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# Adaptive Robust Precision Motion Control of a Flexible System with Unmatched Model Uncertainties

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***Abstract*— Flexible systems which function under the influence of friction are very common in many applications such as flexible robot arms. The precision motion control of such systems is a difficult task because of the modelling difficulties involved in modelling friction and various flexible modes of the system. This paper presents the design of a nonlinear adaptive robust controller (ARC) to the precision motion control of a floating oscillator system under the influence of friction. This system consists of two masses moving on a track under the influence of friction and coupled by a spring. The floating oscillator acting under the influence of friction can be used to represent many applications such as hard disk drives and flexible robots where friction is present and the end effector position is to be controlled.**

## I. INTRODUCTION

In many applications such as disk drive heads, precision positioning systems, space structures and flexible robotic manipulators high performance control algorithms are required in order to obtain the high level of performance required in such applications. In the design of controllers for such applications, modelling of the physical system forms a major part of the challenge faced by the control system designer. These modelling problems come from the friction acting on the system and the flexible modes of the system under consideration.

Friction is the most common hard nonlinearity that occurs in most mechanical systems. It is often assumed in the controller design process that the system maintains positive velocity, which enables one to assume that the system is stick-slip free and to design controllers using optimal control techniques on the linearized model of the system [11]. Other designs have been proposed in the literature to account for the presence of friction, these include adaptive methods of feedforward compensation [9], impulsive control [15], and compensating for friction using the linear programming approach [10]. Robust control design methods like $H_\infty$ and $\mu$-synthesis have also been applied to such systems [12], [13]. A robust controller design for flexible robot arms with unknown parameters has also been studied in [14].

In this paper the control of the benchmark floating-oscillator [1] system with friction acting on both masses is considered. One of the masses, called the forced mass is acted upon by a DC motor and the motion of the unforced mass is of interest. This system is representative of many applications such as hard disk drives and flexible robots where frictions is present at both the actuator and the end-effector.

In this paper, a nonlinear model based adaptive robust controller is designed in which the robust controller is designed to attenuate the effect of the various model uncertainties. Also, on-line parameter adaptation is employed to improve the system performance. The ARC design presented in this paper is able to attenuate the effect of unmatched uncertainties, i.e., model uncertainties present in dynamic equations which are not directly related to the control input. Also, the design guarantees a prescribed level of transient performance and final steady state tracking accuracy. The system is able to track a trajectory with a prescribed tracking accuracy as opposed to the simple regulation problem. Experimental results on a floating oscillator are presented to validate the controller design.

This paper is organized as follows: In section II, the problem is formulated highlighting the difficulty of design of control algorithms for the floating oscillator system. In section III, the model based nonlinear ARC design is presented. Section IV presents some performance results and in section V some conclusions are given.

## II. PROBLEM FORMULATION

A floating oscillator under the influence of friction at both the masses is shown in Fig. 1. The mass $m_2$, also called the forced mass is acted upon by a DC motor which provides the actuation effort and the position of mass $m_1$ called the free mass is to be controlled.

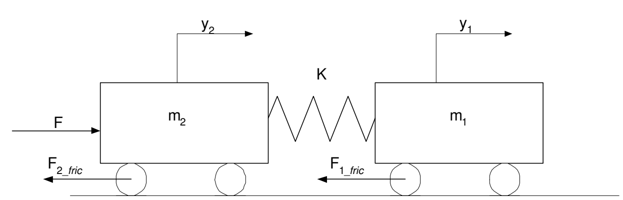


Fig. 1. Floating Oscillator with friction at the mass and cart.

The dynamics of the system are given in (1).

$$m_1\ddot{y}_1 = k(y_2 - y_1) - b_1\dot{y}_1 - F_{f1}sign(\dot{y}_1) - \Delta_1 \quad (1)$$
$$m_2\ddot{y}_2 = F - k(y_2 - y_1) - b_2\dot{y}_2 - F_{f2}sign(\dot{y}_2) - \Delta_2$$

where, $m_1$ and $m_2$ are the masses of the free and forced masses respectively. $y_1$ and $y_2$ are the displacements of

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the free and forced masses. $k$ is the spring constant of the spring connecting the two masses and $F$ is the force that the motor exerts on the cart to move it. $F_{f1}$, $F_{f2}$ represent the coulomb friction constants that act on the two masses respectively. $b_1$, $b_2$ are the viscous/damping coefficients and $\Delta_1$ and $\Delta_2$ are the unmodeled dynamics and disturbances such as unmodeled friction and higher modes which have been neglected.

Neglecting the motor dynamics, the force $F$, that the motor exerts on the forced mass $m_1$ is given by (2):

$$F=\frac{k_t r_0}{R_a r_m}(u-K_e\frac{r_0}{r_m}\dot{x}) \tag{2}$$

where, $u$ is the input voltage to the motor and the control input to the floating oscillator system, $k_t$ is the torque constant of the motor, $r_0$ is the radius of the encoder gear on the forced mass, $r_m$ is the radius of the motor gear, $R_a$ is the resistance of the motor armature and $K_e$ is the back emf constant of the motor.

Define a set of state variables as $x=[x_1,x_2,x_3,x_4]=[y_1,\dot{y}_1,y_2,\dot{y}_2]$, then the entire system of equations (1)-(2) with the control input $u$ can be written as:

$$\begin{aligned}\dot{x}_1&=x_2\\ m_1\dot{x}_2&=kx_3-kx_1-b_1x_2-F_{f1}sign(x_2)-\Delta_1\\ \dot{x}_3&=x_4\\ m_2\dot{x}_4&=a_1u+kx_1-kx_3-a_2x_4-b_2x_4-F_{f2}sign(x_4)-\Delta_2\end{aligned} \tag{3}$$

where, $a_1=\frac{k_t r_0}{R_a r_m}$ and $a_2=\frac{k_t k_e r_0^2}{R_a r_m^2}$ are known constants.

*Given the desired motion trajectory $y_d(t)$, the objective is to synthesize a control input u, such that the trajectory of the free mass $y_1(t)$ tracks $y_d(t)$ as closely as possible in spite of the presence of various model uncertainties.*

### A. DESIGN MODEL AND ISSUES TO BE ADDRESSED

The floating oscillator system described in (3) is subjected to parametric uncertainties due to the unknown parameters $F_{f1}$, $F_{f2}$, $b_1$, $b_2$ as well as unknown nonlinearities $\Delta_1$ and $\Delta_2$. Practically, $\Delta_1$ and $\Delta_2$ may be composed of two parts, a nominal part denoted by $\Delta_{1n}$ and $\Delta_{2n}$ which is constant or slowly changing, and a fast changing part. The nominal parts of the model uncertainties can be combined into parametric uncertainties and dealt with by parameter adaptation, and the fast changing parts have to be attenuated by the robust feedback.

In order to use parameter adaptation to reduce parametric uncertainties to improve the performance of the control algorithm, it is necessary to linearly parameterize the state-space equation (3) in terms of a set of unknown parameters. Defining the set of unknown parameters as $\theta=[\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6]^T$ where $\theta_1=b_1$, $\theta_2=F_{f1},\theta_3=\Delta_{1n},\theta_4=b_2,\theta_5=F_{f2}$, and $\theta_6=\Delta_{2n}$. Also, the nonlinearity represented by $sign(x_2)$ and $sign(x_4)$ are hard discontinuous nonlinearities, and in the design of a backstepping algorithm, we approximate this nonlinearity using a smooth differentiable function $S_f(x_2)$ and $S_f(x_4)$. Using this approximation the state-space equation (3) can be linearly parameterized in terms of $\theta$ as

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=\frac{k}{m_1}(x_3-x_1)-\frac{\theta_1}{m_1}x_2-\frac{\theta_2}{m_1}S_f(x_2)-\frac{1}{m_1}\theta_3-\tilde{\Delta}_1\\ \dot{x}_3&=x_4\\ \dot{x}_4&=\frac{a_1}{m_2}u+\frac{k}{m_2}(x_1-x_3)-\frac{a_2}{m_2}x_4-\frac{\theta_4}{m_2}x_4\\ &\quad-\frac{\theta_5}{m_2}S_f(x_4)-\frac{1}{m_2}\theta_6-\tilde{\Delta}_2\end{aligned} \tag{4}$$

where,

$$\begin{aligned}m_1\tilde{\Delta}_1&=(\Delta_1-\Delta_{1n})-(sign(x_2)-S_f(x_2))F_{f1}\\ m_2\tilde{\Delta}_2&=(\Delta_2-\Delta_{2n})-(sign(x_4)-S_f(x_4))F_{f2}\end{aligned}$$

Since the extents of the parametric uncertainties and uncertain nonlinearities are normally known, the following practical assumptions are made

*Assumption 1:* Parametric uncertainties and uncertain nonlinearities satisfy

$$\begin{aligned}&\theta\in\Omega_\theta=\{\theta:\theta_{min}\le\theta\le\theta_{max}\}\\ &|\tilde{\Delta}_i|\le\delta_i(x,t)\end{aligned}$$

where, $\theta_{min}=[\theta_{1min},\cdots,\theta_{6min}]^T$, $\theta_{max}=[\theta_{1max},\cdots,\theta_{6max}]^T$, the operator $\le$ when acting on vectors means entry by entry, and $\delta_i(x,t)$ are known.

As can be observed from the dynamics in equation (4), the major difficulties in controlling (4) are:

1. The model uncertainties are mismatched, i.e., both the parametric uncertainties and the uncertain nonlinearities appear in the dynamic equations which are not directly related to the control input $u$.
2. The system has parametric uncertainties represented by the lack of knowledge of the damping coefficients $b_1$, $b_2$ and the coulomb friction constants $F_{f1}$, $F_{f2}$.
3. As opposed to most designs available in literature, the frictions acting on the both masses are considered in the design of the controller.
4. The system also has a large extent of lumped modeling error (represented by $\tilde{\Delta}_i$) including external disturbance, unmodeled friction force, etc.

To address the above challenges, the following strategies are adopted in the design of the control algorithm:

1. The controller design will be based on the nonlinear physical model of the system to directly deal with the nonlinear system dynamics and flexible modes of the system.
2. The adaptive robust control (ARC) technique [2], [3] will be used to handle the parametric uncertainties and the uncertain nonlinearities effectively to obtain high performance. Robust feedback is employed to attenuate the effect of the model uncertainties and parameter

adaptation is used to improve the performance of the control design.

3. Backstepping design [6] via ARC Lyapunov functions [4], [3] is used to overcome the design difficulties caused by unmatched model uncertainties.

### *B. Notations and Discontinuous Projection Mapping*

Let $\hat{\theta}$ denote the estimate of $\theta$ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta}=\hat{\theta}-\theta$). A discontinuous projection mapping, $Proj_{\hat{\theta}}(\bullet)=[Proj_{\hat{\theta}_1}(\bullet_1),\ldots,Proj_{\hat{\theta}_n}(\bullet_n)]^T$, can be defined [7],[8] as :

$$Proj_{\hat{\theta}_i}(\bullet_i)=\begin{cases} 0 & \text{if } \hat{\theta}_i=\theta_{imax} \text{ and } \bullet_i>0 \\ 0 & \text{if } \hat{\theta}_i=\theta_{imin} \text{ and } \bullet_i<0 \\ \bullet_i & \text{otherwise} \end{cases} \quad (5)$$

By using the adaptation law given by

$$\dot{\hat{\theta}}=Proj_{\hat{\theta}}(\Gamma\tau) \quad (6)$$

where $\Gamma>0$ is a diagonal matrix and $\tau$ is an adaptation function to be synthesized later. It can be shown [2], [7], [8] that for any adaptation function $\tau$, the projection mapping used in (6) guarantees

P1. $\hat{\theta}\in\Omega_\theta=\{\hat{\theta}:\theta_{min}\le\theta\le\theta_{max}\}$

P2. $\tilde{\theta}^T(\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\tau)-\tau)\le 0, \quad \forall\tau$

## III. ARC Controller Design

The design is parallel to the backstepping design procedure via ARC Lyapunov functions in [3], [4]:

### *A. Step 1:*

Noting that the first equation of (4) does not have any uncertainties, an ARC Lyapunov function can be constructed. Define a switching-function-like quantity as

$$z_2=\dot{z}_1+k_1z_1=x_2-\alpha_1, \quad \alpha_1=\dot{y}_d-k_1z_1 \quad (7)$$

where $z_1=x_1-y_d$ is the trajectory tracking error and $k_1$ is a positive feedback gain. If $z_2$ converges to a small value or zero, the output tracking error $z_1$ will converge to a small value since $G_s(s)=\frac{z_1(s)}{z_2(s)}=\frac{1}{s+k_1}$ is a stable transfer function. So the rest of the design is to make $z_2$ as small as possible.

Differentiating (7) and noting (4)

$$\begin{aligned}\dot{z}_2&=\dot{x}_2-\dot{\alpha}_1 \\ &=\frac{k}{m_1}(x_3-x_1)-\frac{x_2}{m_1}\theta_1-\frac{S_f(x_2)}{m_1}\theta_2-\frac{1}{m_1}\theta_3-\tilde{\Delta}_1-\dot{\alpha}_1\end{aligned} \quad (8)$$

where $\dot{\alpha}_1=\ddot{y}_d-k_1\dot{z}_1$ is calculable. In (8), if we treat $x_3$ as the input to (8), we can synthesize a virtual control law $\alpha_2$ for $x_3$ to make $z_2$ as small as possible. Since (8) has both parametric uncertainties $\theta_1$, $\theta_2$ and $\theta_3$ and uncertain nonlinearity $\tilde{\Delta}_1$, the ARC technique in [3] will be used to accomplish this objective. The control function $\alpha_2$ consists of two parts given by

$$\begin{aligned}&\alpha_2(x_1,x_2,\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_3,t)=\alpha_{2a}+\alpha_{2s} \\ &\alpha_{2a}=x_1+\frac{1}{k}(x_2\hat{\theta}_1+S_f(x_2)\hat{\theta}_2+\hat{\theta}_3)+\frac{m_1}{k}\dot{\alpha}_1-\frac{m_1}{k}z_1\end{aligned} \quad (9)$$

in which $\alpha_{2a}$ functions as the adaptive control law for improved model compensation using on-line parameter adaptation given by (6), and $\alpha_{2s}$ is a robust control law to be synthesized later. If $x_3$ were the actual control input, then $\tau$ in (6) would be

$$\tau_2=w_2\phi_2z_2, \quad \phi_2=[\frac{-x_2}{m_1},\frac{-S_f(x_2)}{m_1},\frac{-1}{m_1},0,0,0]^T \quad (10)$$

where $w_2>0$ is a constant weighting factor.

Due to the use of discontinuous projection (5), the adaptation law (6) is discontinuous and cannot be used in the control design at each step contrary to the tuning function based backstepping adaptive control [6]; backstepping design needs the control law to be smooth in order to obtain its partial derivatives. To compensate for this loss of information, the robust control law is strengthened. So the robust control function $\alpha_{2s}$ consists of two terms given by

$$\begin{aligned}&\alpha_{2s}=\alpha_{2s1}+\alpha_{2s2} \\ &\alpha_{2s1}=\frac{-m_1}{k}(k_{2s1})z_2, \quad k_{2s1}\ge\|C_{\phi_2}\Gamma\phi_2\|^2+k_2\end{aligned} \quad (11)$$

where $k_{2s1}$ is a positive nonlinear control gain function and $\alpha_{2s2}$ is a robust control function synthesized as follows. Let $z_3=x_3-\alpha_2$ denote the input discrepancy. Substituting (9) and (11) into (8) while noting (10),

$$\dot{z}_2=\frac{k}{m_1}z_3-\tilde{\theta}^T\phi_2-z_1-k_{2s1}z_2+\frac{k}{m_1}\alpha_{2s2}-\tilde{\Delta}_1 \quad (12)$$

The robust control function $\alpha_{2s2}$ is chosen to satisfy the following conditions

i. $z_2(\frac{k}{m_1}\alpha_{2s2}-\theta^T\phi_2-\tilde{\Delta}_1)\le\varepsilon_2$

ii. $z_2\alpha_{2s2}\le 0$

where $\varepsilon_2$ is a positive design parameter which can be arbitrarily small. Condition i shows that $\alpha_{2s2}$ is synthesized to dominate model uncertainties from both parametric uncertainties and uncertain nonlinearities, and condition ii is to make sure that $\alpha_{2s2}$ is dissipating in nature. The existence of such $\alpha_{2s2}$ for arbitrarily $\varepsilon_2$ is proved in [5].

For the positive semi-definite (p.s.d) function $V_2$ defined by

$$V_2=\frac{1}{2}z_1^2+\frac{1}{2}z_2^2 \quad (13)$$

from (12), its time derivative is

$$\dot{V}_2=-k_1z_1^2-k_{2s1}z_2^2+\frac{k}{m_1}z_2z_3+z_2(\frac{k}{m_1}\alpha_{2s2}-\tilde{\theta}^T\phi_2-\tilde{\Delta}_1) \quad (14)$$

### *B. Step 2:*

As seen from (14), if $z_3=0$, then the output tracking can be achieved in viewing condition i. So step 2 is to synthesize a virtual control input so that $z_3$ converges to zero or a small value with a guaranteed transient performance. From (4),

$$\begin{aligned}\dot{z}_3&=\dot{x}_3-\dot{\alpha}_2 \\ &=x_4-\dot{\alpha}_{2c}-\dot{\alpha}_{2u}\end{aligned} \quad (15)$$

where

$$\dot{\alpha}_{2c}=\frac{\partial\alpha_2}{\partial x_1}x_2+\frac{\partial\alpha_2}{\partial x_2}\hat{\dot{x}}_2+\frac{\partial\alpha_2}{\partial t}$$
$$\dot{\alpha}_{2u}=\frac{\partial\alpha_2}{\partial x_2}(\dot{x}_2-\hat{\dot{x}}_2)+\frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} \quad (16)$$

In (16), $\dot{\alpha}_{2c}$ is calculable and can be used in the design of control functions, but $\dot{\alpha}_{2u}$ cannot be calculated due to various uncertainties. Therefore, $\dot{\alpha}_{2u}$ has to be dealt with via certain robust feedback in this step design.

Now design a virtual control law $\alpha_3$ such that $z_3$ converges to zero or a small value. Since $\dot{\alpha}_{2c}$ is calculable, and noting (15),

$$\alpha_3=\alpha_{3a}+\alpha_{3s}$$
$$\alpha_{3a}=\dot{\alpha}_{2c}-\frac{k}{m_1}z_2 \quad (17)$$

where $\alpha_{3a}$ is used for model compensation. In order to take care of the uncertainties in $\dot{\alpha}_{2u}$ we design the robust control law $\alpha_{3s}$ consisting of the two terms

$$\alpha_{3s}=\alpha_{3s1}+\alpha_{3s2}$$
$$=-k_3z_3+\alpha_{3s2} \quad (18)$$

where $k_3$ is any positive feedback gain and $\alpha_{3s2}$ is a robust control function to be synthesized.

Let $z_4=x_4-\alpha_3$ denote the input discrepancy between $x_4$ and $\alpha_3$. Substitute (17),(18) into (15),

$$\dot{z}_3=-k_3z_3+z_4+\alpha_{3s2}-\frac{k}{m_1}z_2-\dot{\alpha}_{2u} \quad (19)$$

The robust control function $\alpha_{3s2}$ is chosen to satisfy the following conditions:

i. $z_3(\alpha_{3s2}-\dot{\alpha}_{2u})\leq\varepsilon_3$
ii. $z_3\alpha_{3s2}\leq 0$

where $\varepsilon_3$ is a positive design parameter which can be arbitrarily small.

For the positive semi-definite function $V_3$ defined by

$$V_3=V_2+\frac{1}{2}z_3^2 \quad (20)$$

from (19), its time derivative is

$$\dot{V}_3=-k_1z_1^2-k_{2s1}z_2^2-k_3z_3^2+z_3z_4$$
$$+z_2(\tfrac{k}{m_1}\alpha_{2s2}-\tilde{\theta}^T\phi_2-\tilde{\Delta}_1)+z_3(\alpha_{3s2}-\dot{\alpha}_{2u}) \quad (21)$$

### C. Step 3:

As seen from (21), if $z_4$=0, then the output tracking can be achieved. So step 3 is to synthesize a control input so that $z_4$ converges to zero or small value. From (4)

$$\dot{z}_4=\dot{x}_4-\dot{\alpha}_3$$
$$=\frac{a_1}{m_2}u+\frac{k}{m_2}(x_1-x_3)-\frac{a_2}{m_2}x_4-\frac{x_4}{m_2}\theta_4 \quad (22)$$
$$-\frac{S_f(x_4)}{m_2}\theta_5-\frac{1}{m_2}\theta_6-\tilde{\Delta}_2-\dot{\alpha}_{3c}-\dot{\alpha}_{3u}$$

where

$$\dot{\alpha}_{3c}=\frac{\partial\alpha_3}{\partial x_1}x_2+\frac{\partial\alpha_3}{\partial x_2}\hat{\dot{x}}_2+\frac{\partial\alpha_3}{\partial x_3}x_4+\frac{\partial\alpha_3}{\partial t}$$
$$\dot{\alpha}_{3u}=\frac{\partial\alpha_3}{\partial x_2}(\dot{x}_2-\hat{\dot{x}}_2)+\frac{\partial\alpha_3}{\partial\hat{\theta}}\dot{\hat{\theta}} \quad (23)$$

In (23), $\dot{\alpha}_{3c}$ is calculable and is used in the design of the controller, but $\dot{\alpha}_{3u}$ cannot due to the presence of various uncertainties. Therefore $\dot{\alpha}_{3u}$ is dealt with using robust feedback.

Now we design control law $u$ such that $z_4$ converges to zero or a small value with guaranteed performance. Since, $\dot{\alpha}_{3c}$ is calculable and noting (23),

$$u=u_a+u_s$$
$$u_a=\frac{1}{a_1}\{-k(x_1-x_3)+a_2x_4+\hat{\theta}_4x_4 \quad (24)$$
$$+\hat{\theta}_5S_f(x_4)+\hat{\theta}_6+m_2\dot{\alpha}_{3c}\}-\frac{m_2}{a_1}z_3$$

where $u_a$ is used along with on-line parameter adaptation using (6) for improved model compensation and $u_s$ is a robust control law to be synthesized later. Then $\tau$ in (6) would be

$$\tau_4=\phi_4z_4,\quad \phi_4=[0,0,0,-\frac{x_4}{m_2},-\frac{S_f(x_4)}{m_2},-\frac{1}{m_2}]^T \quad (25)$$

In order to take care of the different model uncertainties, we use the robust control law $u_s$ consisting of two terms,

$$u_s=u_{s1}+u_{s2}$$
$$u_{s1}=\frac{-m_2}{a_1}k_{4s1}z_4,\quad k_{4s1}\geq\|C_{\phi_4}\Gamma\phi_4\|+k_4 \quad (26)$$

where $k_{4s1}$ is a positive nonlinear control gain function and $u_{s2}$ is a robust control function synthesized as follows. Substituting (25), (26) into (23),

$$\dot{z}_4=\dot{x}_4-\dot{\alpha}_3$$
$$=\frac{a_1}{m_2}(u_a+u_s)+\frac{k}{m_2}(x_1-x_3)-\frac{a_2}{m_2}x_4 \quad (27)$$
$$-\frac{\theta_4}{m_2}x_4-\frac{\theta_5}{m_2}S_f(x_4)-\frac{1}{m_2}\theta_6-\tilde{\Delta}_2$$
$$=-z_3-\tilde{\theta}^T\phi_4-\tilde{\Delta}_2-\dot{\alpha}_{3u}-k_{4s1}z_4+\frac{a_1}{m_2}u_{s2}$$

The robust control function is now chosen to satisfy the following conditions

i. $z_4(\frac{a_1}{m_2}u_{s2}-\tilde{\theta}^T\phi_4-\tilde{\Delta}_2-\dot{\alpha}_{3u})\leq\varepsilon_4$
ii. $z_4u_{s2}\leq 0$

where $\varepsilon_4$ is an arbitrarily small positive design parameter. For the positive semi-definite Lyapunov function $V_4$ defined by

$$V_4=\sum_{i=1}^{4}\frac{1}{2}z_i^2 \quad (28)$$

from (28), its time derivative is

$$\dot{V}_4=-k_1z_1^2-k_{2s1}z_2^2-k_3z_3^2-k_{4s1}z_4^2$$
$$+z_2(\tfrac{k}{m_1}\alpha_{2s2}-\tilde{\theta}^T\phi_2-\tilde{\Delta}_1)+z_3(\alpha_{3s2}-\dot{\alpha}_{2u}) \quad (29)$$
$$+z_4(\tfrac{a_1}{m_2}u_{s2}-\tilde{\theta}^T\phi_4-\tilde{\Delta}_2-\dot{\alpha}_{3u})$$

## IV. Main Performance Results

### A. Theoretical Performance Results

With the above controller design, the following theoretical results on the performance of the ARC controller can be obtained.

*Theorem 1:* Let the parameter estimates be updated using the adaptation law in (6) in which $\tau$ is chosen as

$$\tau = z_2\phi_2 + z_4\phi_4 \tag{30}$$

If the nonlinear control gains are chosen such that $k_{2s1} \geq \|C_{\phi_2}\Gamma\phi_2\|^2 + k_2$ and $k_{4s1} \geq \|C_{\phi_4}\Gamma\phi_4\| + k_4$ respectively, then, the control law given in (25), (26) guarantees that

$$V_4(t) \leq -exp(\lambda_V t)V_4(0) + \frac{\varepsilon_V}{\lambda_V}[1 - exp(-\lambda_V t)] \tag{31}$$

where $\lambda_V = 2min\{k_2, k_3, k_4\}$ and $\varepsilon_V = \varepsilon_2 + \varepsilon_3 + \varepsilon_4$

**Proof.** Substituting (11), (26) into (29), while noting condition i in step 1, step 2 and step 3,

$$\begin{aligned} \dot{V}_4(t) &\leq -k_1z_1^2 - k_2z_2^2 - k_3z_3^2 - k_4z_4^2 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 \\ &\leq -\lambda_V V_4(t) + \varepsilon_V \end{aligned} \tag{32}$$

The result in equation (31) can be obtained by using the comparison lemma. This proves Theorem 1.

*Theorem 2:* If after a finite time $t_0$, $\tilde{\Delta}_1 = \tilde{\Delta}_2 = 0$, i.e., the model uncertainties are only due to parametric uncertainties, asymptotic output tracking or zero final tracking error (i.e., $z_1 \to 0$ as $t \to \infty$) is obtained for the positive gains $k_i$ and $\varepsilon_i$.

**Proof.** The Theorem can be proved in the same way as in [5].

*Remark 1.* The reference trajectory $y_d(t)$ is assumed to be smooth up to order 4, i.e., bounded derivatives up to 4th order. The derivatives of the reference trajectory are used in each step of backstepping design for model compensation.

*Remark 2.* If a step or discontinuous position command is given to the system, a low pass filter can be applied to the position command to generate smooth reference trajectory. Properly chosen filter initial states can make $V_4(0)$ equal to zero and practically eliminate transient error in (31).

### B. Experimental Results

To test the effectiveness and validate the proposed ARC design, the controller was implemented on a floating oscillator testbed which consists of the two masses each of which weigh $m_1 = 1.039$ Kg and $m_2 = 2.0911$ Kg respectively. They are connected using a spring with spring constant $K = 130$ Nm$^{-1}$. This leads to the system having a vibration mode at 16.5 rad/sec. The forced mass has a 6 volt DC motor powered by a linear amplifier. The amplifier is connected to the computer through a PCI board and Wincon software. The Wincon software enables Simulink to send control signals to the motor through MATLABs Real-time Workshop toolbox. Incremental encoders measure the rotation of gears as the carts move along the track and this enables the measurements of positions and velocities of both the forced and the unforced masses. Both masses are affected by Coulomb friction which is equivalent to about 0.6 volt (1/10 of the maximal input force).

In the implementation of the controller, the initial values of the parameter estimates are all chosen to be zero. The tracking performance is shown in Fig. 2, with the first subplot showing the reference and actual free mass trajectory and second subplot showing the tracking error. The control signal is shown in Fig. 3.

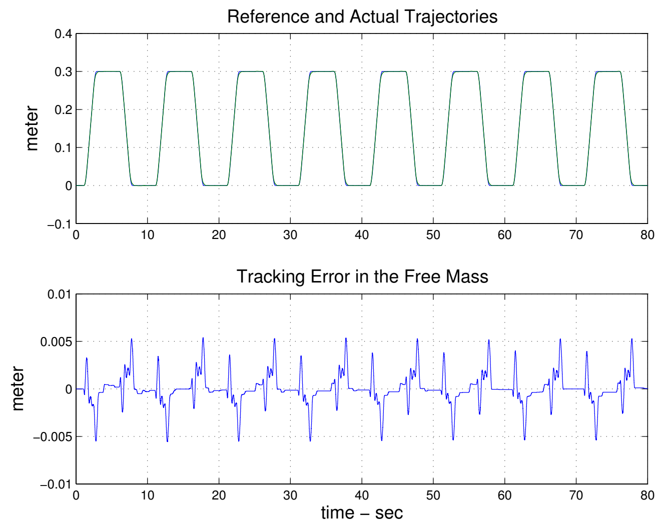


Fig. 2. Tracking Performance of the ARC Based Controller.

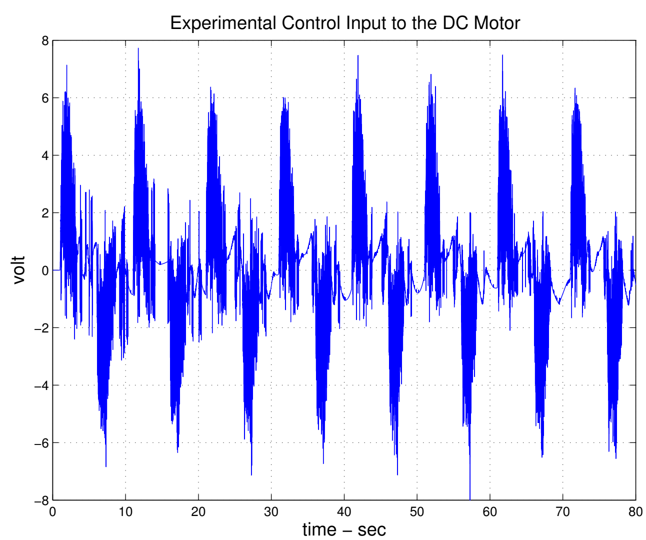


Fig. 3. Control Effort of the ARC Based Control.

## V. Conclusions

In this paper, a discontinuous projection based ARC controller is constructed for the precision motion control of a benchmark problem in control, the floating oscillator system. The controller takes into account the system nonlinearities, such as the frictions in the system and is also able to tackle unmatched model uncertainties. In addition, we are able to obtain a prescribed level of transient performance when we are tracking a smooth trajectory. Experimental results verify the effectiveness and the high-performance nature of the proposed ARC algorithm to the control of flexile systems.

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