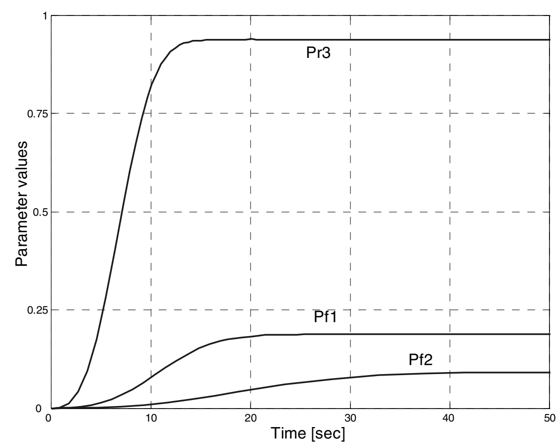

Fig. 6 Adaptation of parameters(Pr3,Pf1,Pf2)

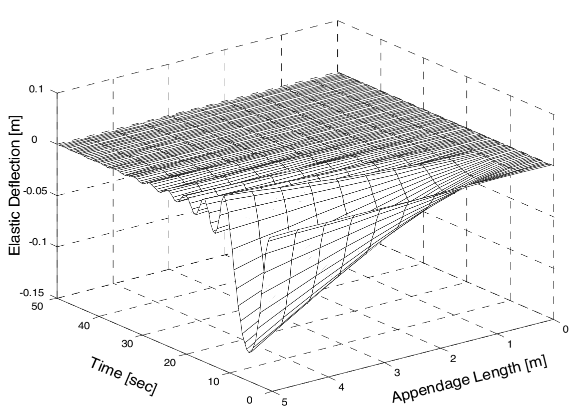

Fig. 4 3D plot for vibration suppression

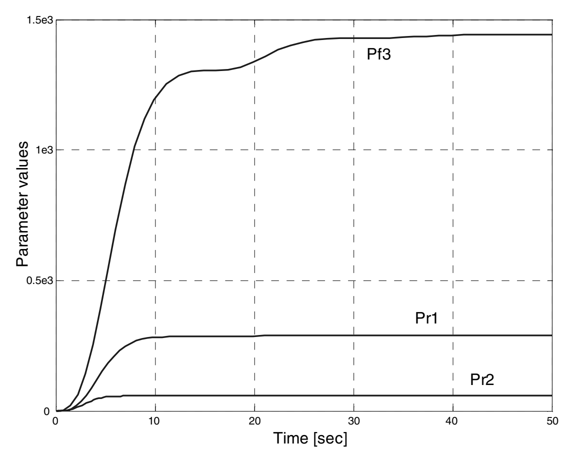

Fig. 7 Adaptation of parameters(Pr1,Pr2,Pf3)

V. Conclusion

An adaptive control law has been proposed for attitude tracking control of a flexible spacecraft. It is assumed that the parameters of the systems are completely unknown. Asymptotical tracking of the reference trajectory for rigid body motion and simultaneously suppression of the elastic motions is accomplished. Stability of the closed-loop system in the presence of model uncertainty has been guaranteed by using the Lyapunov stability theory. The efficiency of the proposed method has been demonstrated through numerical simulations.

Appendix 1: Elements of the Mathematical Model

$$q_r = \theta$$

$$\mathbf{q}_f = [q_{f1}(t) \quad q_{f2}(t) \quad \ldots \quad q_{fm}(t)]^T$$

$$M_{rr} = J_h + \frac{2}{3} mL^2 + 2\mathbf{q}_f{}^T \boldsymbol{\psi} \mathbf{q}_f$$

$$\boldsymbol{\psi} = \frac{m}{L} \int_0^L \boldsymbol{\varphi}(x) \boldsymbol{\varphi}^T(x) dx$$

$$\mathbf{M}_{rf} = 2\frac{m}{L} \int_0^L x \boldsymbol{\varphi}^T(x) dx$$

$$\mathbf{M}_{fr} = \mathbf{M}_{rf}^T$$

$$\mathbf{M}_{ff} = 2\boldsymbol{\psi}$$

$$h_{rr} = -4\mathbf{q}_f^T \boldsymbol{\psi} \dot{\mathbf{q}}_f$$

$$\mathbf{h}_{rf} = \mathbf{0}$$

$$\mathbf{h}_{fr} = 2\boldsymbol{\psi} \mathbf{q}_f \dot{\theta}$$

$$\mathbf{h}_{ff} = 2\mu \int_0^L \boldsymbol{\varphi}(x) \boldsymbol{\varphi}^T(x) dx$$

$$K_{rr} = \mathbf{0}$$

$$\mathbf{K}_{rf} = \mathbf{0}$$

$$\mathbf{K}_{fr} = \mathbf{0}$$

$$\mathbf{K}_{ff} = 2EI \int_0^L \frac{d^2 \boldsymbol{\varphi}(x)}{dx^2} dx$$

Where the following are the parameters of the system:

θ Hub angular position

J_h Hub inertia

m Mass of the flexible appendage

L Length of the flexible appendage

EI Flexural rigidity of the flexible appendage

μ Structural damping of the flexible appendage

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