Shape Change Maneuvers for Attitude Control of Underactuated Satellites

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Abstract— **An asymptotically stable set point control law is introduced for attitude control of the underactuated spacecraft of a satellite system through its appendage shape changes. It is proposed that rotational maneuvers of the spacecraft are possible while simultaneously achieving a desired shape if the total angular momentum is conserved and the system is initially motionless. In addition, it is also proposed that shape changes can be used to stabilize a tumbling satellite system with only one reaction wheel. The approach assumes that appendage shape changes can result in effective changes in inertia properties. A robust control algorithm based on sliding mode approach was proposed for stabilization and tracking control of underactuated multibody mechanical systems in an earlier work. It was shown that the construction of first order sliding surfaces leads only to marginally stable control law when angular momentum is conserved and equals zero. Here, we propose that the marginally stable control laws can be put together to achieve an asymptotically stable discontinuous control law. The control law essentially uses shape changes leading to changes in effective moments of inertia of the system about the axis of rotation. It is proposed that repetitive application of such maneuvers will lead to asymptotic convergence of the shape to the desired configuration. The controller is applied to the model of an existing complex satellite system and the relevant maneuvers are discussed.**

I. INTRODUCTION

Set point control of underactuated spacecrafts [1-5] and robots [6-8] has been the subject of considerable research in the past decade. Underactuated systems are normally those that have fewer actuators than rigid body degrees of freedom. An example is an underactuated spacecraft where one or more of its reaction wheels have been lost. Stabilization and attitude control of such a spacecraft with only two reaction wheels has been addressed in several studies [9-11]. Researchers have also suggested shape change actuation as a means to overcome such a problem [12-15]. Shape change actuators can be used to maneuver spacecrafts thus saving fuel consumption and extending the duration of missions. Serial type robotic links have also been suggested for reorienting space structures [12-13]. Complex and expensive satellite systems are prime candidates for shape change actuation through their articulated appendages. The underactuated problem for such systems results in a set of nonholonomic constraint equations, which may have useful control properties [3].

Sliding mode control is a robust control approach for systems with parameter uncertainties and disturbances [16]. The approach is based on defining exponentially stable (sliding) surfaces from the position and velocity tracking errors and using Lyapunov theory to insure all system trajectories reach these surfaces in finite time. Sliding mode controllers have been successfully developed for detumbling of underactuated single body spacecrafts [10] and nonholonomic integrators [17].

In a previous study, we applied the sliding mode control approach to underactuated nonlinear multibody systems [18]. We defined first order sliding surfaces as linear combinations of the actuated and unactuated position and velocity tracking errors and were able to determine the control laws by defining as many as surfaces as actuators. Using the squared of the surfaces as Lyapunov functions, we were able to guarantee all system trajectories would reach the sliding surfaces in finite time and stay there. It was also shown that the sliding surfaces could be made asymptotically stable when the presence of potential energy constraints the equilibrium points to a subspace with dimension equal to the number of actuators. However, in absence of potential energy, stability cannot be guaranteed unless momentum is conserved at zero, which leads to marginally stable surfaces and hence marginally stable controllers.

In this study, we will demonstrate that several marginally stable sliding mode control laws can be put together to achieve an asymptotically stable discontinuous control law for satellite systems which are capable of changing their

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effective mass moments of inertia through shape changes as long as the total angular momentum is conserved at zero. We will also propose shape changes that can stabilize a tumbling satellite (nonzero angular momentum) as long as at least one of the three reaction wheels is functional. The control law is further articulated for attitude control of the spacecraft of a communication satellite system through its antenna dish shape changes. The specific maneuvers required for roll, pitch, and yaw rotations are discussed. It is shown that the control law is effective in rotating the satellite system about any of its body-fixed reference frame axes and within a reasonable amount of time with its existing appendage actuator saturation limits while simultaneously achieving any desired shape.

II. Sliding Model Control of Underactuated Multibody Systems

The sliding mode controller developed in our earlier work guaranteed that all system trajectories reach the surface by selecting appropriate surface parameters [18]. However, asymptotic stability of the surface was not guaranteed and depended on the selection of surface parameters and additional properties such as presence of potential energy and conservation of momentum.

For nonlinear multibody mechanical systems with n degrees of freedom, the surfaces are asymptotically stable if the equilibrium points are constrained to an m dimensional space where m is the number of actuated coordinates. In other words, if there are $r = n - m$ (number of unactuated coordinates) independent constraint equations, then the system will be locally asymptotically stable near the equilibrium points. Otherwise, there will be between 2 and $2r$ poles of the linearized system exactly equal to zero independent of our choice of surface parameters. In the later case, the system will be marginally stable if angular momentum is conserved and equals to zero for rotational motion [18].

III. STABILIZATION AND CONTROL VIA INERTIA CHANGES

In this section, we will limit our discussion to satellite systems with appendages where the articulated coordinates are actuated but some or all three attitude controls (reaction wheels) are not available. It is assumed that the angular momentum is conserved. We divide the problem into two parts. First, the system is initially motionless (zero total angular momentum) and the objective is to go from one configuration to another. Next, we will consider how to stabilize a tumbling satellite (nonzero total angular momentum).

A. Zero Angular Momentum

As mentioned in Section II, the system will be marginally stable if angular momentum is conserved and is equal to zero. This assumption is valid if a multi-rigid body system is initially motionless in space and the total impulse of external moments is zero. Hence, the total angular momentum for a system of nb rigid bodies is:

$$\sum_{i=1}^{nb} J_i \boldsymbol{\omega}_i + \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i = \boldsymbol{0} \tag{1}$$

where J_i is the (3×3) inertia matrix, ω_i is the angular velocity vector, m_i is the mass, and $\boldsymbol{r}_i$ & $\dot{\boldsymbol{r}}_i$ are the position and velocity vectors of body i. We may rewrite the above equations in terms of actuated and unactuated generalized coordinates as:

$$I_a(\boldsymbol{q})\dot{\boldsymbol{q}}_a + I_u(\boldsymbol{q})\dot{q}_u = \boldsymbol{0} \tag{2}$$

where I_a (3×m) and I_u (3×r) are "effective" inertia matrices corresponding to the actuated and unactuated generalized coordinates, respectively. In the above equation, let's assume that the changes in $\boldsymbol{q}_a$, $\Delta\boldsymbol{q}_a$, are selected (through actuation) such they all contribute to changes in $\boldsymbol{q}_u$, $\Delta\boldsymbol{q}_u$, in the same direction (positively or negatively). Recall that the matrices I_a and I_u are constructed from positive definite rigid body inertia matrices. Hence, if we effectively decrease the value of components of I_a, then $\Delta\boldsymbol{q}_u$ decreases as a result of the same $\Delta\boldsymbol{q}_a$. Note that, Eq. (2) represents nonholonomic constraint equations and thus we cannot trivially relate $\Delta\boldsymbol{q}_u$ to $\Delta\boldsymbol{q}_a$ even in the scalar case where simple planar rotations are considered. The following example is useful in understanding the maneuvers required for achieving a desired configuration for such a system.

1) *Simple Example:* Consider an initially motionless system with two planar rigid bodies connected through a revolute joint. We model the first body as the base body and assume it can only rotate (no translation) and is unactuated with moment of inertia I_u. The second body is considered an appendage with moment of inertia I_a and is actuated through a motor located at the revolute joint. Thus, the base body rotation in the unactuated coordinate and the joint angle is the actuated coordinate. The total angular momentum of the system is written as:

$$(I_a + I_u)\dot{q}_a + I_u\dot{q}_u = 0 \tag{3}$$

Integrating the angular momentum equation:

$$\Delta q_u = -\frac{I_a}{I_u + I_a}\Delta q_a \tag{4}$$

Now, lets assume that we have reduced the second body's inertia by factor α ($0 < \alpha < 1$). We can rewrite equation (4) for the new system as:

$$\Delta q_u{}' = -\frac{\alpha I_a}{I_u + \alpha I_a}\Delta q_a \tag{5}$$

Comparing equations (4) and (5), it is clear that for the same Δq_a, we get smaller change in q_u, when the appendage inertia is reduced. The difference in the rotation is the relative motion or “progress” toward the goal:

$$\Delta q_u - \Delta q_u' = -\frac{(1-\alpha)I_a I_u}{(I_u + I_a)(I_u + \alpha I_a)}\Delta q_a \qquad (6)$$

Equation (6) demonstrates that the appendage inertia must be of significance relative to the base body, otherwise the “progress” toward the goal will be too small and possibly impractical.

Next, consider the case where we would like to achieve a desired change in the unactuated coordinate, Δq_u, while the appendage remains at the same relative position to the base body (i.e. $\Delta q_a = 0$) and we are somehow capable of changing I_a to αI_a and vise versa. The following maneuvers will asymptotically achieve our desired configuration:

1- Maneuver the system to achieve the desired Δq_u using Eq. (4)
2- Reduce I_a to αI_a and return the appendage back to its initial position using Eq. (5). The main body now returns toward its initial position but with a net change toward its goal as determined by Eq. (6)
3- Repeat steps 1-2 from the current configuration until Δq_a approaches zero. The rate of convergence after each maneuver (steps 1-2) k is:

$$(\Delta q_u)_k = \left[\frac{\alpha(I_u + I_a)}{I_u + \alpha I_a}\right]^k (\Delta q_u)_0 \qquad (7)$$

Note that, the rate of convergence will be faster as $\alpha \to 0$; i.e. larger changes in moment of inertia.

Clearly in a real system, reduction of I_a to αI_a requires at additional an appendage either in the form a proof mass [14-15] or by a second appendage connected through a revolute joint to the first one which allows it to rotate in and out of the plane and change inertia (or shape). This can be done very effectively through an in-plane circular disk in the plane, which by rotating 90 degrees out of plane can reduce its in-plane inertia by 50% ($\alpha = 1/2$).

B. Constant Nonzero Angular Momentum

Consider the case when the system is not initially motionless, in other words the case where the total angular momentum is nonzero though conserved. In such a cases the controller must first stop the motion (detumble), which requires at least one reaction wheel as discussed below. The algorithm required for detumbling and reorienting the system is outlined here. We consider two cases. First we discuss how to detumble and reorient a satellite system with two reaction wheels and next move to the more difficult case where only one is available.

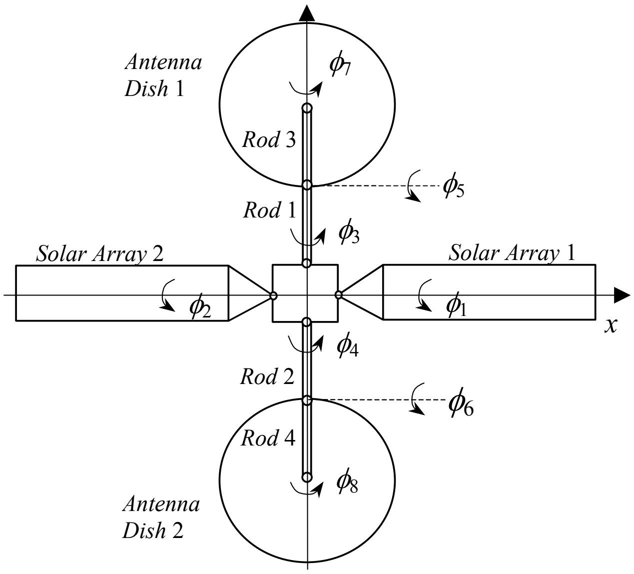

Fig. 1: Top view of a satellite system kinematic model

1) Satellite with Two Reaction Wheels

1- Since the appendages are actuated, stop their motion and shape them as desired.
2- Keep the appendages motionless relative to the main body such that the whole system acts as one rigid body. Detumble the rigid body motion according to the control law suggested in [10].
3- Once the system is motionless, rotate the satellite and change the shape as described in the zero angular momentum case.

2) Satellite with One Reaction Wheel

1- Stop the appendages and reorient them such that the largest rigid body moment of inertia of the whole system is about the axis of the one working reaction wheel. Since momentum is conserved and internal frictional losses always exist, the system will reach its minimum kinetic energy and will rotate about the axis with the largest moment of inertia [19] after possibly several hours since we are only relying on internal friction.
2- Stop the rotation using the reaction wheel.
3- Rotate the satellite and change the shape as described in the zero angular momentum case.

IV. ATTITUDE CONTROL OF A SATELLITE SYSTEM

Consider the satellite system shown in Fig. 1 consisting of nine rigid bodies, the spacecraft, two solar arrays, two antenna dishes, and four rods, as. The objective is attitude control of the spacecraft through its appendages when it has lost one, two, or all three of its reaction wheels. The assumption is that the system is initially motionless or can be made motionless as discussed in Section 3.2. The objective of the controller is to take the system from one set point to another.

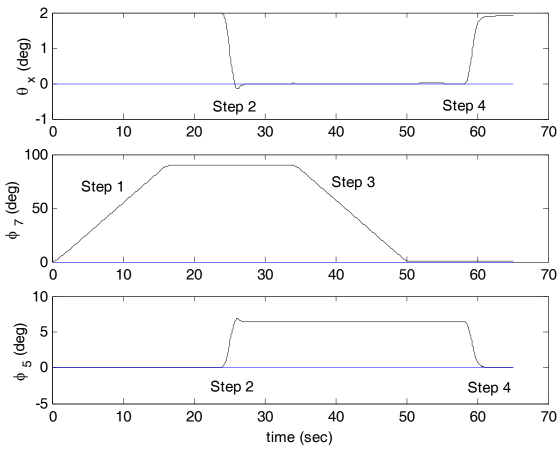

Fig. 2: The suggested maneuvers for roll motion

A. Equations of Motion

There are a total of 14 DOF, 6 for the spacecraft (body 0) and 8 articulated DOF for the 8 appendages (bodies 1 – 8). The articulated generalized coordinates are collected into a vector as $\boldsymbol{\phi} = [\phi_1 \;\; \phi_2 \; \ldots \;\; \phi_8\,]^T$. The spacecraft motion is represented by 3 translations $\boldsymbol{r} = [x\; y\; z]^T$ which are actuated by thrusters and 3 Euler angles $\boldsymbol{\theta} = [\theta_x \;\; \theta_y \;\; \theta_z\,]^T$. The equations of motion of this system have derived and were presented in [18].

B. Controller Construction and Simulations

The system has 14 DOF (n = 14) and 11 (m = 11, r = 3), 12 (m = 12, r = 2), or 13 (m = 13, r = 1), controllers. The main idea in this study is rotate the spacecraft about one of its body-fixed axes that does not have a reaction wheel via its appendages while maintaining a desired shape.

Since the system can have very complex 3D shapes, we consider the appendages initial and desired configuration to be all at zero, as shown in Fig. 1. This allows us to articulate the maneuvers in a simpler form. The current version of our controller also assumes that only one reaction wheel is lost. The system must start from rest and go back to rest at the desired attitude and shape. All appendage control inputs are saturated at the magnitude of 6 Nm according to the limits of the current gimbals used in communication satellites.

We have considered each rotation task (roll: θ_x, pitch: θ_y, yaw: θ_z), separately, and hence a few comments are necessary: a) Since only simple planar rotations are used, the rate of convergence discussed for the simple example in Section III holds for this example as well; b) The solar arrays are not used since they can only rotate about the spacecraft roll axis and thus provide very small moment of inertia changes ($\alpha \approx .96$); c) The antenna dishes comprise about 15% considerable inertia and can provide significant changes in the inertia about the spacecraft body-fixed axes (α = 1/2). Therefore, only the rods and the antenna dishes are used to reorient the spacecraft.

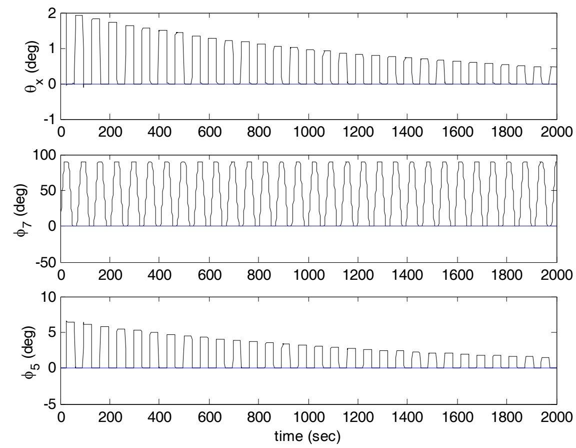

Fig. 3*:* Roll angle and appendage maneuvers

1) *Roll:* The first case considers rotating the spacecraft 2^o about its x-axis. The required maneuvers in order to rotate the spacecraft toward its goal while simultaneously achieving the desired appendage shape are summarized as follows:

1- Rotate the two dishes (ϕ_7 & ϕ_8) 90^o each such that they lie in the *yz*-plane (orthogonal to *z*-axis. This approximately doubles the dish moment of inertia about the *x*-axis.
2- Rotate θ_x by allowing rods 3 & 4 (ϕ_5 & ϕ_6) to freely stabilize at some angle [18].
3- Rotate back the two dishes (ϕ_7 & ϕ_8) 90^o each such that they are orthogonal to the *yz*-plane. This reduces the dish moment of inertia about the *x*-axis by 50%.
4- Rotate rods 3 & 4 (ϕ_5 & ϕ_6) back to their initial values. This will rotate the angle θ_x back towards the initial value but not as much because of the shape change, as discussed in Section III.
5- Repeat steps 1 – 4 until θ_x reaches its desired value.

Figure 2 illustrates the maneuvers described in steps 1 – 4. Figure 3 shows the time history of the roll angle, θ_x, along with the required maneuvers by the antenna dishes and the rods. It is clear that the convergence is very slow such that after 2000 seconds only 1.5^o rotation is achieved.

In order to make the process faster, we made the maneuvers adaptive such that amount of rotation of θ_x after the first maneuver is based on how large of step toward the goal was taken. However, we need to saturate the required rotations since actuators are not capable of such large maneuvers. The result of this adaptive maneuvering with saturation value at -10^o is shown in Fig. 4. The convergence process takes less than 8 minutes with the currently available gimbals.

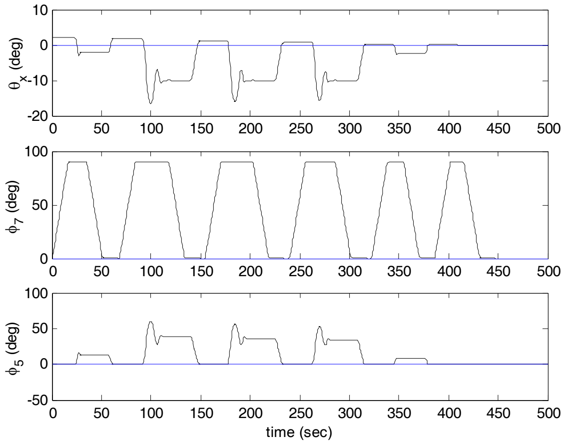

Fig. 4: Adaptive roll angle and appendage maneuvers

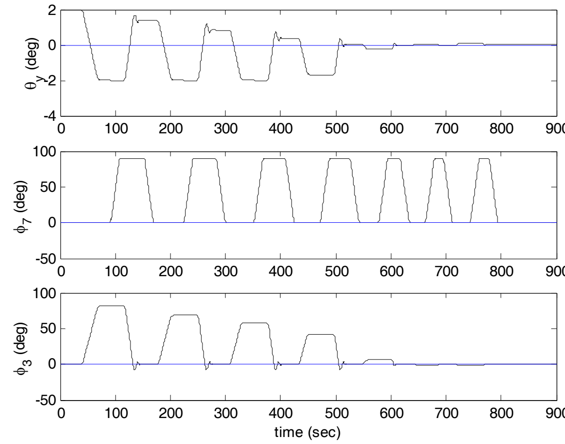

Fig. 6: Adaptive pitch angle and appendage maneuvers

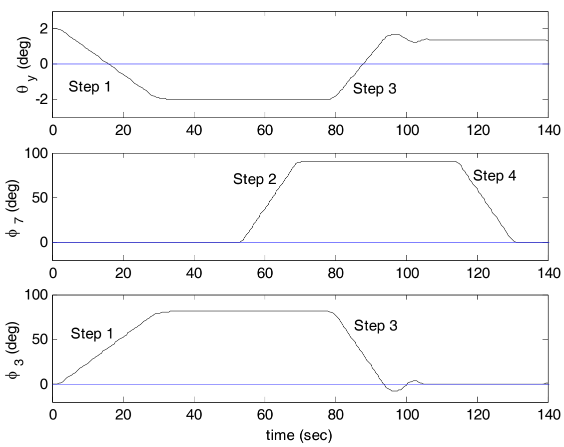

Fig. 5: The suggested maneuvers for pitch motion

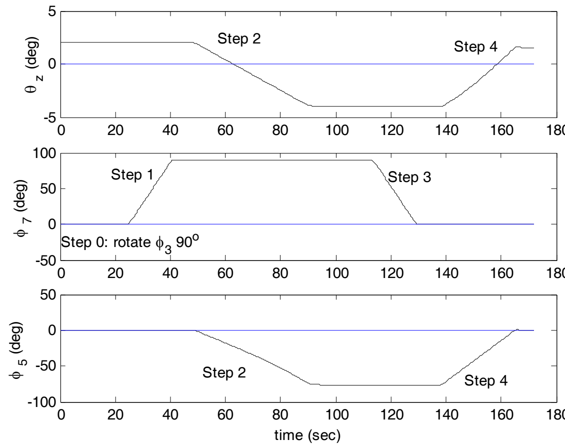

Fig. 7: The suggested maneuvers for yaw motion

2) *Pitch:* The second case considers rotating the spacecraft 2^{o} about its y-axis. The required maneuvers in this case are:

0- Initially rotate rods 3 & 4 90^{o} each so that the subsequent dish rotations will actually be able to place them in and out of plane of rotation.
1- Rotate θ_y by allowing rods 1 & 2 (ϕ_3 & ϕ_4) to freely stabilize at some angle [18].
2- Rotate the two dishes 90^{o} each such that they are orthogonal to the *xz*-plane. This reduces the dish moment of inertia about the *y*-axis by 50%.
3- Rotate rods 1 & 2 back to their initial values. This will rotate the angle θ_y back towards the initial value but not as much, as discussed in Section III.
4- Rotate the two dishes 90^{o} each such that they lie in the *xz*-plane. This approximately doubles the dish moment of inertia about the *y*-axis.
5- Repeat steps 1 – 4 until θ_y reaches its desired value. But step 4 must be skipped in the last maneuver.
6- Rotate rods 3 & 4 90^{o} back to their initial values.

Figure 5 illustrates the maneuvers described in steps 1 – 4. Only one dish and one rod motion is shown since all the maneuvers are assumed to be symmetric. The adaptive form of the controller is again used with saturation value of -2^{o} and the convergence process takes less than 14 minutes, as shown in Fig. 6.

3) *Yaw:* The third case considers rotating the spacecraft 2^{o} about its z-axis. The required maneuvers in this case are:

0- Initially rotate rods 1 & 2 90^{o} each so that the subsequent rod 5 & 6 rotations will actually be able to rotate the spacecraft about *z*-axis.
1- Rotate the two dishes 90^{o} each such that they lie in the *xy*-plane. This approximately doubles the dish moment of inertia about the *z*-axis.
2- Rotate θ_z by allowing rods 3 & 4 to freely stabilize at some angle [18].
3- Rotate back the two dishes 90^{o} each such that they are orthogonal to the *xy*-plane. This reduces the dish moment of inertia about the *z*-axis by 50%.

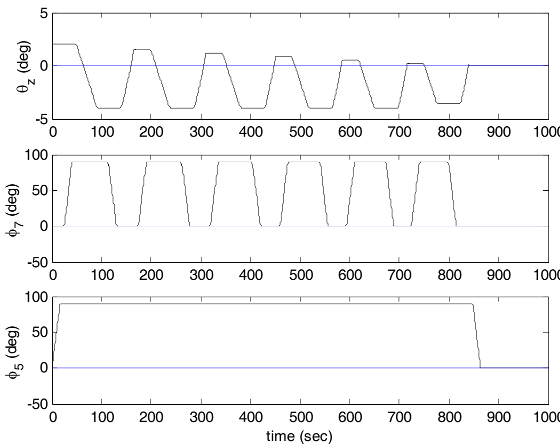

Fig. 8: Adaptive yaw angle and appendage maneuvers

4- Rotate rods 3 & 4 back to their initial values. This will rotate the angle θ_z back towards the initial value but not as much, as discussed in Section III.
5- Repeat steps 1 – 4 until θ_z reaches its desired value.
6- Rotate rods 1 & 2 90^o back to their initial values.

Figure 7 illustrates the maneuvers described in steps 1 – 4. Only one dish and one rod motion is shown since all the maneuvers are assumed to be symmetric. The adaptive form of the controller is again used with saturation value of -4^o and the convergence process takes less than 15 minutes, as shown in Fig. 8.

V. CONCLUSIONS

Stabilization and attitude control of underactuated satellite systems via their appendages was discussed. It was established that shape changes could be used to stabilize a tumbling satellite with one reaction wheel by making the rigid body moment of inertia about the actuated axis the largest of the three. An asymptotically stable set point control law was introduced for attitude control of initially motionless underactuated satellite systems while simultaneously achieving a desired shape. The control law was only applicable if the shape changes resulted in changes in effective moment of inertia of the appendages about the axis of rotation. The controller was specifically adapted and applied to the model of an existing complex satellite system and the relevant appendage maneuvers were discussed. It was shown that the control law is effective in rotating the satellite system within a reasonable amount of time while taking into account existing actuator saturation limits. Future research on the subject would include formal proof of asymptotic stability the shape change control law extension to system with no reaction wheels, and inclusion of low frequency deformable body structural vibration modes, which could be excited as a result of discontinuity in the actuator torques.

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