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Impulsive Control of Networked Systems with Communication Delays

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Abstract— **In this paper, the control problem of networked control systems (NCSs) with communication delays is considered. An impulsive control scheme for NCSs with communication delays is presented. This scheme converts the design of NCSs with delays to a control design problem of a linear time-invariant (LTI) system via output feedback. Necessary and/or sufficient conditions that guarantee global exponential stability of the closed-loop systems are presented. The results suggest a simple procedure for designing state/output feedback controllers of the systems. Numerical examples are worked out to demonstrate the feasibility and efficiency of the proposed methods.**

I. INTRODUCTION

Networked control systems (NCSs) have received increasing attention because of the advantages to use real-time networks in control systems, e.g., lower cost and more convenience for installation and maintenance, flexibility and distributed nature in architectures. However, due to network bandwidth restriction, the insertion of communication network in feedback control loops inevitably leads to communication delays and makes the analysis and design of NCSs complex. Communication delays can deteriorate the performance of NCSs and even can destabilize the systems when they are not considered in the design of NCSs. Therefore, communication delay problems have raised greatly interest in recent years.

So far, a variety of efforts have been devoted to issues on communication delays (see, e.g., [1], [3]–[11], [18], [19], [21], and the references therein). Various techniques have been adopted for analyzing NCSs with communication delays. For example, in [7] and [18], the authors analyzed NCSs' stability and proposed stability regions using a hybrid systems technique. The stability of NCSs was analyzed using dual-locus diagram and the stability region was presented in [10]. Ref. [3] presented linear matrix inequality (LMI) conditions for obtaining maximum allowable delay bounds, which guarantee the stability of NCSs. Based on Lyapunov-Razumikhin function method, conditions on the admissible bounds of data packet loss and of delays for NCSs were given in terms of LMIs in [8]. Based on stochastic control theory, optimal controller design of NCSs with stochastic network delays was investigated in [4] and [11]. Works in this direction also can been seen in [1] and [6]. For other control schemes, we refer readers to the survey [20].

To reduce network traffic load, the authors of [5] and [6] proposed a model-based control scheme for NCSs without/with communication delays and the authors of [13], [14], and [17] further investigated this issue later. In [5], necessary and sufficient conditions for exponential stability of continuous-time/discrete-time NCSs without communication delays were established in both cases of state feedback and output feedback. When communication delays were considered, similar results were also presented for continuous-time NCSs via state feedback in [5] and at this time, a propagation unit was used to compensate delays. However, the authors did not present any method for controller design when communication delays were considered. Moreover, it is in general not an easy task to design the controller based on their conditions. Recently, an impulsive control scheme for continuous-time/discrete-time NCSs without communication delays was proposed in [12] and [15]. Also, controller design procedures were presented.

In this paper, we extend the method in [12] and [15] to the case in which communication delays are considered. We consider the case that communication delays only occur in the process of samplings passing through the network. We assume that network delay τ is constant and does not exceed a sampling period h of the sensor. Concretely, if the sensor samples plant output at time t_k, then the sampling $y(t_k)$ passes through the network and arrives at the model at time $t_k+\tau$. When network delays are considered, an impulsive control scheme transfers the controller design problem of NCSs into a control design problem for a continuous-time linear time-invariant (LTI) system via output feedback. The advantages of the scheme for NCSs with network delays are as follow: It introduces additional freedom and hence flexibility in designing the NCSs with network delays. This first makes it possible to design state/output feedback controllers for NCSs, even to design controllers based on lower order models for NCSs. Next, sampling period of the sensor can be increased so as to reduce the network traffic load. Moreover, the present scheme does not require a propagation unit used in [5] to compensate delays for NCSs. These are of practical interest in applications. The NCS configuration is shown in Fig. 1.

The paper is organized as follows. Section II gives the problem formulation and some preliminaries. Section III

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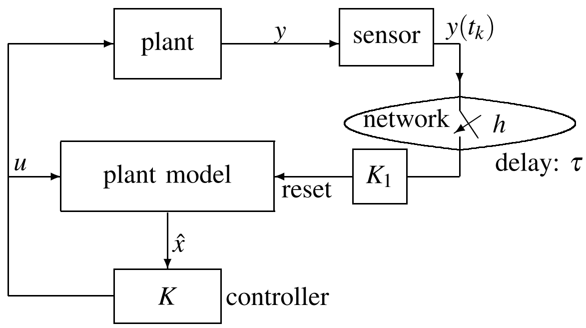

Fig. 1. Proposed configuration of networked control systems

presents conditions for the global exponential stability of the closed-loop system and a design procedure for controllers. Numerical simulations are presented in Section IV and some conclusions are drawn in Section V.

II. PROBLEM FORMULATION AND PRELIMINARIES

The NCS we consider is described as follows.

Plant: $\dot{x}(t) = Ax(t) + Bu(t),\ \ y(t) = Cx(t),$

Model: $\begin{cases} \dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}u(t), & t \in (t_k+\tau,\ t_{k+1}+\tau), \\ \hat{x}(t_k+\tau) = K_1 y(t_k), & k = 0,1,2,..., \end{cases}$

Control law: $u(t) = K\hat{x}(t),$

where $t_{k+1} - t_k = h > 0$ is constant, h is the sampling period of the sensor, τ is constant network delay satisfying $0 < \tau \le h$, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$, and $\hat{x}(t) \in \mathbb{R}^q$ are the plant state, the plant input, the plant output, and the model state, respectively, K_1 is a $q \times p$ gain matrix, K is an $m \times q$ state feedback gain matrix, $A, B, C, \hat{A}$, and $\hat{B}$ are known real constant matrices with appropriate dimensions. A is not Hurwitz stable. C is of full row rank. Assume $p \ge q$ and $pq \ge n+q$. The network is switched off at the initial time t_0. The initial states $x(t_0)$ and $\hat{x}(t_0)$ are arbitrarily selected.

Remark 1. Since τ is constant, the period h is also the period of model state updated.

Define $z(t) = [x(t)^T\ \hat{x}(t)^T]^T$. The dynamics of overall system for $t \in (t_k+\tau,\ t_{k+1}+\tau)$ can be described as

$$\begin{aligned} \dot{z}(t) &= \Lambda z(t), \qquad t \in (t_k+\tau,\ t_{k+1}+\tau), \\ z(t_k+\tau) &= [(x(t_k+\tau)^-)^T\ \ (K_1 C x(t_k))^T]^T, \\ z(t_0) &= [x(t_0)^T\ \ \hat{x}(t_0)^T]^T, \end{aligned} \tag{1}$$

where

$$\Lambda = \begin{bmatrix} A & BK \\ 0 & \hat{A} + \hat{B}K \end{bmatrix}.$$

Clearly, the overall system (1) is an impulsive control system via state feedback and with delays.

Our goal is to establish conditions for the trivial solution of system (1) to be globally exponentially stable and based on the conditions, to present design approach for the controller gain matrix K and the gain matrix K_1.

We will make use of the following preliminary results in the sequel. For the sake of convenience, denote

$$S_1 = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix},\ \ S_2 = \begin{bmatrix} 0 & 0 \\ K_1 C & 0 \end{bmatrix},$$
$$M = S_1 e^{\Lambda h} + S_2 e^{\Lambda(h-\tau)},\ \ M_0 = S_1 e^{\Lambda\tau} + S_2,$$

where I_n is the $n \times n$ identity matrix. From (1),

$$\begin{aligned} z(t_k+\tau) &= S_1 z((t_k+\tau)^-) + S_2 z(t_k) \\ &= \left(S_1 e^{\Lambda h} + S_2 e^{\Lambda(h-\tau)}\right) z(t_{k-1}+\tau) \\ &= M^k z(t_0+\tau) \\ &= M^k \left(S_1 e^{\Lambda\tau} + S_2\right) z(t_0) \\ &= M^k M_0 z(t_0). \end{aligned}$$

Particularly, for $\tau = h$, one has $M = M_0$ and

$$z(t_k+\tau) = z(t_{k+1}) = M_0^{k+1} z(t_0).$$

This leads to the following result.

Lemma 1. The response of the system (1) is

$$z(t) = \begin{cases} e^{\Lambda(t-t_0)} z(t_0), & t \in [t_0,\ t_0+\tau), \\ e^{\Lambda(t-(t_k+\tau))} M^k M_0 z(t_0), & t \in [t_k+\tau,\ t_{k+1}+\tau), \end{cases}$$

for $0 < \tau < h$ and

$$z(t) = e^{\Lambda(t-t_k)} M_0^k z(t_0),\ \ t \in [t_k,\ t_{k+1}),$$

for $\tau = h$, where $k = 0,1,....$

Lemma 2. The trivial solution of the system (1) is globally exponentially stable if and only if M is Schur stable.

We will convert the design problem to a control problem of an LTI system via output feedback. For related results, we refer to [2] where it was shown that under certain conditions, the desired pole set of the closed-loop system can be assigned by assigning eigenstructure. For the sake of completeness, we recall some useful results of [2] for us as follows.

Remark 2. Given a controllable and observable LTI system

$$\dot{x} = \bar{A}x + \bar{B}u,\ y = \bar{C}x, \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^r$, $\bar{A}, \bar{B}$, and $\bar{C}$ are constant matrices of appropriate dimensions with $\text{rank}(\bar{B}) = m$, $\text{rank}(\bar{C}) = r$, and $mr \ge n$. Under the output feedback law $u = \bar{K}y$, the closed-loop system is

$$\dot{x} = (\bar{A} + \bar{B}\bar{K}\bar{C})x. \tag{3}$$

Let $\bar{C}_1$ is any $(n-r) \times n$ constant matrix such that $[\bar{C}^T\ \bar{C}_1^T]^T$ is nonsingular and $[\bar{C}^T\ \bar{C}_1^T]^T = [T_1\ T_2]^{-1} = T^{-1}$ with T_2

being an $n\times(n-r)$ matrix. Denote $A_{22}=\bar{C}_1\bar{A}T_2$. Let $\bar{\Lambda}=\{\Lambda_1,\Lambda_2\}$ be an arbitrarily selected set subject to the following constraints.

a) $\bar{\Lambda}$ contains distinct values,
b) Λ_1 and Λ_2 are self-conjugated sets,
c) Λ_1 contains no eigenvalues of $\bar{A}$ and Λ_2 contains no eigenvalues of A_{22},

where $\Lambda_1=\{\lambda_1,\lambda_2,...,\lambda_r\},\Lambda_2=\{\lambda_{r+1},\lambda_{r+2},...,\lambda_n\}$. Necessary and sufficient conditions which contain two coupled Sylvester matrix equations for assigning a desired eigenvalue set $\bar{\Lambda}$ to (3) were established in [2], which reduce the design of output feedback gain matrix $\bar{K}$ to solving following bilinear algebraic equations

$$a_i^T M_{ij} a_j=0 \text{ for } i=1,2,...,r, \text{ and } j=r+1,r+2,...,n, \quad (4)$$

where for each i and each j,

$$a_i=[a_{i1},a_{i2},...,a_{i(m-1)},1]^T,$$
$$a_j=[a_{j1},a_{j2},...,a_{j(r-1)},1]^T,$$

are m-order and r-order parametric vectors, respectively, and

$$M_{ij}^T=\bar{C}[I_n+\bar{A}T_2[\lambda_j I_{n-r}-\bar{C}_1\bar{A}T_2]^{-1}\bar{C}_1][\lambda_i I_n-\bar{A}]^{-1}\bar{B}$$

is an $r\times m$ constant matrix. Particularly, for the case of $m+r\geq n+1$, by preselecting the vectors a_j arbitrarily, (4) are reduced to a set of linear algebraic equations with the vectors a_i's as unknown variables. Denote

$$\Psi_r=[a_1,a_2,...,a_r],\ U_r=[V_1a_1,V_2a_2,...,V_ra_r], \quad (5)$$

where a_i, $i=1,2,...,r$, verify (4) and

$$V_i=(\lambda_i I_n-\bar{A})^{-1}\bar{B},\quad i=1,2,...,r.$$

Lemma 3 [2]. For system (2) and $\bar{\Lambda}$ subject to constraints a)–c) described above, if the output feedback matrix K is taken as $\bar{K}=\Psi_r(\bar{C}U_r)^{-1}$ with Ψ_r and U_r determined by (5), then the pole set $\bar{\Lambda}$ is assigned to (3).

III. STABILITY ANALYSIS AND CONTROL DESIGN

In this section, we will establish conditions for the trivial solution of system (1) to be globally exponentially stable and, based on the results, present a design procedure for K and K_1 with h and τ satisfying certain conditions. In the following, we will establish the main results for $0<\tau<h$. For the case $\tau=h$, similar results can be obtained.

We start from analyzing the structure of M and then, present conditions for M to be Schur stable. For convenience, denote

$$e^{\Lambda\delta}=\begin{bmatrix} A_1(\delta) & A_2(\delta)\\ 0 & A_3(\delta)\end{bmatrix},\quad \delta\in\mathbb{R},$$

where $A_1(\delta)=e^{A\delta}$, $A_3(\delta)=e^{(\hat{A}+\hat{B}K)\delta}$, and $A_2(\delta)$ is certain matrix which depends on $K,h,$ and τ. Particularly, for $\delta=0$, $e^{\Lambda\delta}$ is the identity matrix. The matrix M is rewritten as

$$\begin{aligned} M &= \begin{bmatrix} I_n & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix} A_1(h) & A_2(h)\\ 0 & A_3(h)\end{bmatrix}+\begin{bmatrix} 0 & 0\\ K_1C & 0\end{bmatrix}\\ &\quad\times \begin{bmatrix} A_1(h-\tau) & A_2(h-\tau)\\ 0 & A_3(h-\tau)\end{bmatrix}\\ &= \begin{bmatrix} A_1(h) & A_2(h)\\ 0 & 0\end{bmatrix}+\begin{bmatrix} 0\\ I_q\end{bmatrix}K_1\begin{bmatrix} A_1^T(h-\tau)C^T\\ A_2^T(h-\tau)C^T\end{bmatrix}^T\\ &\triangleq A_1+B_1K_1C_1, \end{aligned} \quad (6)$$

where I_q is the $q\times q$ identity matrix. Based on the method in Remark 2 and Lemma 3, we have the following result.

Theorem 1. For given $K,h,$ and τ, if the system with a triple (A_1,B_1,C_1) as coefficient matrices is controllable and observable, then K_1 can be designed such that M is Schur stable.

Proof. From (6), we can view M as the closed-loop system matrix of (A_1,B_1,C_1) and K_1 as its output feedback matrix. Hence, M is Schur stable if and only if the closed-loop system of (A_1,B_1,C_1) via output feedback is Schur stable. In the following, we will prove that gain matrix K_1 can be designed such that $A_1+B_1K_1C_1$ is Schur stable by the method in Remark 2 and Lemma 3. To see this, we will show that the system (A_1,B_1,C_1) verifies the corresponding conditions of (2).

By assumption of Theorem 1, (A_1,B_1,C_1) is controllable and observable for given $K,h,$ and τ. Note that B_1 is of full column rank and that C_1 are of full row rank because C is of full row rank and $A_1(h-\tau)$ is invertible. Also from the assumptions on the described NCS, the inequality $pq\geq n+q$ holds. For the triple (A_1,B_1,C_1) with such $K,h,$ and τ, select the eigenvalues of $A_1+B_1K_1C_1$ subject to corresponding constraints a)–c) in Remark 2 and to be strictly lying in unit disk. Therefore according to the method in Remark 2 and Lemma 3, by solving a bilinear system of algebraic equations, K_1 can be found such that $A_1+B_1K_1C_1$ is Schur stable and so is M. □

Theorem 1 shows that if we have found $K,h,$ and τ such that (A_1,B_1,C_1) is controllable and observable, then we can find further K_1 such that M is Schur stable. Therefore, we first try to find $K,h,$ and τ such that (A_1,B_1,C_1) is controllable and observable. Generally speaking, it is not easy to find such $K,h,$ and τ directly because of the complex expressions of A_1 and C_1. In the sequel, we give a constraint on choosing K.

Theorem 2. If M is Schur stable for given $K,h,\tau,$ and K_1, then $BK\neq 0$.

Proof. It can be proved by contradiction. If $BK=0$, then $A_2(\delta)=0$ and

$$M=\begin{bmatrix} A_1(h) & 0\\ CA_1(h-\tau) & 0\end{bmatrix}.$$

Thus, M is Schur stable if and only if $A_1(h)$ is Schur stable. Since $A_1(h) = e^{Ah}$, then M is Schur stable if and only if A is Hurwitz stable which is in contradiction with A given in the system. $\square$

Theorem 2 imposes a constraint on choosing K. Without loss of generality, we can choose a K with full row rank. We also note that a necessary condition for observability of (A_1, C_1).

Theorem 3. If (A_1, C_1) is observable, then $p \geq q$.

Proof. Since (A_1, C_1) is observable if and only if $\mathrm{rank}(Q_o) = n+q$, where

$$Q_o = \begin{bmatrix} C_1 \\ C_1A_1 \\ \vdots \\ C_1A_1^{n+q-1} \end{bmatrix}.$$

From (6), substituting the expressions of A_1 and C_1 into Q_o, we can obtain

$$Q_o = \begin{bmatrix} CA_1(h-\tau) & CA_2(h-\tau) \\ Q & P \end{bmatrix}$$

with

$$Q = \begin{bmatrix} CA_1(h-\tau)A_1(h) \\ \vdots \\ CA_1(h-\tau)A_1^{n+q-1}(h) \end{bmatrix},$$

$$P = \begin{bmatrix} CA_1(h-\tau)A_2(h) \\ \vdots \\ CA_1(h-\tau)A_1^{n+q-2}(h)A_2(h) \end{bmatrix}.$$

Considering $A_1(h) = e^{Ah}, A_1(h-\tau) = e^{A(h-\tau)}$, and $P = QA_1(-h)A_2(h)$, we have

$$Q_o \begin{bmatrix} I & -A_1(-h)A_2(h)] \\ 0 & I \end{bmatrix} = \begin{bmatrix} CA_1(h-\tau) & W \\ Q & 0 \end{bmatrix} \triangleq \bar{Q}_o$$

with $W = C[A_2(h-\tau) - A_1(-\tau)A_2(h)]$. Hence $\mathrm{rank}(Q_o) = \mathrm{rank}(\bar{Q}_o)$. Therefore (A_1, C_1) is observable if and only if $\mathrm{rank}(\bar{Q}_o) = n+q$. In the expression of $\bar{Q}_o$, the orders of $CA_1(h-\tau)$ and W are $p \times n$ and $p \times q$, respectively. Therefore, the inequalities

$$\mathrm{rank}[\bar{Q}_o \times (I_n \quad 0_{n\times q})^T] \leq n, \quad \mathrm{rank}[\bar{Q}_o \times (0_{q\times n} \quad I_q)^T] \leq p,$$

hold. So one can get $\mathrm{rank}(\bar{Q}_o) \leq n+p$ and further, $q \leq p$. $\square$

Theorem 3 shows that $q \leq p$ is necessary for the observability of (A_1, C_1). Besides, since C is a $p \times n$ matrix of full row rank, then one has $p \leq n$ and further $q \leq p \leq n$, which shows that the order of the model used for generating control signal is not larger than that of the plant. Particularly, in the case of $q < n$, controllers based on lower order models are used for controlling the plants. This is of great interest in practical applications. Therefore, it is necessary and reasonable to assume $q \leq p$ in the system.

In order to determine h and τ such that (A_1, B_1, C_1) is controllable and observable for a chosen gain matrix K, a necessary and sufficient condition for the controllability and observability will be established in following theorem.

Theorem 4. If $q \leq p$ and gain matrix K is such that $BK \neq 0$, then (A_1, B_1, C_1) is controllable and observable if and only if the following three equalities hold:

$$\mathrm{rank}(Q_1) = n, \quad \mathrm{rank}(Q_2) = n, \tag{7}$$

$$\mathrm{rank}(Q_3) = q, \tag{8}$$

where $Q_1 = [A_2(h) \quad A_1(h)A_2(h) \quad \cdots \quad A_1^{n+q-2}(h)A_2(h)]$,

$$Q_2 = \begin{bmatrix} C \\ CA_1(h) \\ \cdots \\ CA_1^{n+q-2}(h) \end{bmatrix}, \quad Q_3 = CA_2(-\tau).$$

Proof. First, (A_1, B_1) is controllable if and only if $\mathrm{rank}(Q_c) = n+q$, where $Q_c = [B_1 \quad A_1B_1 \quad \cdots \quad A_1^{n+q-1}B_1]$. From (6), inserting the expressions of A_1 and B_1 into Q_c, we get

$$Q_c = \begin{bmatrix} 0 & Q_1 \\ I_q & 0 \end{bmatrix}.$$

Clearly, $\mathrm{rank}(Q_c) = n+q$ is equivalent to $\mathrm{rank}(Q_1) = n$.

Next, we will prove (A_1, C_1) is observable if and only if $\mathrm{rank}(Q_2) = n$ and $\mathrm{rank}(Q_3) = q$. From Theorem 3, (A_1, C_1) is observable if and only if $\mathrm{rank}(\bar{Q}_o) = n+q$. Since $\bar{Q}_o$ is a matrix with $n+q$ columns, then $\mathrm{rank}(\bar{Q}_o) = n+q$ if and only if all column vectors are linear independent, and further from the expression of $\bar{Q}_o$, the latter is equivalent to $\mathrm{rank}(W) = q$ and $\mathrm{rank}(Q) = n$. Since $A_1(h-\tau)A_1(h) = A_1(h)A_1(h-\tau)$ and $A_1(h-\tau)A_1(h)$ is invertible, then $Q = Q_2A_1(h-\tau)A_1(h)$ and further $\mathrm{rank}(Q) = \mathrm{rank}(Q_2)$. Also note that $e^{\Lambda(h-\tau)} = e^{\Lambda(-\tau)}e^{\Lambda h}$, i.e.,

$$\begin{aligned} &\begin{bmatrix} A_1(h-\tau) & A_2(h-\tau) \\ 0 & A_3(h-\tau) \end{bmatrix} \\ = &\begin{bmatrix} A_1(-\tau) & A_2(-\tau) \\ 0 & A_3(-\tau) \end{bmatrix} \begin{bmatrix} A_1(h) & A_2(h) \\ 0 & A_3(h) \end{bmatrix}, \end{aligned}$$

from which it yields $A_2(h-\tau) - A_1(-\tau)A_2(h) = A_2(-\tau)A_3(h)$. Since that $A_3(h) = e^{(\hat{A}+\hat{B}K)h}$ is invertible, then $\mathrm{rank}(W) = \mathrm{rank}[CA_2(-\tau)]$. Therefore, (A_1, C_1) is observable if and only if $\mathrm{rank}(Q_2) = n$ and $\mathrm{rank}(Q_3) = q$. $\square$

Theorem 4 shows that for a chosen K, if we can find h and τ satisfying conditions (7) and (8), then (A_1, B_1, C_1) is controllable and observable. Again according to Theorem 1, gain matrix K_1 can be designed such that M is Schur stable. From the expressions of Q_1, Q_2, and Q_3, $\mathrm{rank}(Q_1)$ and $\mathrm{rank}(Q_2)$ depend on K and h while $\mathrm{rank}(Q_3)$ is related to K and τ. Hence, once K is given, the determination of h is independent of τ and the determination of τ is also independent of h except for $0 < \tau \leq h$. By Theorem 2, we first choose a K with full row rank. Then we can plot the evolutions of rank(Q_1) and rank(Q_2) with h and rank(Q_3) with τ using MATLAB and from the plots, determine a positive value h satisfying (7) and a positive value τ satisfying (8) and $\tau \le h$. If no suitable h and τ can be found in $(0, +\infty)$, we need to choose another K and run again. This suggests the following procedure.

Step 1. Choose K with full row rank.
Step 2. Plot the graphs of rank(Q_1) and rank(Q_2) vs h and rank(Q_3) vs τ. Then find from the plots positive real numbers h, τ $(\le h)$ such that h satisfies (7) and τ satisfies (8). If such h and τ are available, then go to Step 3. Or else return to Step 1.
Step 3. Select the eigenvalues of $A_1 + B_1K_1C_1$ subject to corresponding constraints a)–c) in Remark 2 and strictly lying in unit disk, determine Ψ_r and U_r by solving corresponding system of algebraic equations (4) and let $K_1 = \Psi_r(C_1U_r)^{-1}$.

In Section IV, we will give numerical examples to demonstrate the procedure.

Remark 3. Similar results have been established for discrete-time NCSs in [16].

IV. NUMERICAL EXAMPLES

Example 1. In this example, the plant full state is available and the model is a reduced order model. The parameters are as follows.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

We choose

$$K = \begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}.$$

The evolutions of rank(Q_1) and rank(Q_2) vs h and of rank(Q_3) vs τ are plotted in Fig. 2, from which we can find that (7) and (8) hold simultaneously for any $h \in (0, 8]$ and any $\tau \in (0, h]$. So we can choose a larger h if it is desired. Here we select $h = 8, \tau = 7$. Let the poles of $A_1 + B_1K_1C_1$ be $0.1, 0.2, 0.3, -0.1$, and -0.2. According to the method introduced in Remark 2 and Lemma 3, we get

$$U_r = \begin{bmatrix} 0.7747 & 0.0063 & 0.0029 \\ 0.0685 & -0.0005 & -0.0005 \\ -1.5659 & -0.0082 & -0.0027 \\ 0 & 0.0013 & 0.0009 \\ 0.0100 & 0.0050 & 0.0033 \end{bmatrix} \times 10^3,$$

$$\Psi_r = \begin{bmatrix} 0 & 0.2623 & 0.2637 \\ 1.0000 & 1.0000 & 1.0000 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 1.2004 & 0.0395 & 0.5544 \\ 4.5282 & 0.1491 & 2.0912 \end{bmatrix} \times 10^3,$$

where $r = 3$. A simulation of the system with $h = 8$, $\tau = 7$, K, and K_1 determined above, and the initial state of the plant $z(0) = [1\ 4\ -2\ -3\ 5]^T$ is shown in Fig. 3.

Example 2. Now we consider another case: The plant full state is not available and the model is a reduced order model. The parameters are as follows.

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, C^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

We choose $K = [-1\ 1]$. The evolutions of rank(Q_1) and rank(Q_2) with h and rank(Q_3) with τ are plotted in Fig. 4, from which we can find that (7) and (8) hold for any $h \in (0, 5.5]$ and $\tau \in (0, h]$. Here we take $h = 1.5$ and $\tau = 1.2$. Let the poles of $A_1 + B_1K_1C_1$ be $0.4, 0.5, 0.6, 0.7, 0.8$, and 0.9. According to Remark 2 and Lemma 3, we get

$$U_r = \begin{bmatrix} 1.3819 & 0.8206 & 0.5343 \\ -2.8240 & -1.9579 & -1.4848 \\ -2.1528 & -1.3275 & -0.9056 \\ -1.1752 & -0.7856 & -0.5689 \\ -0.3433 & -0.3153 & -0.2874 \\ 1.4286 & 1.2500 & 1.1111 \end{bmatrix},$$

$$\Psi_r = \begin{bmatrix} -0.2403 & -0.2522 & -0.2587 \\ 1.0000 & 1.0000 & 1.0000 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 4.0063 & 1.6448 & 1.2652 \\ -14.4436 & -5.9649 & -4.5850 \end{bmatrix},$$

where $r = 3$. A simulation of the system with $h = 1.5$, $\tau = 1.2$, K and K_1 determined as above, and the initial state of the plant $z(0) = [10\ 4\ -12\ -3\ 15\ 8]^T$ is shown in Fig. 5.

V. CONCLUSIONS

We have extended the impulsive control scheme proposed in [12] and [15] to networked systems with communication delays and established sufficient and/or necessary conditions for the global exponential stability of the close-loop systems. Based on the results, a simple procedure for desinging controllers has been presented for NCSs. Numerical results have shown the feasibility and efficiency of the proposed method. Compared with recent work on model-based network systems in [5], the present approach introduces additional freedom and hence flexibility in designing an NCS. Moreover, our method allows of a larger samplings period and a reduced order model based control of the plant. Also, the propagation unit used in [5] for compensating delays is no longer needed here. These are of practical interest in applications.

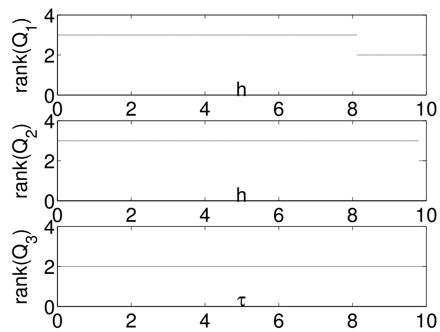

Fig. 2. The evolutions of rank(Q_1), rank(Q_2) vs h and rank(Q_3) vs τ

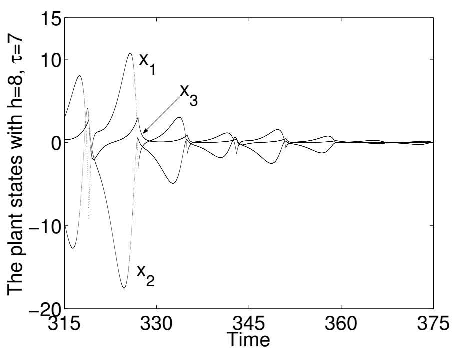

Fig. 3. The evolution of the plant state with time

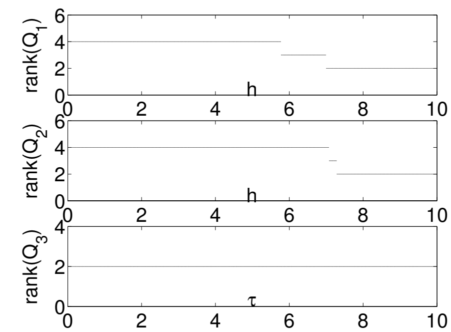

Fig. 4. The evolutions of rank(Q_1), rank(Q_2) vs h and rank(Q_3) vs τ

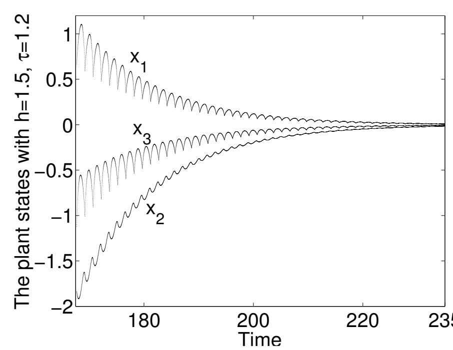

Fig. 5. The evolution of the plant state with time

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