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H_2 networked servo control systems with time-varying delays

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***Abstract*— An H_2 servo controller is proposed for networked control systems. The network-induced delay is assumed to be time-varying and vary in the known range. The proposed controller guarantees stability and H_2 performance for all time-varying delay in the known range. The proposed controller is verified using a simple networked motor control system.**

I. INTRODUCTION

Control systems in which control loops are closed through a serial network is called networked control systems (NCSs). Recently, NCSs have received a lot of attention due to their flexibility and easy maintenance [1], [2]. The main disadvantage of NCSs is network-induced time delay in the control loop. Since the time delay problem is unavoidable in NCSs, the problem has been studied extensively.

Depending on the network type and scheduling methods, the time delay characteristics in NCSs can be modelled as constant, time-varying, and stochastic. In the case of constant time delay [3], it is relatively easy to design controllers. In [4], dynamic scheduling methods are proposed and network-induced delay is assumed to be time-varying. And maximum allowable delay bound (MADB) for a given controller is derived: if the network-induced time delay is smaller than MADB, the closed-loop system is stable. The derived bound is rather conservative and less conservative bound is derived in [5]. In both cases, controller synthesis problems are not considered. In [6], an LQG controller is proposed for a NCS where time delay is a stochastic process. It is assumed that the network-induced time delay is measurable, for example, by using time-stamped packet. We note that control of time varying delay systems is also considered in a general framework of time delay systems [7]. We also note that time varying delay of packets can be modelled as intermittent transmission and this approach is pursued in [8].

In this paper, we propose a controller for a NCS with time-varying delay, where the delay is known to vary in the known range. The time-varying delay is treated as parameter variation in the system and robust control technique is used to design a controller. An H_2 servo control problem is formulated in the framework of NCSs.

Notations : For a wide matrix A, $A^{\perp}$ denotes a matrix of full column rank whose columns span the kernel space of A. For a tall matrix A, $A^{\perp}$ denotes a matrix of full row rank such that the columns of $A^{\perp'}$ span the kernel space of A'. For a matrix A, $A^{\dagger}$ denotes the pseudo-inverse of matrix A.

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II. PROBLEM FORMULATION

Consider a networked control system in Fig. 1, where sampled outputs and actuator commands are sent through a single serial communication channel.

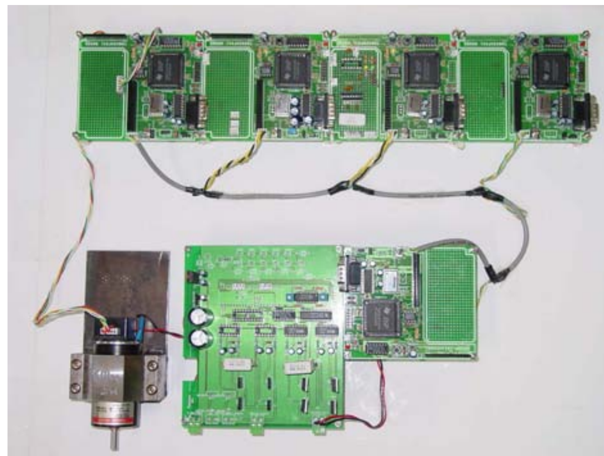

Fig. 1. Networked Control Systems

The configuration in Fig. 1 should be interpreted as generic and other configurations are also possible. For example, sensor 1 and actuator 3 can be in the same hardware board and in that case they are connected to the network through a single network interface.

A timing diagram of the networked control system is shown in Fig. 2, where sensor outputs are periodically transmitted to the controller (with the period T). Then the controller computes actuator commands and transmits them to actuators.

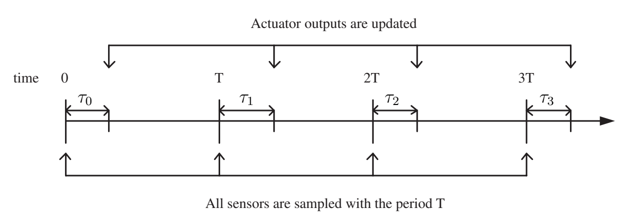

Fig. 2. Timing diagram of the networked control system

Time-varying delay τ_k includes communication delay (sensors-to-controller plus controller-to-actuators) and controller computation time (see Fig. 3). Controller computation time can be considered constant; however, communication delay is time-varying depending on the network traffic.

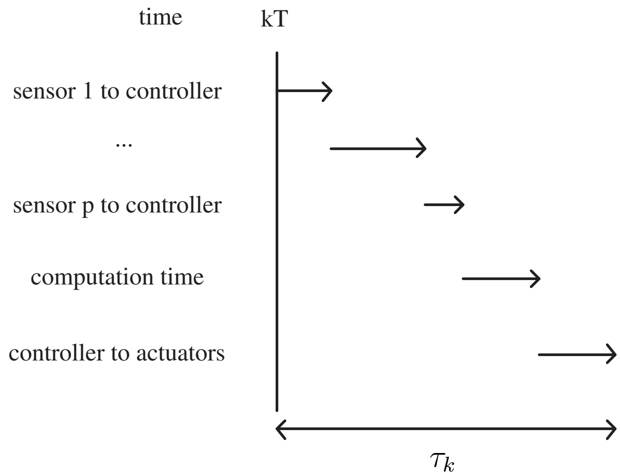

Fig. 3. time-varying delay τ_k = communication delay + controller computation time

In this paper, the following network assumptions are made.

(N1) Plant outputs are sampled with the fixed period T and the sampling is synchronized.

(N2) Delay τ_k is time-varying and its bounds are known:

$$\tau_{min} \leq \tau_k \leq \tau_{max} < T \tag{1}$$

(N3) Actuator updates are synchronized.

Periodic synchronized sampling of plant outputs can be achieved in many ways: for example, the controller can send a message periodically for sampling synchronization. Assumption 3 can be achieved by broadcasting actuator commands to all actuators.

The networked servo control problem can be formulated in a nonstandard sampled-data control framework (see Fig. 4).

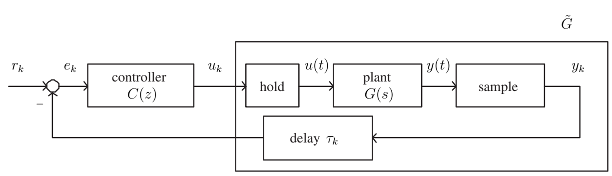

Fig. 4. Networked servo control problem as a nonstandard sampled-data control problem

We assume that the continuous time plant $G(s)$ is a linear, time-invariant system given by

$$G(s): \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) \end{array} \tag{2}$$

where $x \in R^n$ is the state, $y \in R^p$ is the output and $u \in R^m$ is the control.

As the controller $C(z)$, we use a linear time-invariant discrete controller:

$$C(z): \begin{array}{rcl} \zeta_{k+1} & = & A_c\zeta_k + B_c e_k \\ u_k & = & C_c\zeta_k + D_c e_k \end{array} \tag{3}$$

where $\zeta \in R^{n_c}$.

The ideal sampler is assumed with the period T. If the delay τ and the hold time is constant, the problem is just a standard sampled-data control problem. However, τ_k is time-varying and the hold-time is also time-varying depending on τ_k. For example, the actuator command applied at τ_0 is held until $T + \tau_1$; then the hold time is $T - \tau_0 + \tau_1$. Similarly, the next step hold time is $T - \tau_1 + \tau_2$.

Throughout the paper, the plant (A, B, C) is assumed to satisfy the followings:

(P1) (A, B) is controllable and (C, A) is observable.

(P2) The sampling period T is non-pathological [9]: i.e., A does not have two eigenvalues with equal real parts and imaginary parts that differ by an integral multiple of $\frac{2\pi}{T}$.

(P3)

$$\text{rank} \begin{bmatrix} \exp(AT) - I & \int_0^T \exp(A(T-r))B\,dr \\ C & 0 \end{bmatrix} = n + p \tag{4}$$

Assumption (P1) is standard and assumption (P2) is to ensure the discrete system of the plant is controllable and observable. Assumption (P3) is for servo controller existence [10].

We formulate the control problem of Fig. 4 in the discrete-time framework: i.e., the closed-loop system of $C(z)$ and $\tilde{G}$. The discrete-time system $\tilde{G}$ includes $G(s)$, the sampler, hold and network delay τ_k. Defining

$$y_k \triangleq y(kT),\ u_k \triangleq u(kT),\ \tilde{x}_k \triangleq \begin{bmatrix} x(kT) \\ u_{k-1} \end{bmatrix},$$

we have a state-space representation of $\tilde{G}$:

$$\tilde{G}: \begin{array}{rcl} \tilde{x}_{k+1} & = & \tilde{A}_k\tilde{x}_k + \tilde{B}_k u_k \\ y_k & = & \tilde{C}\tilde{x}_k \end{array} \tag{5}$$

where

$$\tilde{A}_k \triangleq \begin{bmatrix} \exp(AT) & \int_0^{\tau_k} \exp(A(T-r))B\,dr \\ 0 & 0 \end{bmatrix}$$

$$\tilde{B}_k \triangleq \begin{bmatrix} \int_{\tau_k}^T \exp(A(T-r))B\,dr \\ I \end{bmatrix}$$

$$\tilde{C} \triangleq \begin{bmatrix} C & 0 \end{bmatrix}.$$

Note that $\tilde{G}$ is a time-varying system due to time-varying delay τ_k.

The servo control objective in this paper is to find a stabilizing controller $C(z)$, which minimizes

$$\sum_{k=0}^{\infty}(||e_k||_2^2 + \beta^2||u_{k-1}||_2^2)$$

when a step input is applied as a reference command. This problem can be formulated into a discrete time-varying H_2 control problem (see Fig. 5).

Note that u_{k-1} instead of u_k is used in the performance index. This is to simplify the generalized plant (7) for the H_2 problem. Since u_{k-1} is included in $\tilde{x}_k$, D-part of the generalized plant becomes zero (see (7)).

The performance index minimization problem can be formulated as the following problem, where comprehensive stability (see Definition 1) is used.

main problem : find a comprehensively stabilizing controller $C(z)$, which minimizes $\|T_{zw}\|_2$, where T_{zw} is a system from w to z and $\alpha \in R^p$ is a constant vector.

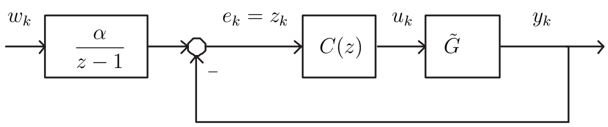

Fig. 5. Network servo problem in discrete time H_2 framework

Since $\frac{1}{z-1}\alpha$ in Fig. 5 cannot be stabilized by $C(z)$, the closed-loop system cannot be internally stabilizable. However the feedback system $(C(z), \tilde{G})$ and T_{zw} can be stabilized. For these kinds of systems, comprehensive stability notion is used [11] and employed in this paper.

Definition 1 [11] If the feedback system $(C(z), \tilde{G})$ is internally stable and T_{zw} is stable, the overall system is said to be comprehensively stable. Such $C(z)$ is called a comprehensively stabilizing controller.

Since T_{zw} is a time-varying system, definition of $\|T_{zw}\|_2$ needs attention. There are several ways to define H_2 norms [12]. For time-invariant systems, they are all identical; however, for time-varying systems, they are not equal. H_2 norm in this paper is defined by

$$\|T_{zw}\|_2^2 \triangleq \sum_{k=0}^{\infty} \|z_k\|_2^2 \tag{6}$$

where z_k is the output when we apply the impulse δ_k to the system. For time-varying systems, it is not easy to compute (6) and usually an upper bound of (6) is used [12]. Thus we will find a controller minimizing an upper bound of H_2 norm.

The generalized plant for the H_2 problem is given by

$$G_g: \begin{array}{rcl} \bar{x}_{k+1} &=& \bar{A}_k \bar{x}_k + \bar{B}_1 w_k + \bar{B}_{2,k} u_k \\ z_k &=& \bar{C}_1 \bar{x}_k \\ y_k &=& \bar{C}_2 \bar{x}_k \end{array} \tag{7}$$

where

$$\bar{A}_k \triangleq \begin{bmatrix} 1 & 0 \\ 0 & \tilde{A}_k \end{bmatrix}, \ \bar{B}_1 \triangleq \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \bar{B}_{2,k} = \begin{bmatrix} 0 \\ \tilde{B}_{2,k} \end{bmatrix}$$

$$\bar{C}_{11} \triangleq \begin{bmatrix} \alpha & -\tilde{C} \end{bmatrix}, \ \bar{C}_{21} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & \beta I_m \end{bmatrix}$$

$$\bar{C}_1 \triangleq \begin{bmatrix} \bar{C}_{11} \\ \bar{C}_{21} \end{bmatrix}, \ \bar{C}_2 \triangleq \bar{C}_{11}.$$

Note that $(\bar{A}_k, \bar{B}_{2,k})$ is not stabilizable: pole 1 of G_g is not controllable. Thus if T_{zw} is to be stable, the pole 1 should not be observable: that is, pole 1 should be cancelled out by a zero 1. To achieve this, we will use techniques in [13], where a controller $C(z)$ is designed to have a discrete integrator so that the pole 1 of G_g is cancelled out by the controller pole 1.

Note that the closed-loop system T_{zw} is given by

$$T_{zw}: \begin{array}{rcl} \begin{bmatrix} \bar{x}_{k+1} \\ \zeta_{k+1} \end{bmatrix} &=& A_{cl,k} \begin{bmatrix} \bar{x}_k \\ \zeta_k \end{bmatrix} + B_{cl} w_k \\ z_k &=& C_{cl} \begin{bmatrix} \bar{x}_k \\ \zeta_k \end{bmatrix} \end{array} \tag{8}$$

where

$$A_{cl,k} \triangleq \begin{bmatrix} \bar{A}_k + \bar{B}_{2,k} D_c \bar{C}_2 & \bar{B}_{2,k} C_c \\ B_c \bar{C}_2 & A_c \end{bmatrix}, \ B_{cl} \triangleq \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix}$$

$$C_{cl} \triangleq \begin{bmatrix} \bar{C}_1 & 0 \end{bmatrix}.$$

Lemma 1 and 2 are technical results to derive the constraints on the controller so that T_{zw} is stable.

Lemma 1: There exists a constant $[u_0' \ u_1']' \neq 0$ such that

$$\begin{bmatrix} \bar{A}_k & \bar{B}_{2,k} \\ \bar{C}_{11} & 0 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \end{bmatrix} = \begin{bmatrix} u_0 \\ 0 \end{bmatrix} \text{ for all } k. \tag{9}$$

and furthermore, if we partition u_0 as follows:

$$u_0 = \begin{bmatrix} u_{0,1} \\ u_{0,2} \\ u_{0,3} \end{bmatrix} \in \begin{bmatrix} R^{1\times(m+1-p)} \\ R^{n\times(m+1-p)} \\ R^{m\times(m+1-p)} \end{bmatrix}, \tag{10}$$

then u_0 and u_1 can be obtained from the following:

$$\begin{bmatrix} 0 & \exp(AT) - I & \int_0^T \exp(Ar) B dr \\ \alpha & -C & 0 \end{bmatrix} u_1 = 0 \tag{11}$$
$$u_1 = u_{0,3}.$$

If $(\bar{A}_k, \bar{B}_{2,k})$ is time-invariant, (9) implies that uncontrollable mode 1 of G_g is an invariant zero of $(\bar{A}_k, \bar{B}_{2,k}, \bar{C}_{11}, 0)$. It is important to note that even if $(\bar{A}_k, \bar{B}_{2,k})$ is time-varying, u_0 and u_1 are constants.

Using u_0 and u_1 in (9), we derive constraints that the controller should satisfy for comprehensive stability. We note that Lemma 2 is an extension from the result in [11], where $\tau_k = 0$ case is considered for H_∞ problem.

Lemma 2: Let T be defined by

$$T \triangleq \begin{bmatrix} u_0 & u_0^\perp & 0 \\ -u_0 & -u_0^\perp & I \end{bmatrix} \tag{12}$$

where u_0 and $u_0^\perp$ are from (9) and satisfy

$$\begin{bmatrix} u_0' \\ u_0^{\perp\prime} \end{bmatrix} \begin{bmatrix} u_0 & u_0^\perp \end{bmatrix} = \begin{bmatrix} I_{m+1-p} & 0 \\ 0 & I_{n-m+p} \end{bmatrix}.$$

If the controller (3) satisfies the following constraints:

$$\begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \begin{bmatrix} -u_0 \\ 0 \end{bmatrix} = \begin{bmatrix} -u_0 \\ u_1 \end{bmatrix}, \tag{13}$$

then

$$T_{zw} = (S_2' A_{cl,k} T_2, S_2' B_{cl}, C_{cl} T_2), \tag{14}$$

where

$$T_2 \triangleq \begin{bmatrix} u_0^\perp & 0 \\ -u_0^\perp & I \end{bmatrix}, \quad S_2 \triangleq \begin{bmatrix} u_0^\perp & I \\ 0 & I \end{bmatrix}.$$

.

Lemma 2 states that if a controller satisfies the constraints (13), then the uncontrollable mode 1 of G_g is unobservable. Thus T_{zw} is stable if $(S_2' A_{cl,k} T_2, S_2' B_{cl}, C_{cl} T_2)$ is stable. The constraints (13) means that the controller has 1 as its pole: i.e., the controller has a discrete integrator.

Using the result in Lemma 2, an upper bound of $\|T_{zw}\|_2$ is considered in the next lemma.

Lemma 3: If (A_c, B_c, C_c, D_c) satisfies (13) and there exists $Q = Q' > 0$ satisfying

$$S_2' A_{cl,k} T_2 Q (S_2' A_{cl,k} T_2)' - Q + S_2' B_{cl} B_{cl}' S_2 < 0 \quad (15)$$

for all $\tau_{\min} \leq \tau_k \leq \tau_{\max}$, then the system is comprehensively stable and

$$\|T_{zw}\|_2^2 \leq \mathrm{Tr}(C_{cl} T_2 Q (C_{cl} T_2)'). \quad (16)$$

Finding Q satisfying (15) is not easy: you have to check whether Q satisfies (15) for all $\tau_{\min} \leq \tau_k \leq \tau_{\max}$. We will tackle this problem using robust control techniques, where time-varying elements are treated like norm-bounded uncertainty.

Time-varying elements in $\bar{A}_{cl,k}$ depending on τ_k are as follows:

$$\int_0^{\tau_k} \exp(A(T-r))B\, dr. \quad (17)$$

Defining a nominal value τ_{nom} of τ_k and

$$\Delta(\tau_k, \tau_{nom}) \triangleq \int_{\tau_{nom}}^{\tau_k} \exp(A(T-r)B)\, dr, \quad (18)$$

we can express (17) as follows:

$$\begin{aligned} \textstyle\int_0^{\tau_k} \exp(A(T-r))B\, dr = \int_0^{\tau_{nom}} \exp(A(T-r))B\, dr \\ +\Delta(\tau_k, \tau_{nom}). \end{aligned} \quad (19)$$

Replacing τ_k with τ_{nom}, we define $\tilde{A}_{nom}$ and $\tilde{B}_{nom}$:

$$\tilde{A}_{nom} \triangleq \begin{bmatrix} \exp(AT) & \int_0^{\tau_{nom}} \exp(A(T-r))B\, dr \\ 0 & 0 \end{bmatrix}$$

$$\tilde{B}_{nom} = \begin{bmatrix} \int_{\tau_{nom}}^{T} \exp(A(T-r))B\, dr \\ I \end{bmatrix}.$$

Similarly, we can define $\bar{A}_{nom}$, $A_{cl,nom}$ and $\bar{B}_{2,nom}$ from $\bar{A}_k$, $A_{cl,k}$ and $\bar{B}_{2,k}$, respectively.

Note that all time-varying elements are in $\Delta(\tau_k, \tau_{nom})$ and $\Delta(\tau_k, \tau_{nom})$ will be treated as norm-bounded uncertainty, where B_Δ is a matrix bound satisfying

$$\begin{aligned} B_\Delta = B_\Delta' > 0, \quad \Delta(\tau_k, \tau_{nom})\Delta(\tau_k, \tau_{nom})' \leq B_\Delta \\ \text{for all } \tau_{\min} \leq \tau_k \leq \tau_{\max}. \end{aligned} \quad (20)$$

To use (20), we need to find τ_{nom} and B_Δ such that $\bar{\sigma}(B_\Delta)$ is as small as possible, where $\bar{\sigma}(B_\Delta)$ is the maximum singular value of B_Δ. The value τ_{nom} can be found by solving the following optimization problem.

$$\min_{\tau_{\min} \leq \tau_{nom} \leq \tau_{\max}} \max_{\tau_{\min} \leq \tau \leq \tau_{\max}} \bar{\sigma}(\Delta(\tau, \tau_{nom})\Delta(\tau, \tau_{nom})').$$

The optimization problem can be solved by using a simple mesh search. Once τ_{nom} is found, B_Δ can be found by solving

$$\min_{B_\Delta} \bar{\sigma}(B_\Delta) \quad (21)$$

subject to

$$\begin{aligned} B_\Delta = B_\Delta' > 0, \quad \Delta(\tau, \tau_{nom})\Delta(\tau, \tau_{nom})' \leq B_\Delta \\ \text{for all } \tau_{\min} \leq \tau \leq \tau_{\max}. \end{aligned}$$

Since to solve (21) is not easy, we will find a suboptimal solution: we only check a few τ values in (21) instead of checking all $\tau_{\min} \leq \tau \leq \tau_{\max}$.

$$\min_{B_\Delta} \gamma \quad (22)$$

subject to

$$\begin{aligned} 0 < B_\Delta = B_\Delta' < \gamma I, \quad \Delta(\tau_i, \tau_{nom})\Delta(\tau_i, \tau_{nom})' \leq B_\Delta \\ \text{for all } \tau_{\min} \leq \tau_i \leq \tau_{\max}, \ 1 \leq i \leq N. \end{aligned}$$

Note that (22) can be formulated in linear matrix inequalities and can be solved efficiently by, for example, MATLAB LMI Toolbox. Once B_Δ is found from (22), we can check the conditions in (21). If the conditions are not satisfied, we can increase N to add more τ_i values in (22) and solve the problem again until the conditions in (21) are satisfied.

The next lemma shows that particular choice of τ_{nom} does not affect controllability and observability of $(\tilde{A}_{nom}, \tilde{B}_{nom}, \tilde{C})$.

Lemma 4: If assumptions (P1) and (P2) are satisfied, $(\tilde{A}_{nom}, \tilde{B}_{nom})$ is controllable and $(\tilde{C}, \tilde{A}_{nom})$ is observable for all $\tau_{\min} \leq \tau_{nom} \leq \tau_{\max}$.

Using (19) and (20), we derive an upper bound of $\|T_{zw}\|_2$ in the next lemma.

Theorem 1: Let F_1, F_2, $F_{1,cl}$, and $F_{2,cl}$ be defined by

$$F_1 \triangleq \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, \quad F_2 \triangleq \begin{bmatrix} 0 & 0 & I \end{bmatrix}$$

$$F_{1,cl} \triangleq \begin{bmatrix} F_1 \\ 0 \end{bmatrix}, \quad F_{2,cl} \triangleq \begin{bmatrix} F_2 - D_c \bar{C}_2 & -C_c \end{bmatrix}.$$

If (A_c, B_c, C_c, D_c) satisfies (13) and there exist $P = P'$ and $W = W'$ satisfying

$$\begin{bmatrix} -P & P S_2' A_{cl,nom} T_2 & P S_2' B_{cl} & P S_2' F_1 & 0 \\ T_2' A_{cl,nom}' S_2 P & -P & 0 & 0 & T_2' F_2' \\ B_{cl}' S_2 P & 0 & -I & 0 & 0 \\ F_1' S_2 P & 0 & 0 & -B_\Delta^{-1} & 0 \\ 0 & F_2 T_2 & 0 & 0 & -I \end{bmatrix} < 0 \quad (23)$$

$$\begin{bmatrix} P & T_2' C_{cl}' \\ C_{cl} T_2 & W \end{bmatrix} > 0, \tag{24}$$

then the system is comprehensively stable and

$$\|T_{zw}\|_2^2 \leq \mathrm{Tr}(W) \tag{25}$$

for all $\tau_{\min} \leq \tau_k \leq \tau_{\max}$.

Now we are ready to derive an H_2 controller in the next theorem.

Theorem 2: If there exist $X = X'$, $Z = Z'$, $\hat{A}$, $\hat{B}$, $\hat{C}$ and $\hat{D}$ satisfying

$$\begin{bmatrix} -X & -u_0^{\perp\prime} & (1,3) & (1,4) & u_0^{\perp\prime} B_1 & u_0^{\perp\prime} F_1 & 0 \\ \star & -Z & \hat{A} & (2,4) & ZB_1 & ZF_1 & 0 \\ \star & \star & -X & -u_0^{\perp\prime} & 0 & 0 & X u_0^{\perp\prime} F_2' + \hat{C}' \\ \star & \star & \star & -Z & 0 & 0 & u_0^{\perp} u_0^{\perp\prime} F_2' + u_0 u_1' - u_0^{\perp} u_0^{\perp\prime} \bar{C}_2' \hat{D}' \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -B_\Delta^{-1} & 0 \\ \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} X & u_0^{\perp\prime} & X u_0^{\perp\prime} \bar{C}_1' \\ \star & Z & u_0^{\perp} u_0^{\perp\prime} \bar{C}_1' \\ \star & \star & W \end{bmatrix} > 0 \tag{27}$$

where

$$\begin{aligned} (1,3) &\triangleq u_0^{\perp\prime} \bar{A}_{nom} u_0^{\perp} X - u_0^{\perp\prime} \bar{B}_{2,nom} \hat{C} \\ (1,4) &\triangleq u_0^{\perp\prime} \bar{A}_{nom} u_0^{\perp} u_0^{\perp\prime} - u_0^{\perp\prime} \bar{B}_{2,nom} u_1 u_0' \\ &\quad + u_0^{\perp\prime} \bar{B}_{2,nom} \hat{D} C_2 \\ (2,4) &\triangleq Z \bar{A}_{nom} u_0^{\perp} u_0^{\perp\prime} - Z \bar{B}_{2,nom} u_1 u_0' + Z u_0 u_0' \\ &\quad + \hat{B} C_2 \end{aligned}$$

then there exists a controller (A_c, B_c, C_c, D_c) satisfying (13), (23) and (24).

Lemma 5: If (26) is satisfied, $(-X + u_0^{\perp\prime} M')$ and $(u_0^{\perp} N + U)$ are nonsingular.

Controllers can be computed as stated in the proof of Theorem 2 and details are omitted.

III. Experiment

To verify the proposed controller, a simple networked control system of a DC motor is constructed. Fig. 1 shows our experiment system. The system consists of five DSP boards (TMS320F241 DSP), which are connected through the CAN network. The CAN network is one of most popular fieldbus networks with relatively small packet size [1]. Only two boards (sensor board and controller board) are actually involved in the DC motor control and the others are used for dummy traffic generation and monitoring.

The sensor board sends the motor speed to the controller board through CAN network with the period $T = 4ms$. The controller board generates control commands and the DC motor is controlled by the motor driver board. Note that the controller board and the motor driver board is hard-wired.

The transmission speed of the CAN network is 1 Mbps and physical transmission of one message packet is about 150 μs. Dummy board 1 and 2 are used to generate dummy message packets. The priority of dummy message packets is higher than that of sensor message packets. Thus sensor message packet transmission is delayed when there are dummy message packets. Sensor data transmission delay τ is time-varying depending on dummy message generation rate. Note that network traffic load is arbitrarily specified by adjusting dummy message generation rate.

One example of τ is given in Fig. 6, where $\tau_{min} = 0.15ms$ and $\tau_{max} = 2.5ms$. Note that we can change τ_{max} by adjusting dummy message generation rate.

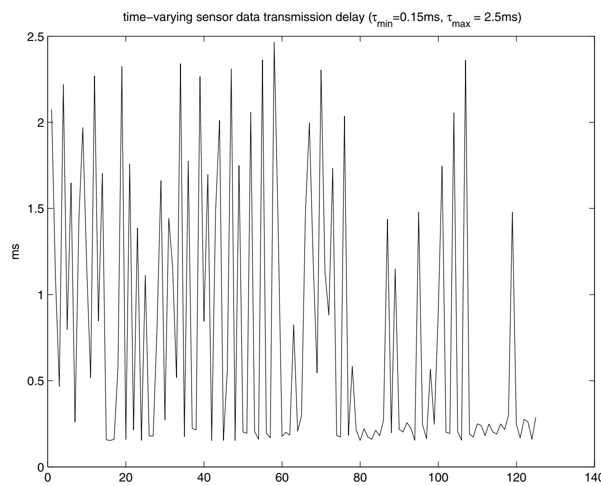

Fig. 6. Time-varying delay of networked control systems

The DC motor is modeled as 1st order system as follows:

$$\begin{aligned} \dot{x}(t) &= -28.48x(t) + 40.14u(t) \\ y(t) &= 40.14x(t). \end{aligned}$$

This model is derived from a standard DC model with parameters from experiments. The dimension is reduced using the balanced realization.

H_2 servo controller is obtained for different τ_{max} values with $\beta = 100$. The computed H_2 norm and τ_{nom} are given in Table I.

τ_{max}	H_2 norm	τ_{nom}
0.6ms	4.1614	0.372 ms
0.9ms	4.2029	0.521 ms
1.5ms	4.3769	0.818 ms
2.5ms	4.9331	1.3 ms

TABLE I

H_2 NORM AND τ_{nom} VALUES FOR DIFFERENT τ_{max}

The performance (H_2 norm) is degraded as τ_{max} increases; this is not surprising since the system is generally more difficult to control if there is larger delay in the control loop.

As an example of a controller, the controller computed for $\tau_{max} = 2.5ms$ is given as follows:

$$\begin{aligned}
\zeta_{k+1} &= \begin{bmatrix} 0.3654 & -21.6549 & 5.3818 \\ -0.0093 & 0.5033 & -0.1724 \\ 0.0064 & -0.3899 & 0.0870 \end{bmatrix} \zeta_k \\
&\quad + \begin{bmatrix} -0.9691 \\ -0.0211 \\ -0.0257 \end{bmatrix} e_k \\
u_k &= \begin{bmatrix} -0.0064 & 0.3863 & -0.0911 \end{bmatrix} \zeta_k \\
&\quad +0.0214 e_k
\end{aligned}$$

Note that poles of the controller are $1.0000, -0.0005, -0.0438$. As expected, the controller has pole at 1; i.e., the controller has a discrete time integrator.

Step response of different τ_{max} values are given in Table II. It can be seen that the step response are relatively insensitive to time-varying delays for all cases.

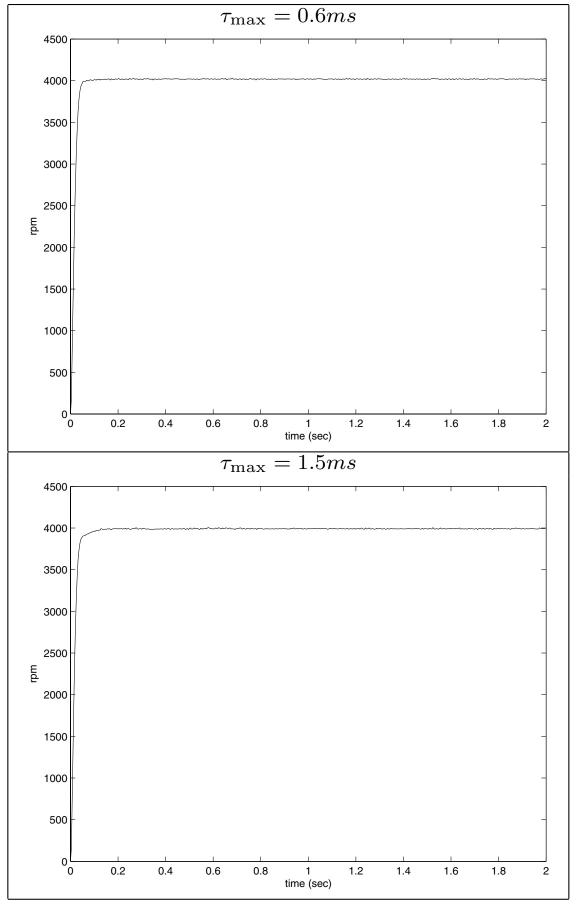

TABLE II

STEP RESPONSES FOR DIFFERENT τ_{max} VALUES

IV. CONCLUSION

In this paper, we proposed a servo controller for networked control systems with time-varying delays. In networked control systems, there is inevitable time delay in data transmission and the delay in many cases is time-varying depending on the network delay. The proposed servo controller guarantees the closed-loop stability for all time-varying delays belonging to a certain interval. As the performance index, H_2 norm is used. The controller can be computed easily by solving linear matrix inequalities.

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