2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Observer Control in a Tracking Problem via Model Predictive Control

Vincent Winstead and Ilya V. Kolmanovsky

Abstract— **The paper describes a Model Predictive Control (MPC) approach for optimally (in a receding horizon sense) controlling the trajectory of a mobile tracker to minimize errors in estimating the state of another, moving object. With the use of the MPC approach, pointwise-in-time state and control constraints can be enforced for the mobile tracker, and its control system can react dynamically to changing operating conditions (such as new obstacles appearing or disappearing). After a discussion of this problem in a general setting, the paper focuses on a case study of the mobile tracker and moving object on a plane. The mobile tracker can measure the distance to the moving object and this measurement is affected by noise. The effect of the noise may increase if the line of sight from the mobile tracker to the moving object deviates from the orientation of the moving tracker or it passes through interference zones and, in addition, the mobile tracker has to stay clear of the obstacles.**

I. INTRODUCTION

Model Predictive Control (MPC) has found a widespread use in the chemical process industry and its benefits have been explored for other applications, including automotive systems, see e.g., [1] and [3]. In the MPC approach, at each time instant the control input trajectory is optimized over a finite prediction horizon into the future, and the first element of the resulting optimal control sequence is applied to control the system. This on-line optimization approach is also referred to as receding horizon or moving horizon optimal control. For an overview of MPC techniques, see references [6] and [7]. The receding horizon approach has also been used in the state estimation problems [8]; in these problems past disturbance or uncertainty sequence, together with an estimate of the initial state, are optimized over a finite horizon to best match the observed output time history.

In the present paper we discuss a class of problems where the MPC is applied to control the trajectory of a mobile tracker with the purpose of optimally estimating the state of another, moving object. While these problems have been previously studied in the optimal control context (see e.g., [4]), with the MPC approach pointwise-in-time state and control constraints can be handled and the control system of the mobile tracker can respond to and accommodate changing operating environment (such as appearance of new obstacles).

A. Fixed Tracker

The tracking problem with a fixed tracker is well known in the literature and has practical utility in many industries, most notably in aviation where the use of radar systems is prevalent. Here the aircraft travels through the waves of energy emitted by the radar system and reflects a portion of that energy back to a receiver. Ground based radar systems have a finite field of view (FOV) and if the aircraft exits the FOV of the radar, it becomes invisible to the radar system.

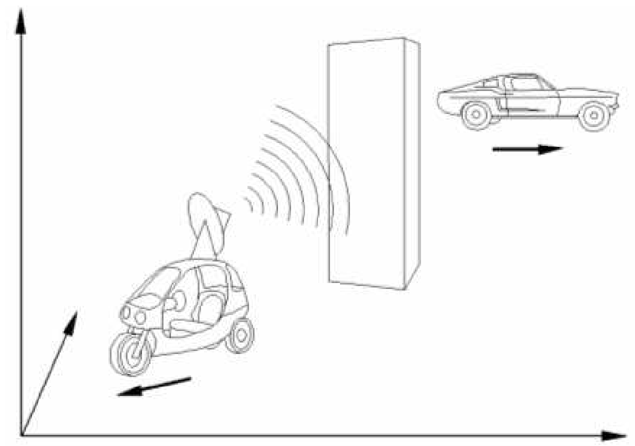

Fig. 1. Mobile tracker and moving object on a plane. The mobile tracker on the bottom left near the origin is moving left and the moving object is moving right with an obstacle separating the two.

There are also parallel applications in the automotive industry which may involve the use of optical, radar or other technology based ranging measurements for the purpose of automated vehicle maneuvering. For example, in adaptive cruise control and automated highway systems applications these type of measurements by the follower vehicle may be used to maintain safe vehicle spacing from the leading vehicle. As long as two vehicles are aligned (in the same lane), this provides a capable method for measuring vehicle spacing. However, for situations in which the vehicles are not well aligned and lane excursions occur, the fixed measurement system with limited FOV may be less effective at measuring vehicle spacing.

B. Mobile Tracker

A mobile tracker has immediate advantages over a fixed tracker including the ability to re-orient itself such that limitations in FOV are reduced. See Figure 1[1]. In the automotive applications, this implies the capability to modify the trajectory of the following vehicle to improve spacing

The authors are with Ford Research & Advanced Engineering, Ford Motor Company, Dearborn, Michigan, USA. E-mails: vwinstea@ford.com, ikolmano@ford.com.

[1]The authors wish to acknowledge John Posa of Ann Arbor, Michigan for his assistance in drawing the figure.

measurements between it and the leading vehicle. In the case of tracking over varying terrain and obstacles for ground or air based tracking, it is obvious that the use of a mobile tracker allows the capability for reducing limitations caused by a finite FOV when obstructions are present. For example, a controller may be designed to move the tracker vehicle to a location where an unobstructed line of sight is achieved.

Depending on the sensor technology involved (radar, optical/camera-based, infra-red, etc.) and operating environment (obstacles, clutter, etc.) the orientation and distance of the mobile tracker with respect to the moving object can also determine the *quality of observations*. This quality is related to the quantity of uncertainty in the measurements. The measurement quality degradation can be modeled as an increase in the measurement noise.

C. General Problem Formulation

Turning now to a more concrete description of the problem in a general setting, suppose the dynamics of the moving observer are described by the following discrete-time equations

$$x(t+1) = f(x(t), u(t)), \tag{1}$$

where $x(t)$ is the moving observer state at the time instant t and $u(t)$ is the control input applied to the mobile tracker at the time instant t. The moving object is described by the following discrete-time equations,

$$z(t+1) = g(z(t), v(t), \omega(t)), \tag{2}$$

where $z(t)$ is the state of the moving object at the time instant t, $v(t)$ is a known input to the moving object at the time instant t, and $\omega(t)$ is an unknown input (disturbance or uncertainty) to the moving object at the time instant t. The mobile tracker uses a known estimation algorithm and measurements defined by

$$y_m(t) = \bar{h}(x(t), z(t), \nu(t)), \tag{3}$$

where $\nu(t)$ is the measurement noise, to generate an estimate of the moving object state, $\hat{z}(t)$.

With the MPC approach, the basic idea is that at each time instant t a control sequence $u(t), u(t+1), \cdots, u(t+T)$, is found that minimizes a weighted sum of a measure of deviation of $\hat{z}(t+k)$ from $z(t+k)$, $k = 0, 1, \cdots, T$, and of the control effort involved. During the minimization, pointwise-in-time state and control constraints can be enforced. In particular, *obstacles* or *exclusion zones* that the mobile tracker is not allowed to enter may be represented by the following constraints,

$$\mathcal{R}_i = \{(x, z) : \ r_i(x, z) \le 0\}, \ i = 1, \cdots, N_o, \tag{4}$$

where N_o is the total number of exclusion zones. An exclusion zone may surround the moving object (e.g., to avoid detection or to force the mobile tracker to always "stay behind" or "stay hidden" relative to the moving object), hence constraint (4) may explicitly depend on $z(t)$. Although the control input cannot affect the trajectory of the moving object, if there are known exclusion zones that the latter is known not to enter they may be accounted for in the update laws for the estimate, $\hat{z}(t)$.

The problem formulation may involve one or more interference zones,

$$\mathcal{S}_i = \{(x, z) : \ s_i(x, z) \le 0\}, \ i = 1, \cdots, N_I, \tag{5}$$

where N_I is the total number of the interference zones. The presence of interference zones leads to deterioration of the quality of the measurement and increase in the measurement noise in subsets of the state space of the combined system obstructed by the interference zones. This effect of the interference zones is captured via an appropriate modification of (3),

$$y_m(t) = h(x(t), z(t), \nu(t)), \tag{6}$$

where

$$h(x, z, \nu) = \bar{h}\left(x, z, (1 + \sum_{i=1}^{N_I} q_i(\pi_x(x), \pi_z(z))) \cdot \nu\right).$$

Here the functions π_x and π_z are projections and q_i models the increase in the effect of the measurement noise due to the ith interference zone. For example, $\pi_x(x)$ and $\pi_z(z)$ may determine the positions of the mobile tracker and of the moving object, respectively, while q_i can be a portion of the total length of the line segment from $\pi_x(x)$ to $\pi_z(z)$. See Figure 2. The interference zones may be associated with the obstacles or exclusion zones for either the mobile tracker or the moving object or both. But there also may be interference zones that are not obstacles or exclusion zones. For example, a cloud may obstruct the aircraft visibility but the aircraft can fly into the cloud.

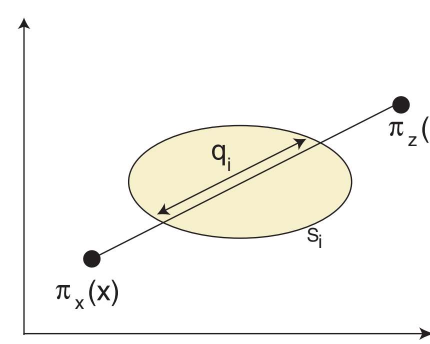

Fig. 2. Interference zone.

The problem may be handled either in deterministic setting if $\omega(t)$ and $\nu(t)$ are set-bounded inputs, or in stochastic setting if $\omega(t)$ and $\nu(t)$ are stochastic processes.

In the latter case, if the constraints in (4) depend on $z(t)$ they need to be replaced by

$$\text{Prob}\{r_i(x(t), z(t)) \leq 0\} > 1 - \epsilon, \quad i = 1, \cdots, N_o,$$

where $\epsilon > 0$ is a threshold, while the deviation of $\hat{z}(t)$ from $z(t)$ may be measured by an estimate of the error covariance. Given a control input sequence over the prediction horizon the error covariance can be estimated using the Extended Kalman Filter (EKF) propagation equations, and unlike in the deterministic case it is not necessary to bound the predicted system trajectories for all possible disturbance sequences over the prediction horizon. We employ the stochastic formulation and the EKF-based approach for predicting the error covariance in the subsequent case study.

D. Further Generalizations

The problem formulation can be generalized to the case of multiple mobile trackers and moving objects. The Model Predictive Control can be used to coordinate these moving trackers to best estimate the states of the moving objects.

Another important class of related problems is when $z(t)$ is an unknown constant parameter, i.e., $z(t+1) = z(t)$, and it affects the dynamics of the mobile tracker (1), i.e.,

$$x(t+1) = f(x(t), z(t), u(t)). \tag{7}$$

In this case, the objective is to determine the input $u(t)$ to minimize the errors in estimating z. This can be viewed as a problem in design of experiments [5] and it may be solved using Model Predictive Control (MPC).

Finally, as another generalization of the original problem, the mobile tracker may have a choice of different observation channels with different usage cost and level of noise. The MPC may be used to optimally control the channel selection.

II. A Two Object Dynamic Model

While the ideas are rather general, we further discuss them in a context of a concrete case study of two objects that move on a plane. Their dynamics are represented by the following equations:

$$\begin{aligned}
x_1(t+1) &= x_1(t) + u_1(t)\cos x_3(t), \\
x_2(t+1) &= x_2(t) + u_1(t)\sin x_3(t), \\
x_3(t+1) &= x_3(t) + u_2(t), \\
z_1(t+1) &= z_1(t) + z_2, \\
z_2(t+1) &= z_2(t) + \omega(t),
\end{aligned} \tag{8}$$

where $\{x_1(t), x_2(t)\} \in \mathbf{R}^2$ are planar coordinates of the mobile tracker. The angular orientation of the mobile tracker is $x_3(t)$. The moving object heads along the x_1-axis of the plane and its position along this axis is z_1. The velocity of the moving object is z_2 and the acceleration of the moving object is $\omega(t)$, which is a zero mean gaussian white noise,

$$\text{Prob}\{\omega(t)\} \sim \mathcal{N}(0, \sigma_\omega).$$

Thus the planar coordinates of the moving object are $(z_1(t), 0)$.

The equations (8) represent a discretization of the continuous time dynamics of the mobile tracker and moving object with the sample period of 1 sec. Note that in continuous-time the mobile tracker behaves as a kinematic nonholonomic system.

We assume that the following control constraints are imposed on the motion of the mobile tracker,

$$0 \leq u_1(t) \leq 5, \quad -\pi/4 \leq u_2(t) \leq \pi/4. \tag{9}$$

Thus the mobile tracker can only move forward with up to a maximum velocity of 5 m/sec and it can rotate in any direction by no more than $\pi/4$ rad/sec.

The measurement vector consists of mobile tracker states and of the distance between the mobile tracker and moving object. These measurements are contaminated by measurement noise so that

$$\begin{aligned}
y_m(t) &= h(x(t), z(t), \nu(t)) \\
&= \begin{bmatrix} x_1(t) + \nu_1(t) \\ x_2(t) + \nu_2(t) \\ x_3(t) + \nu_3(t) \\ \big(\sqrt{(z_1(t) - x_1(t))^2 + (x_2(t))^2} \\ +(1 + \phi(x(t), z(t)) \cdot \nu_4(t)\big) \end{bmatrix},
\end{aligned} \tag{10}$$

where ν_i, $i = 1, \cdots, 4$, represent stochastic gaussian zero mean white noise terms with

$$\text{Prob}\{\nu_i(t)\} \sim \mathcal{N}(0, \sigma_{\nu_i}),$$

and ϕ is a nonlinear function which is a square of the angle difference between x_3 and the angle of the line of sight from the mobile tracker to the moving object.

III. Application of Extended Kalman Filter

Since the system dynamics are not deterministic, an observer must be utilized which takes into account the statistics of the uncertainties while providing a "deterministic" result. A classical observer design that has existed since the 1960s is the Kalman Filter. This optimal linear filter is constructed by estimating system states using a recursive function of the error covariance [2]. This produces an observer which can be used to compute a state estimate. The Kalman Filter, though, produces a linear filter and hence assumes linear system dynamics. In our case, the system dynamics and the measurement function are nonlinear. Thus they are linearized to enable the application of the Extended Kalman Filter (EKF) approach [2]. Formally, the EKF does not produce an optimal linear filter in the same sense that the Kalman Filter generates an optimal linear filter, but one can use the EKF to generate a filter having similar characteristics and producing filter gains and recursive equations for the covariance which allow useful application.

Denoting the combined state of the mobile tracker and moving object by

$$X = \begin{bmatrix} x_1 & x_2 & x_3 & z_1 & z_2 \end{bmatrix}^{\mathrm{T}},$$

we can write (8) in the form

$$X(t+1) = F(X(t), u(t)) + B_\omega \omega(t), \tag{11}$$

where

$$B_\omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}.$$

Let

$$G(X) = \begin{bmatrix} \sigma_{\nu_1} & 0 & 0 & 0 \\ 0 & \sigma_{\nu_2} & 0 & 0 \\ 0 & 0 & \sigma_{\nu_3} & 0 \\ 0 & 0 & 0 & (1+\phi(X))\sigma_{\nu_4} \end{bmatrix}, \tag{12}$$

$$C(X) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \sqrt{(z_1 - x_1)^2 + (x_2)^2}, \end{bmatrix}. \tag{13}$$

Then,

$$y_m(t) = C(X(t)) + G(X(t)) \cdot \begin{bmatrix} \nu_1(t) \\ \nu_2(t) \\ \nu_3(t) \\ \nu_4(t) \end{bmatrix}.$$

Suppose now $\hat{X}(t)$ is an estimate of the combined state at the time instant t, and $\{u(t+k),\ k = 0, \cdots, T\}$, is a control sequence specified over the prediction horizon. Then, (11) may be used to predict the system state as

$$\begin{gathered} \hat{X}^-(t+k+1) = F(\hat{X}^-(t+k), u(t+k)), \\ \hat{X}^-(t) = \hat{X}(t). \end{gathered} \tag{14}$$

The *a posteriori* error covariance matrix, $P^+(t+k)$, can be updated over the prediction horizon according to the EKF equations:

$$\begin{aligned} &P^-(t+k+1) = A(t+k)P^+(t+k)A(t+k)^{\mathrm{T}} \\ &+B_\omega B_\omega^{\mathrm{T}} \cdot \sigma_\omega, \\ &K(t+k) = P^-(t+k)H^{\mathrm{T}}(t+k) \cdot \\ &\cdot \Big\{ H(t+k)P^-(t+k)H^{\mathrm{T}}(t+k) + G(\hat{X}^-(t+k)) \cdot \\ &\cdot G^{\mathrm{T}}(\hat{X}^-(t+k)) \Big\}^{-1} \\ &P^+(t+k) = \big(I - K(t+k)H(t+k)\big)P^-(t+k), \end{aligned} \tag{15}$$

where $P^-(t+k)$ denotes the *a priori* covariance matrix, $A(t+k)$ is the linearized dynamics system matrix, $H(t+k)$ is the linearized output matrix, i.e,

$$A(t+k) = \frac{\partial F(X,u)}{\partial X}|_{X=\hat{X}^-(t+k), u=u(t+k)},$$

and

$$H(t+k) = \frac{\partial C}{\partial X}|_{X=\hat{X}^-(t+k), u=u(t+k)}.$$

The Model Predictive Control (MPC) approach can now be realized as follows. At each time instant, t, an optimal control sequence $\{u(t+k),\ k = 0, \cdots, T\}$ is determined based on minimizing a cost function which penalizes the *a posteriori* error covariance matrix, $P^+(t_k)$, and the control effort involved (see next section). Then, $u(t)$, is applied to control the system between the time instants t and $t+1$. The measurement $y_m(t+1)$, once obtained during the on-line operation, is applied to condition $\hat{X}^-(t+1)$ and arrive at $\hat{X}(t+1)$, i.e.,

$$\hat{X}(t+1) = \hat{X}^-(t+1) + K(t+1)(y_m(t+1) - C(\hat{X}^-(t+1))). \tag{16}$$

The process is repeated at the time instant $t+1$ and so on.

IV. The Optimization Problem

We consider the minimization of a cost functional that penalizes a measure of the *predicted* estimation error and of the control effort involved:

$$J(t) = \sum_{k=0}^{T} \left\{ u^T(t+k)Ru(t+k) + \alpha^T P^+(t+k)\alpha \right\}, \tag{17}$$

where

$$\begin{aligned} R &= \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}, \\ \alpha &= \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \end{bmatrix}^{\mathrm{T}}. \end{aligned}$$

The cost (17) is minimized at each time instant t subject to constraints (8), (9), (14), (15) and subject to additional state constraints representing various obstacles.

For our case study we assumed that the knowledge of moving object position was more important than knowledge of its velocity. This led us to select α as

$$\alpha = [0 \ \ 0 \ \ 0 \ \ 1 \ \ 0]^{\mathrm{T}}.$$

V. Simulation Study

The simulation study is based on the following noise and disturbance statistics: $\sigma_\omega = \sigma_{\nu_1} = \sigma_{\nu_2} = \sigma_{\nu_3} = \sigma_{\nu_4} = 0.1$. The control weights r_1 and r_2 are first set to 10^{-8} in order to study the *cheap control* case. The prediction horizon T is set to 5, since longer horizons do not appear to produce substantially different behavior in the trajectories. The optimization problem formulated in Section IV is solved numerically at each discrete-time instant.

The moving object initial state is $z_1(0) = 30$, $z_2(0) = -3$. The mobile tracker initial state is $x_1(0) = 20$, $x_2(0) = 20$ and $x_3(0) = \pi/2$. The initial estimate of the combined state of the system is

$$\hat{X}(0) = [20 \;\; 20 \;\; \pi/2 \;\; 22 \;\; 0]^{\mathrm{T}},$$

and we set $P^{+}(0) = I$, the identity matrix. Thus the initial estimates of the moving object position and velocity are quite a bit off.

A pointwise-in-time constraint of the form (4) has been added on the mobile tracker position on the plane,

$$\frac{x_1(t)}{2} - x_2(t) \leq 0. \tag{18}$$

We also introduce an interference zone (5) which is a disk on the (x_1, x_2) plane centered at the origin with the radius equal to 10. It is assumed that if the line of sight from the mobile tracker to the moving object passes through this interference zone, the effect of the measurement noise on the distance measurement is increased. Specifically, the function $\phi(X)$ is (12) is replaced by the function $\bar{\phi}(X) = \phi(X) + \psi(X)$, where $\psi(X)$ is the length of the segment of the line of sight from the mobile tracker to the moving object within the interference zone. Figures 3-5 show, respectively, the planar trajectories of the mobile tracker $(x_1(t), x_2(t))$ and of the moving object $(z_1(t), 0)$, the time history of actual $(z_1(t), z_2(t))$ and estimated $(\hat{z}_1(t), \hat{z}_2(t))$ states of the moving object, and the time history of the mobile tracker orientation angle $(x_3(t))$. The mobile tracker stays clear of violating (18) and optimally (in a receding horizon sense) handles the noise effects due to the interference zone and other sources.

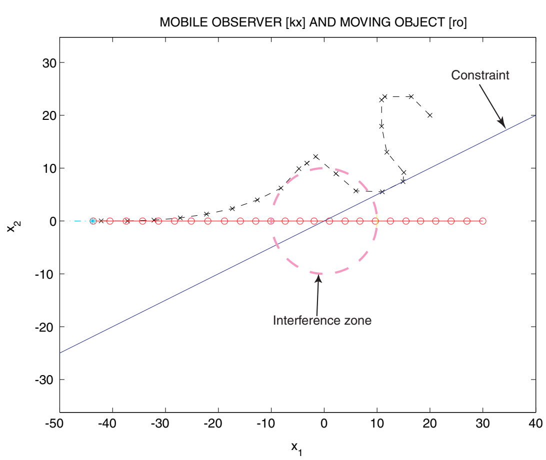

Fig. 3. Planar trajectories of the moving object (solid, "o") and mobile tracker (dashed, "x") for the case with an obstacle and an interference zone.

Note that the trajectory of the mobile tracker in Figure 3 enters into the interference zone. If the interference zone is also an obstacle so that the mobile tracker is not allowed to enter it, an additional pointwise-in-time constraint can be imposed to enforce this requirement. See Figure 6.

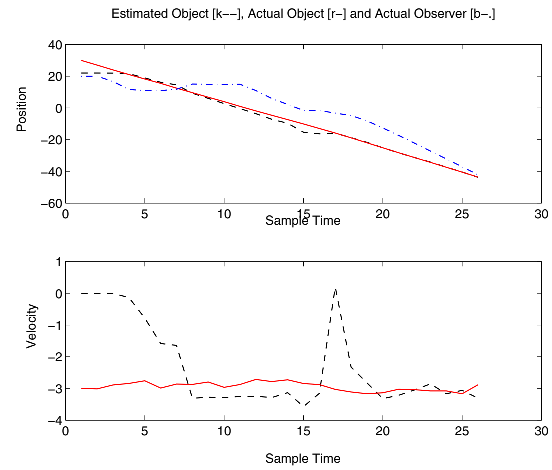

Fig. 4. Actual (solid) and estimated (dashed) position and velocity of the moving object for the case with an obstacle and an interference zone, and actual moving observer position (dash-dot).

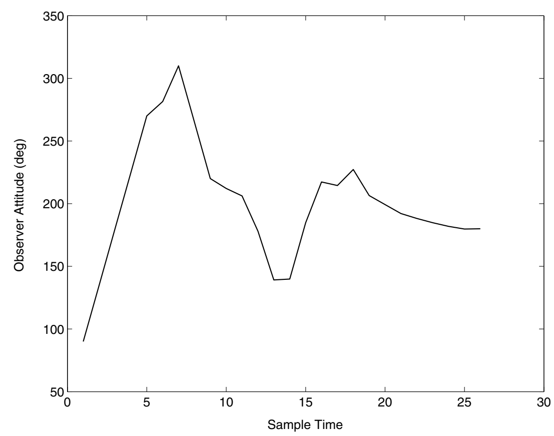

Fig. 5. Orientation of the mobile tracker for the case with an obstacle and an interference zone.

We finally illustrate the effect of the increase in the control cost to $r_1 = r_2 = 10^{-3}$ for the case when the interference zone is also an obstacle. In this case, the mobile tracker performs an initial maneuver but then ceases to follow the moving object while continuing to monitor it via the adjustment of the orientation angle, see Figures 7-9.

VI. Concluding Remarks

The paper described how a Model Predictive Control (MPC) approach can be applied to optimally (in a receding horizon sense) control the dynamics of a mobile tracker to best estimate the state of another, moving object. The use of the MPC allows input and state constraints to be enforced and the control system of the mobile tracker can respond and reconfigure in the event of changing operating conditions (such as new obstacles or interference zones appearing or disappearing). The basic problem and several of its generalizations have been discussed. The approach

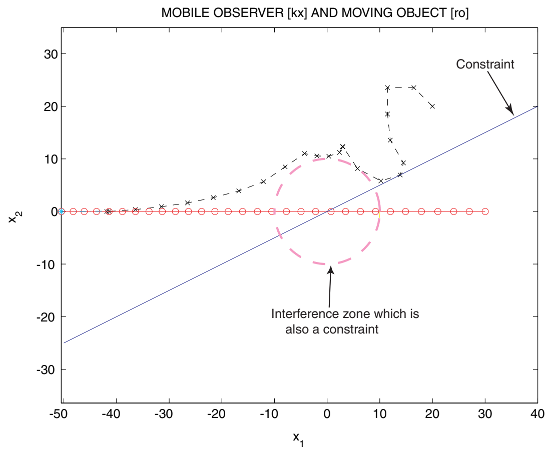

Fig. 6. Planar trajectories of the moving object (solid, "o") and mobile tracker (dashed, "x") for the case when the interference zone is also an obstacle.

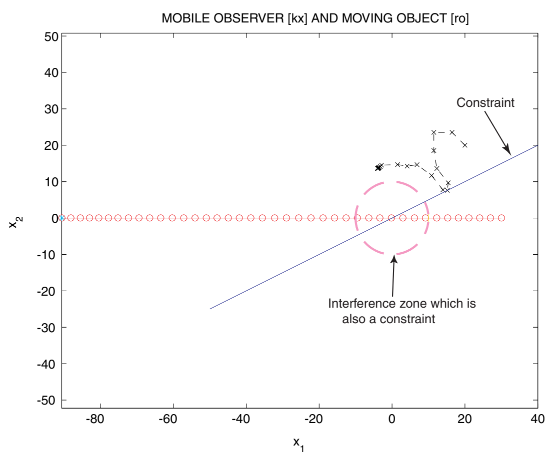

Fig. 7. Planar trajectories of the moving object (solid, "o") and mobile tracker (dashed, "x") for the case when the interference zone is also an obstacle and with higher control cost.

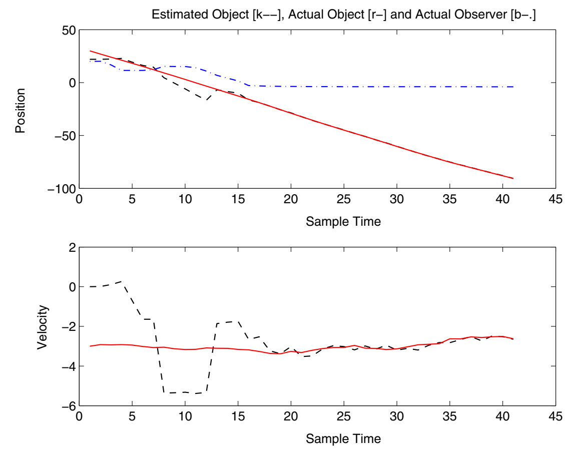

Fig. 8. Actual (solid) and estimated (dashed) position and velocity of the moving object for the case when the interference zone is also an obstacle with higher control cost, and the actual position of the moving observer (dash-dot).

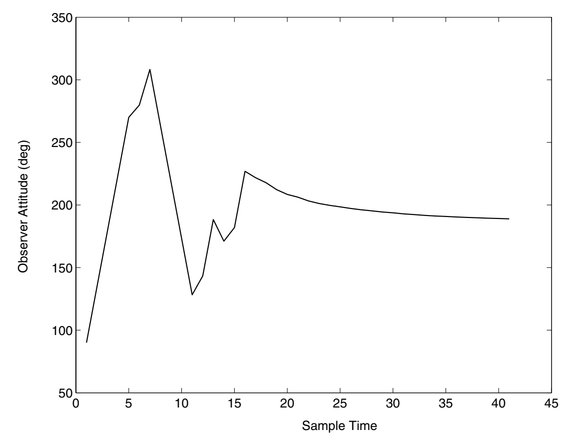

Fig. 9. Orientation of the mobile tracker for the case when the interference zone is also an obstacle and with higher control cost.

was illustrated via a simulation case study of two objects moving on a plane.

Continuing research is directed at understanding the theoretical and numerical properties of the approach, in particular, the asymptotic properties of the mobile tracker trajectory and of the state estimation error, and the structure and efficient ways to solve the optimization problem at each time instant.

Other applications of the approach to multiple air or sea-based moving objects, operating in an environment with numerous obstacles and interference zones, are also of interest and will be studied in greater detail in the future. The potential of the approach for experiment design in systems identification problems will be explored as well.

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