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Self-Tuning Sub-Optimal Control of Time-Invariant Systems With Bounded Disturbance

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Abstract— **In this paper, a sub-optimal controller is studied for a class of time-invariant systems subject to bounded disturbances. The controller is based on the singular perturbation and simultaneous perturbation methods to achieve stable online self-tuning while requiring a little information about the plant model. The suggested controller is an extension of a self-tuning PID controller of Davison by adopting the gradient search scheme and the simultaneous perturbation method to tune the controller parameters. Further optimization effort is added by extending the self-tuning PID controller to a high-order controller with the stable self-tuning property. Both simulation and experimental results are provided to verify the feasibilities of the proposed controller.**

I. INTRODUCTION

Research activities on collaboration of multidisciplinary areas have brought related control and optimization efforts to an increasingly complex level, which emerges from the enhancement of more unknown factors in system dynamic models. Conventionally, the more sufficient plant's information the preassigned model has, the larger possibility to find an effective controller efficiently will be guaranteed. However, in most practical cases, a perfectly matching model can rarely be constructed as a result of the lack of cognition ability for human beings on complex system dynamics. In such cases, some model-based design schemes may not be applicable, and meanwhile, some efforts should be made on the condition of the information scarcity of plant dynamics.

Usually, adaptive control is considered to deal with the unmatched system dynamic models. References [1], [2] promote PID tuning methods. For more sophisticated design scheme, advanced algorithms, such as backstepping, sliding modes and fuzzy neural network control, are well developed in [3], [4], [5] respectively. However, all of these are model-based controllers. It is more challenging to design controllers when little information of the plant model is available. Reference [6] makes contributions on the single-input-single-output(SISO) systems control with automatic

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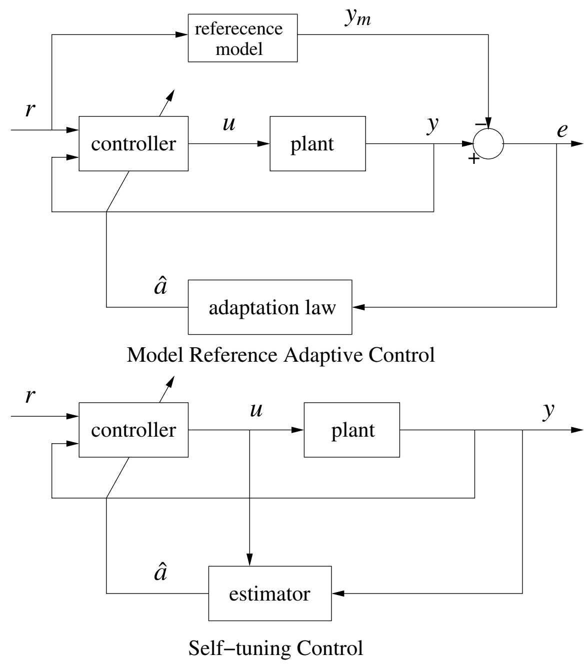

Fig. 1. Two Typical Adaptive Control Systems, $\hat{a}$ is estimated parameters

tuning parameters under no perturbation condition. Reference [7] gives a self-tuning PID control method for the multi-input-multi-output(MIMO) systems with unknown extreme perturbations. Nevertheless, the performance of the controller cannot be ensured to be optimal and the choice of initial parameters for controller must be case-dependent. Thus, it is necessary to find a method to optimize the self-tuning controller.

Fig.1 shows two typical kinds of adaptive control schemes. The approach of this study is based on the second type of adaptive control schemes in Fig.1, which consists of two stages. In the first stage, since the self tuning method for PID controller described in [7] is able to maintain a global stability for linear time invariant systems, one parameter searching scheme, namely the Simultaneous Perturbation Method [8], is incorporated into the controller to search for optimal PID parameters. Moreover, due to extreme disturbance tolerance in [7], this promoted controller must also be stable under bounded disturbance. The second stage, inspired from the idea of tuning method at the

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first stage, is that we also explore an extended switching control and searching scheme for control optimization purpose. In order to get the simplest controller design, one butter-worth-typed controller is well studied.

The objective is to obtain an improved frequency response with the aid of the suggested optimization methods, which are tested in both simulation and experiment. The key point is that little model information is used in the controller design. Therefore, in simulation, arbitrary chosen model transfer functions are selected for validation. As for the experiment, one typical and difficult engineering application described as the control of flow induced vibration [11] is used to study suggested controllers' optimizing possibility.

II. PRELIMINARIES

Consider the linear time-invariant continuous system given by [7]

$$\begin{aligned} \dot{x} &= Ax + Bu + Ew \\ y &= Cx + Fw \\ e &= y_{ref} - y \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is control input, $w \in \mathbb{R}^r$ is the unknown bounded disturbance, and $e \in \mathbb{R}^q$ is the difference between reference $y_{ref} \in \mathbb{R}^q$ and output $y \in \mathbb{R}^q$. Similar to [7], the overall system (1) is assumed to be open-loop stable, which implies the invertibility of A. Also, its DC steady-state gain matrix $\mathscr{T} := -CA^{-1}B$ has full-row rank. While matrices A, B, and C are used for analysis, exact values of these matrices are not available. The only information available to the controller is the DC gain $\mathscr{T}$, which can be identified by many available methods with sufficient accuracy. A model-independent controller is studied here to control such class of plants with partial model information.

Since (1) is open-loop stable, the eigenvalues of matrix A, $eig(A) \subset \mathbb{C}^-$(Left Hand of Complex Plane), and (1) may be regulated by a self-tuning controller given by [7, p.1978]

$$\begin{aligned} \dot{\eta}(t) &= \varepsilon_1(t)e(t) \\ \dot{\xi}(t) &= -N\xi(t) + e(t) \\ \gamma(t) &= -N^2\xi(t) + Ne(t) \\ u(t) &= K(t)(\eta(t) + \rho(t)e(t) + \varepsilon_2(t)\gamma(t)) + u^c \end{aligned} \tag{2}$$

where u^c is the center of the input value set that is bounded by u_{min} and u_{max}, $k \in \{1,2,3,\cdots\}$, $(\eta(t_k^+), \xi(t_k^+)) \equiv (0,0)$, and where

$$\begin{aligned} &(\varepsilon_1(t), \varepsilon_2(t), K(t), \rho(t)) \\ &= (g_1(k), \bar{\varepsilon}_2 g_1(k)^{\beta_\varepsilon+1}, \bar{k}(k), \bar{\rho} g_1(k)^{\beta_\rho+1}), t \in (t_k, t_{k+1}] \end{aligned} \tag{3}$$

with $(\beta_\varepsilon, \beta_\rho, \bar{\varepsilon}_2, N, \bar{\rho}) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$ fixed constants, $\bar{k}$ is K tuning function[1], $g_1(k) = \varepsilon_0/\tau^k$, $(\varepsilon_0, k, \tau - 1) \in \mathbb{R}^+ \times \mathbb{Z}^+ \times \mathbb{R}^+$, is tuning function, whose switching time is set at the boundary of control input's constraint set.

[1] for the definitions of this K tuning function and the following tuning function, please refer to [7, p.1977]

Assume that disturbance w is under ergodic hypothesis, if plant (1) is open-loop stable with $eig(A) \subset \mathbb{C}^-$, then one sub-optimal control is applicable to render the closed-loop system as a convex optimization problem. To solve convex optimization problems, usually, the objective function gradient is used to estimate descendent direction of objective function's value. However, for most optimization problems, a specific model information, such as A, B, and C, is needed by available design methods. When only $\mathscr{T}$ is available, the simultaneous perturbation method ([8], [9], [10]) is used here to estimate gradients. Consider an unknown objective function $J(\chi)$, where $\chi \in R^n$ is a set of parameters of J, the algorithm is described as follows:

$$\begin{aligned} \chi_{t+1} &= \chi_t - \alpha\Delta\chi_t \\ \Delta\chi_t^i &= \frac{J(\chi_t + ps_t) - J(\chi_t)}{p} s_t^i \end{aligned} \tag{4}$$

where p is perturbation magnitude for one searching step, χ^i represents the i-th element of vector χ, and s^i denotes the i-th element of the sign vector s, which is assigned randomly and independent with each other. For the second equation of (4), we expand $J(\chi_t + ps_t)$ with respect to χ_t,

$$J(\chi_t + ps_t) = J(\chi_t) + ps_t^T \frac{\partial J(\chi_t)}{\partial \chi} + \frac{p^2 s_t^T}{2} \frac{\partial^2 J(\chi_{s1})}{\partial \chi^2} s_t \tag{5}$$

then substitute into (4),

$$\Delta\chi_t^i = s_t^i s_t^T \frac{\partial J(\chi_t)}{\partial \chi} + \frac{ps_t^i}{2} s_t^T \frac{\partial^2 J(\chi_{s1})}{\partial \chi^2} s_t \tag{6}$$

the expectation of above equation is [10, p.1125]

$$E(\Delta\chi_t^i) = \frac{\partial J(\chi_t)}{\partial \chi_t^i} + E\left\{ \frac{ps_t^i}{2} s_t^T \frac{\partial^2 J(\chi_{s1})}{\partial \chi^2} s_t \right\} \tag{7}$$

which means that if p is sufficiently small, the second term of the right-hand side of (7) is small. Then $\Delta\chi_t^i \approx \partial J(\chi_t)/\partial \chi_t^i$.The precious point of this method is that objective function's gradients can be evaluated approximately by two estimations $J(\chi + ps)$ and $J(\chi)$. For a typical selection of $J(\chi)$ in practice, the error function is usually used

$$J(\chi_t) = \sum_{k=1}^{\lambda} e^2(k) \tag{8}$$

where k is sampling number in a block interval, and λ is total sampling number for one block interval as illustrated in Fig.2. The performance index in simultaneous perturbation method looks exactly the same as H_2 norm. However, for available H_2 design methods, e.g. LMI design or Riccati equation approach, exact A, B and C are required. While in this paper, we use little knowledge about A, B and C during the H_2 optimizing process.

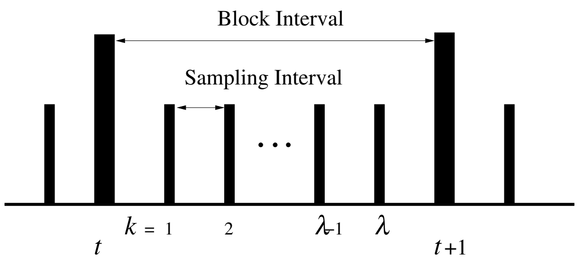

Fig. 2. Block interval and sampling interval

III. MODEL-INDEPENDENT OPTIMAL CONTROLLERS

After describing necessary design tools, now, we will have a look at the controllers design ideas.

A. 1st version: 3-term searching

Given that plant (1) is stabilized by controller (2), we suggest that one additional quadratic optimal control is feasible to be applied onto the system, which is also convex. Recalling the controller (2) in Laplace transformed format [7, p.1976],

$$u(s) = K\left[\frac{\varepsilon_1 I}{s} + \bar{\rho}\varepsilon_1^{\beta_\rho+1} I + \bar{\varepsilon}_2\varepsilon_1^{\beta_\varepsilon+1}\frac{sNI}{s+N}\right]e(s) + \frac{u^c}{s} \quad (9)$$

we notice that it is recognized as a PID tuning controller, which is mainly characterized by its three terms' coefficients, namely $(\varepsilon_0, \bar{\rho}, \bar{\varepsilon}_2)$ in (3). Thus, 1st version of optimal control algorithm can be enumerated as:

1) arbitrarily set initial values of $(\varepsilon_0, \bar{\rho}, \bar{\varepsilon}_2)$
2) examine the control input u for the corresponding initial values of $(\varepsilon_0^0, \bar{\rho}^0, \bar{\varepsilon}_2^0)$
   a) if u is on the boundary or outside of the constraint region, apply controller (2) to the system and get $(\varepsilon_0^i, \bar{\rho}^i, \bar{\varepsilon}_2^i)$
   b) if u is inside of the constraint region, then go on
3) define the total sampling number λ, and calculate $J(\varepsilon_0^i, \bar{\rho}^i, \bar{\varepsilon}_2^i) = \sum_{k=1}^{\lambda} e^2(k)$
4) perform simultaneous perturbation of $\varepsilon_0^i, \bar{\rho}^i, \bar{\varepsilon}_2^i$ to get $\varepsilon_0^{i+1}, \bar{\rho}^{i+1}, \bar{\varepsilon}_2^{i+1}$ and examine their corresponding control input u'
   a) if u' is on the boundary or outside of the constraint region, apply controller (2) to the system and get another set of $(\varepsilon_0^{i+1}, \bar{\rho}^{i+1}, \bar{\varepsilon}_2^{i+1})$
   b) if u' is inside of the constraint region, then go on
5) calculate $J(\varepsilon_0^{i+1}, \bar{\rho}^{i+1}, \bar{\varepsilon}_2^{i+1}) = \sum_{k=1}^{\lambda} e^2(k)$
6) evaluate the gradient of J for the tuning of $\varepsilon_0^i, \bar{\rho}^i, \bar{\varepsilon}_2^i$
7) start from the latest tuned $\varepsilon_0^i, \bar{\rho}^i, \bar{\varepsilon}_2^i$, and go to step 3.

B. 2nd version: extended tuning control and searching

The 3-term PID control sometimes may not achieve high performance for higher order plants, even if the above 1st version optimal controller leads to improved results. Inspired by the controller design concept of (2) in [7], one may extended the PID controller to a high-order controller with additional terms.

Consider an extended control input with $u^c = 0$ as shown at the bottom of this page (10), where $\{a_1, a_2, \cdots, a_h\} \subset \mathbb{C}^+$ and $\{c_1, c_2, \cdots, c_h\} = \{\bar{c}_1\varepsilon_1^{\beta_{c_1}+1}, \bar{c}_2\varepsilon_1^{\beta_{c_2}+1}, \cdots, \bar{c}_h\varepsilon_1^{\beta_{c_h}+1}\} \subset \mathbb{R}$. If one a_k is complex number, then another a_{k+1} must exist to act as a conjugate of a_k. The corresponding c_k and c_{k+1} are equal. Define the original PID switching control input state variables as $[\eta^T \ \ \xi^T]^T$, and the added state variables as $\Psi \in \mathbb{R}^h$. Thus, equations (11a) and (11b) at the bottom of next page give the state-space expression for (10) in partial diagonal form, which means if a_k and a_{k+1} are complex conjugate with each other, then values $-a_k$ and $-a_{k+1}$ in the diagonal form can be replaced by a 2×2 real matrix on the diagonal direction.

$$\begin{bmatrix}\dot{\eta}\\ \dot{\xi}\\ \dot{\Psi}\end{bmatrix} = \left[\begin{array}{cc:cccc} 0 & 0 & 0 & 0 & \cdots & 0\\ 0 & -NI & 0 & 0 & \cdots & 0\\ \hdashline 0 & 0 & -a_1 & & & 0\\ 0 & 0 & & -a_2 & & \\ \vdots & \vdots & & & \ddots & \\ 0 & 0 & 0 & & & -a_h \end{array}\right]\left[\begin{array}{c}\eta\\ \xi\\ \hdashline \psi_1\\ \psi_2\\ \vdots\\ \psi_h\end{array}\right] + \left[\begin{array}{c}\varepsilon_1 I\\ I\\ \hdashline I\\ I\\ \vdots\\ I\end{array}\right] e \quad \text{(11a)}$$

Denote $\Omega = diag\{a_1, a_2, \cdots, a_h\}$, $G = [I, I, \cdots, I]^T$, $H = [\bar{c}_1\varepsilon_1^{\beta_{c_1}+1}, \bar{c}_2\varepsilon_1^{\beta_{c_2}+1}, \cdots, \bar{c}_h\varepsilon_1^{\beta_{c_h}+1}]$, the closed-loop system combined by plant (1) and control (10) is expressed as

$$\begin{aligned}\dot{\tilde{x}} &= \hat{A}\tilde{x} + \hat{B}v\\ \tilde{y} &= \hat{C}\tilde{x} + \hat{D}v\end{aligned} \quad (12)$$

where $\tilde{x} = [x^T \ \ \eta^T \ \ \xi^T \ \ \Psi^T]$, $v = [y_{ref}^T \ \ w^T]$ and $\tilde{y} = [y^T \ \ u^T]$, which are evaluated in (12a). Before proceeding further, the following lemma on singular perturbation theory will be needed.

$$u(s) = K\left\{\overbrace{\left[\frac{\varepsilon_1 I}{s} + \bar{\rho}\varepsilon_1^{\beta_\rho+1} I + \bar{\varepsilon}_2\varepsilon_1^{\beta_\varepsilon+1}\frac{sNI}{s+N}\right]}^{PID\ \ switching} + \overbrace{\frac{c_1 I}{s+a_1} + \frac{c_2 I}{s+a_2} + \cdots + \frac{c_h I}{s+a_h}}^{added\ \ items}\right\}\times e(s) \quad (10)$$

Lemma 3.1: [12, p.57] Consider a singular perturbed system given by

$$\begin{aligned}\dot{x} &= (A_{11} + \varepsilon^{\beta_1}\overline{A_{11}})x + (A_{12} + \varepsilon^{\beta_2}\overline{A_{12}})z \\ \varepsilon\dot{z} &= (A_{21} + \varepsilon^{\beta_3}\overline{A_{21}})x + (A_{22} + \varepsilon^{\beta_4}\overline{A_{22}})z\end{aligned} \tag{13}$$

where $\beta_i \geq 1$ are fixed finite constants for $i \in \{1,2,3,4\}$, and $(x,z,\varepsilon) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^+$. If A_{22}^{-1} exists, then as $\varepsilon \to 0$, n_1 eigenvalues of (13) tend to $eig(A_{11} - A_{12}A_{22}^{-1}A_{21})$, while the remaining n_2 eigenvalues of (13) tend to infinity, with a rate of $1/\varepsilon$, along the asymptotes defined by $eig(A_{22})/\varepsilon$.

We now apply the above Lemma to the stability analysis of the extended self-tuning controller.

Theorem 3.1: If plant (1) is open-loop stable with $eig(A) \in \mathbb{C}^-$ and its DC steady-state gain matrix $\mathscr{T}$ has full-row rank, then the extended controller (10) guarantees the stability of the closed-loop system (12) if ε_1 is sufficiently small.

Proof: By applying similarity transformation, matrix $\hat{A}$ is rewritten as (14) at the bottom of last page. The equivalence of this proof is to verify $eig(\hat{A}^*/\varepsilon_1) \subset \mathbb{C}^-$. Notice that B is one $m \times 1$ matrix and H is one $1 \times h$ matrix, and for the i^{-th} column of H there is an $\varepsilon_1^{\beta_{c_i}+1}$ control action. Therefore, the resultant matrix for BHK must be a $m \times h$ matrix with the control action $\varepsilon_1^{\beta_{c_i}+1}$ in its $i-th$ column. Let $(bhk)_{ij}$ to be element of BHK without the $\varepsilon_1^{\beta_{c_i}+1}$ multiple, then BHK is written as,

$$\begin{aligned}&\begin{bmatrix} \varepsilon_1^{\beta_{c_1}+1}(bhk)_{11} & \varepsilon_1^{\beta_{c_2}+1}(bhk)_{12} & \cdots & \varepsilon_1^{\beta_{c_h}+1}(bhk)_{1h} \\ \varepsilon_1^{\beta_{c_1}+1}(bhk)_{21} & \varepsilon_1^{\beta_{c_2}+1}(bhk)_{22} & \cdots & \varepsilon_1^{\beta_{c_h}+1}(bhk)_{2h} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_1^{\beta_{c_1}+1}(bhk)_{m1} & \varepsilon_1^{\beta_{c_2}+1}(bhk)_{m2} & \cdots & \varepsilon_1^{\beta_{c_h}+1}(bhk)_{mh} \end{bmatrix} \\ &BHK := \begin{bmatrix} \varepsilon_1^{\beta_{c_1}+1}\overrightarrow{bhk_1} & \varepsilon_1^{\beta_{c_2}+1}\overrightarrow{bhk_2} & \cdots & \varepsilon_1^{\beta_{c_h}+1}\overrightarrow{bhk_h} \end{bmatrix}\end{aligned} \tag{15}$$

Now, we define the following sub-matrix,

$$A_{11} = \begin{bmatrix} 0 \end{bmatrix} \tag{16a}$$

$$A_{12} = -\begin{bmatrix} C & 0 & 0 \end{bmatrix} \tag{16b}$$

$$A_{21} = \begin{bmatrix} BK \\ 0 \\ 0 \end{bmatrix} \tag{16c}$$

$$A_{22} = \begin{bmatrix} A & 0 & 0 \\ -C & -NI & 0 \\ -GC & 0 & -\Omega \end{bmatrix} \tag{16d}$$

$$\overline{A_{22}^1} = -\begin{bmatrix} -\bar{\rho}BKC & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{16e}$$

$$\overline{A_{22}^2} = -\begin{bmatrix} \bar{\varepsilon}_2 NBKC & \bar{\varepsilon}_2 N^2 BK & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{16f}$$

$$u = K\left[\begin{array}{cc:cccc} 1 & -\bar{\varepsilon}_2\varepsilon_1^{\beta_\varepsilon+1}N^2 & \bar{c}_1\varepsilon_1^{\beta_{c_1}+1} & \bar{c}_2\varepsilon_1^{\beta_{c_2}+1} & \cdots & \bar{c}_h\varepsilon_1^{\beta_{c_h}+1} \end{array}\right] \left[\begin{array}{c} \eta \\ \xi \\ \hdashline \psi_1 \\ \psi_2 \\ \vdots \\ \psi_h \end{array}\right] + \left[K\bar{\rho}\varepsilon_1^{\beta_\rho+1} + K\bar{\varepsilon}_2\varepsilon_1^{\beta_\varepsilon+1}N\right]e \tag{11b}$$

$$\left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & \hat{D} \end{array}\right] = \left[\begin{array}{cccc|cc} A-(\rho+\varepsilon_2 N)BKC & BK & -\varepsilon_2 N^2 BK & BHK & (\rho+\varepsilon_2 N)BK & E-(\rho+\varepsilon_2 N)BKF \\ -\varepsilon_1 C & 0 & 0 & 0 & \varepsilon_1 I & -\varepsilon_1 F \\ -C & 0 & -NI & 0 & I & -F \\ -GC & 0 & 0 & -\Omega & G & -GF \\ \hline C & 0 & 0 & 0 & 0 & F \\ -(\rho+\varepsilon_2 N)KC & K & -\varepsilon_2 N^2 K & HK & (\rho+\varepsilon_2 N)K & -(\rho+\varepsilon_2 N)KF \end{array}\right] \tag{12a}$$

$$\hat{A}^* = \left[\begin{array}{c|cccc} 0 & -\varepsilon_1 C & 0 & 0 \\ \hline BK & A-(\varepsilon_1^{\beta_\rho+1}\bar{\rho}+\varepsilon_1^{\beta_\varepsilon+1}\bar{\varepsilon}_2 N)BKC & -\varepsilon_1^{\beta_\varepsilon+1}\bar{\varepsilon}_2 N^2 BK & BHK \\ 0 & -C & -NI & 0 \\ 0 & -GC & 0 & -\Omega \end{array}\right] \tag{14}$$

$$\overline{A_{22}^{h1}} = \begin{bmatrix} 0 & 0 & \overrightarrow{bhk_1} & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{16h1}$$

$$\overline{A_{22}^{h2}} = \begin{bmatrix} 0 & 0 & 0 & \overrightarrow{bhk_2} & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{16h2}$$

$$\vdots$$

$$\overline{A_{22}^{hh}} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & \overrightarrow{bhk_h} \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{16hh}$$

and one singularly perturbed system matrix is written as (17) at the bottom of this page. Obviously, from Lemma 3.1, the eigenvalues of system (17) will tend to

$$\left\{ \frac{eig(A)}{\varepsilon_1} \cup \frac{-N}{\varepsilon_1} \cup \frac{eig(-\Omega)}{\varepsilon_1} \cup eig(-\mathscr{T}K) \right\}$$

Since $eig(A) \in \mathbb{C}^-$ and the full-row rank condition of $\mathscr{T}$ are assumed, and for $-\Omega = -diag\{a_1, a_2, \cdots, a_h\} \in \mathbb{C}^-$, the system matrix of (17) is stable, which exactly refers to the stability of $\hat{A}^*/\varepsilon_1$. Thus, the closed-loop system (12) is also stable with $\hat{A} \in \mathbb{C}^-$. ■

Remark 3.1: In fact, in the extended controller (10), if we omit the first three terms for PID switching control, the remaining controller will still be able to stabilize the closed-loop system combined by plant (1) and only 'added items'. The proof can be shown just by reducing those sub-matrix order in (16). ◇

By mathematical manipulations of those 'added items', one transfer function with relative degree of one is concluded. The interesting characters are (1) its poles are all on the negative complex plane; (2) each coefficient of its numerator has such a control action as $\varepsilon_1^{\beta_c+1}$. Now, a generalized version of the extended switching controller can be obtained.

Corollary 3.1: For the plant (1) with pre-assumed properties, one switching controller

$$\begin{aligned} u(s) &= K\frac{b_1 s^{h-1} I + b_2 s^{h-2} I + \cdots + b_h I}{(s+a_1)(s+a_2)\cdots(s+a_h)} e(s) \\ &= K\frac{b_1 s^{h-1} I + b_2 s^{h-2} I + \cdots + b_h I}{s^h + a_1^* s^{h-1} + a_2^* s^{h-2} + \cdots + a_h^*} e(s) \end{aligned} \tag{18}$$

where $\{a_1^*, a_2^*, \cdots, a_h^*\}$ are fixed initial constants which ensures eigenvalues of denominator all on the negative complex plane, $\{b_1, b_2, \cdots, b_h\} = \{\bar{b}_1 \varepsilon_1^{\beta_{b_1}+1}, \bar{b}_2 \varepsilon_1^{\beta_{b_2}+1}, \cdots, \bar{b}_h \varepsilon_1^{\beta_{b_h}+1}\} \subset \mathbb{R}$, can be used to stabilize the closed-loop system combined by the plant and such a switching controller.

Proof: Proof is omitted since it is substantially the same as Theorem 3.1. ■

Remark 3.2: The next design step remains how to select the initial coefficients $\{a_1^*, a_2^*, \cdots, a_h^*\}$ of denominator of (18). In order to give a more optimal selection, observer idea serves the initial requirement. Consider controllable canonical form matrix for u,

$$\dot{\Psi} = \begin{bmatrix} -a_1^* & -a_2^* & \cdots & -a_h^* \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix} \Psi + \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} e \tag{19}$$
$$u = [b_1 I \quad b_2 I \quad \cdots \quad b_h I]\Psi$$

A set of butter-worth filter coefficients can be used to estimate both the initial values of $\overrightarrow{a^*}$ and approximate states of Ψ in certain bandwidth, which will alleviate further optimization effort. One example in section IVB. gives a detailed illumination about the design. ◇

Furthermore, the simultaneous perturbation method (4) can be imported to further optimize parameters $\{\bar{b}_1, \bar{b}_2, \cdots, \bar{b}_h\}$. The steps are omitted due to space limitation, nevertheless, which are also quite similar to those described in 1st version since the only difference is about the stabilization control method.

IV. SIMULATION EXAMPLES

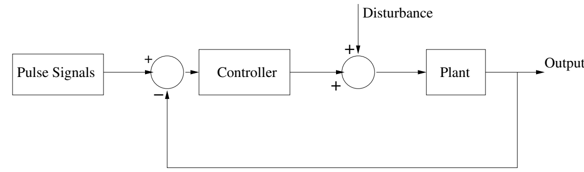

Fig. 3. An example of ordinary control loop with both disturbance and referencesignals

The ordinary control loop with both disturbance and reference signals is shown in Fig.3, where pulse signals are usually considered to be reference signals. In this paper, we concentrate on disturbance rejection and consider reference signal to be zero while disturbance to be the pulse signal as shown in Fig.4 and 6. The purpose of all the demonstrations is to validate the promoted controllers' feasibility of improving plant's frequency response under bounded disturbance with little system information. Thus, all the plant parameters are chosen randomly without any intension. The simulation

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} + \varepsilon_1^{\beta_\rho+1}\overline{A_{22}^1} + \varepsilon_1^{\beta_\varepsilon+1}\overline{A_{22}^2} + \varepsilon_1^{\beta_{c_1}+1}\overline{A_{22}^{h1}} + \varepsilon_1^{\beta_{c_2}+1}\overline{A_{22}^{h2}} + \cdots + \varepsilon_1^{\beta_{c_h}+1}\overline{A_{22}^{hh}} \end{bmatrix} \tag{17}$$

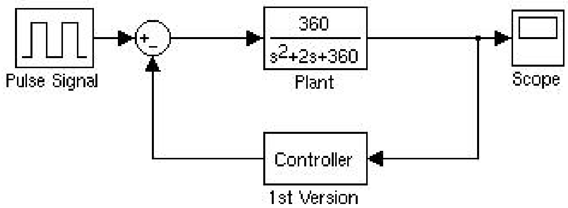

Fig. 4. An example of block diagram of 1st version control

is conducted with a commercial package called MATLAB SIMULINK.

A. 1st version

Recalling the controller described in section IIIA., the plant with the transfer function $\frac{360}{s^2+2s+360}$ is under the disturbance of a pulse signal shown in Fig.4. The initial coefficients of $\{\varepsilon_0, \bar{\rho}, \bar{\varepsilon}_2\}$ are $\{1,1,1\}$. After applying 1st version controller, those parameters are changed to $\{2.286, 3.104, 4.426\}$. Fig.5 gives the bode diagram comparison of the results for original parameters control and optimally tuned parameters control. From the graph, it is evident that the 1st version controller can fulfil the optimal control task as described in this paper.

B. 2nd version

In order to verify the feasibility of the extended version controller under more complicated conditions, one more plant is added into the above tested example Fig.4, which is also randomly chosen as shown in Fig. 6. As mentioned in Remark 3.2, a set of butter-worth filter coefficients $\{a_1^*, a_2^*, \cdots, a_h^*\}$ can be used in the controller design. The controller states can be approximated in certain bandwidth by

$$[\dot{\Psi}] = \begin{bmatrix} -a_1^* & -a_2^* & \cdots & -a_{h-1}^* & -a_h^* \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} [\Psi] + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} e$$

$$[\tilde{\Psi}] \approx \begin{bmatrix} -a_1^* & -a_2^* & \cdots & -a_{h-1}^* & -a_h^* \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} [\Psi] + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} e \quad (20)$$

Thus, for controller (19), the only optimization task is to tune $\{b_1, b_2, \cdots, b_h\}$. In this example, a 5-order extended controller is used. The original set of $\{b_1, b_2, b_3, b_4, b_5\}$ is $\{0,0,0,0,0\}$. After applying the 2nd version optimal control, the tuned set is $\{-0.875, 1.3324, 1.6018, -1.1951, -2.002, 2.02\}$. From Fig.7, the improved frequency response is obtained, which validates such an extended switching optimal controller.

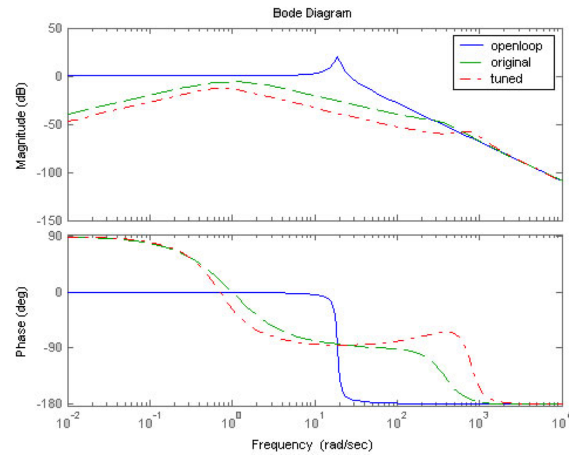

Fig. 5. 1st version control: Bode diagram comparison of original and tuned parameters

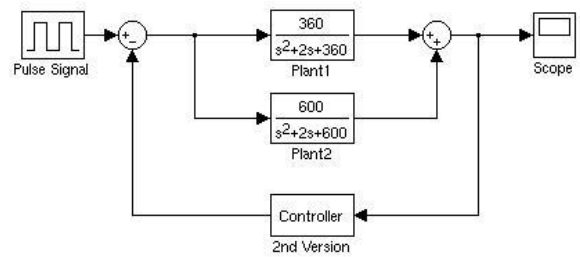

Fig. 6. An example of block diagram of 2nd version control

V. EXPERIMENTAL APPROACH: FLOW INDUCED VIBRATION

In engineering applications, flow-induced vibration has aroused much public attention since last century, especially about its control methodology. When flow is passing through immersed body, the vortex shedding is formed in the wake and excites the body to vibrate. The motion of the body will further affect vortex, and in turn the force acting on the body itself. The experiment is carried out in a closed circuit wind tunnel with a square test section of $0.6 \times 0.6m^2$ and a length of $2.4m$. One square cylinder of height $h = 15.2mm$, which is supported on springs at both ends, is placed in the downstream of wind tunnel and allowed to vibrate vertically as shown in Fig.8. The free-stream velocity $U_\infty = 2.00ms^{-1}$ is considered as unknown bounded disturbance, which excites cylinder to vibrate. The upper side of the cylinder is made of a thin plastic plate, under which there are three curved piezoelectric ceramic actuators used to give out control input signal. All the setup is the same as [11]. As mentioned in [11], a set of PID parameters can be found to achieve high closed-loop control performance, which indicates satisfaction of the stable requirement. Thus, it is possible to apply the optimal controllers described in this paper.

At this moment, only 1st version controller is tested.

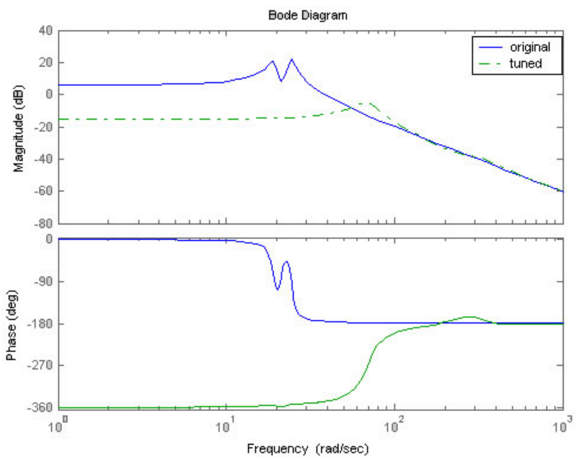

Fig. 7. 2nd version control: Bode diagram comparison of original and tuned parameters

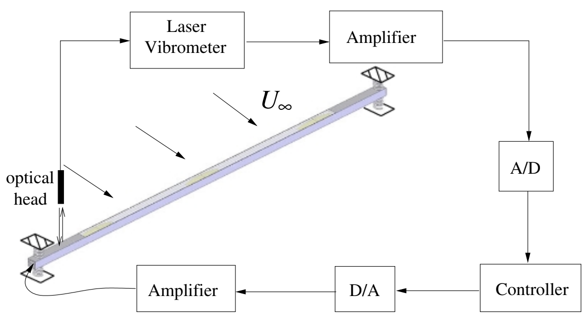

Fig. 8. Experiment setup for control of flow induced vibration

The original PID parameters is randomly set to $\{1, 2, 3\}$. The estimation of J uses the vibration signal from the laser-vibrometer as shown in Fig.8. The block interval is set to 400000 sampling iterations in SIMULINK engine. After applying 1st version control action, the final tuned parameters are $\{1.478, 1.480, 2.887\}$. The comparison of power spectrum density (PSD) plots for original and tuned parameters is shown in Fig.9. It is noticed that the optimal controller really gives improved frequency response at low frequency range, while for high frequency range, the fact that improvement cannot be achieved may mainly result from machine and flow turbulent noise.

VI. CONCLUSION

In this paper, we have discussed about the self-tuning sub-optimal control at two stages, which are on the basis of singular perturbation idea for switching control and the simultaneous perturbation method for gradient searching of objective function's optimizing problem. As it is assumed, the only requirements of the plant are open-loop stable, and with bounded disturbance which needs not to be measured. Then, both the 3-term searching and the extended switching-searching control can give out optimized performance compared with original non-switching and non-searching control. Especially, for the extended version, it simplifies the controller design to employ a set of butter-worth filter coefficients and can get used to more complicated cases. Also, the control experiment of fluid induced vibration has been studied to validate the practical feasibility of such promoted optimal controllers.

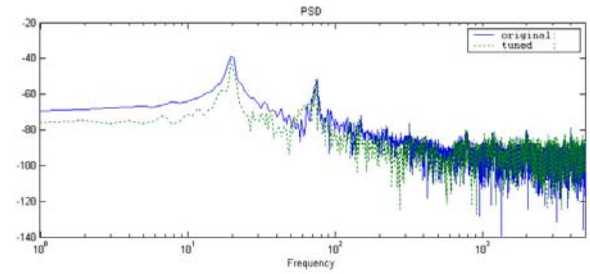

Fig. 9. Power spectrum density of vibration signals for original and tuned parameters control

VII. ACKNOWLEDGEMENTS

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