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Backstepping Control Synthesis for Automated Low Speed Vehicle

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***Abstract*— This paper proposes and evaluates a lateral control law for an automated vehicle. Based on a simple model of coupled longitudinal and lateral modes of a vehicle, we present a solution for the vehicle following by using the backstepping control technique. The performance and robustness of the control law are highlighted by the simulation of various maneuvers.**

***Index Terms*— lateral, longitudinal dynamics, backstepping, robustness, lyapunov.**

NOMENCLATURE

v_x/v_y	Longitudinal/lateral vehicle speeds
T_{net}	Equivalent drive and brake torque
T_{rr}	Rolling resistance torque
C_x/C_y	Longitudinal/lateral aerodynamic drag coefficients
m	Vehicle mass
I_{eff}/I_z	Effective longitudinal inertia/ inertia moment about the yaw axis through the vehicle center of gravity
F_{yf}/F_{yr}	Cornering forces at the front tires/rear tires
L_1/L_2	Distances of the front and rear tires from vehicle's center of gravity
α_f/α_r	Slip angle of the front and rear tires
c_f/c_r	Cornering stiffness of the front and rear tires
δ_f	Steering angle
l_c	Distance from the vehicle center of gravity to the vehicle mounted sensor

I. INTRODUCTION

Nowadays, the vehicle becomes the means of transport the most widespread. It offers several benefits for daily displacements such as comfort, availability and travel time. This is justified by the number of vehicles which frequently circulate around major cities [1]. However, this increasing traffic leads to traffic jams [2], accidents and a driver stress. In order to overcome these drawbacks, a possible solution is to act on the infrastructure by creating additional lanes, but the cost is very high and the available place is limited. Thus another possible solution to increase passengers safety, to reduce the driver's stress [3], and to improve traffic conditions is to adopt an automated driving even at reasonable speed of around 60km/h. In fact the daily travel distances are generally less than 30km/h, and a travel time of about half an hour is acceptable. Among the various tasks that the vehicle has to perform are lateral and longitudinal controls. The first one is the positioning of the vehicle on the road which consists in acting on the steering angle, the second consists in maintaining the safety distance to the preceeding vehicle by generating acceleration or braking.

This paper deals with the synthesis of a control law that makes a vehicle perform a car-following maneuver. It is organized as follows : In section 2, a nonlinear model of the vehicle is presented, it includes longitudinal and lateral dynamics when both are coupled. Section 3 gives the equations of relative positioning between follower vehicle and leader vehicle in both longitudinal and lateral directions. These equations will then be used for the control synthesis in section 4. The purpose of this control law is to make the controlled vehicle follow the leader by generating suitable steering angle. The adopted method is based on the application of the backstepping technique whose choice is motivated by the presence of strong non-linearities in the considered model. The vehicle handling is uncertain (in particular with respect to the state of the roadway : dry, softened, snow-covered, icy...) and can be very disturbed. Then the analysis of the control law robustness is performed with respect to these various elements. This is essential in order to guarantee a satisfactory behavior for different conditions.

II. VEHICLE MODELING

This section allows to introduce the model of the vehicle used for control synthesis. In the following, the indices (l,s,r) will respectively designate the leader vehicle, the follower vehicle and the relative variables. The road is supposed plane with no gradient, no superelevation, and a simplified dynamic model is used with the following assumptions :

1) The pitch, the roll and the vertical dynamics are neglected.
2) The vehicle side slip angle is small.
3) Only translations in the longitudinal and lateral directions and the yaw rotation are allowed.
4) All angles are sufficiently small in order to allow linear approximation.

Under the above assumptions, the overall motion of the vehicle can be effectively described by three equations [7] :

$$\begin{cases} \dot{v}_{x_s} = \frac{T_c - T_{rr}}{I_{eff}} - \frac{C_x v_{x_s}^2}{m} + v_{y_s}\dot{\psi}_s \\ \dot{v}_{y_s} = \frac{F_{y_f} + F_{y_r} - C_y v_{y_s}^2}{m} - v_{x_s}\dot{\psi}_s \\ \ddot{\psi}_s = \frac{1}{I_z}\left[l_f F_{y_f} - l_r F_{y_r} \right] \end{cases}$$

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where the first one characterizes the longitudinal dynamics, the second one represents the dynamics of the lateral motion and the last one describes the yaw motion [4].The quantities $v_{y_s}\dot{\psi}_s$ and $-v_{x_s}\dot{\psi}_s$ denote the coupling terms (see table 1 for definitions of all paarmeters). Longitudinal motion is considered without slipping, controlled by the composite torque, however the lateral motion is controlled by the cornering tire forces which are computed based on the fourth assumption :

$$\begin{cases} F_{y_f} = 2c_f\alpha_f \\ F_{yr} = 2c_r\alpha_r \end{cases}$$

where the side slip angles are given by :

$$\begin{cases} \alpha_f = \delta_f - \frac{v_{y_s}+l_f\dot{\psi}_s}{v_{x_s}} \\ \alpha_r = -\frac{v_{y_s}-l_r\dot{\psi}_s}{v_{x_s}} \end{cases}$$

The model can thus be written in the canonical form :

$$\dot{v}_{x_s} = f_0 \tag{1}$$

$$\dot{v}_{y_s} = f_1 + g_1\delta_f \tag{2}$$

$$\ddot{\psi}_s = f_2 + g_2\delta_f \tag{3}$$

where

$$\begin{cases} f_0 = \frac{T_c - T_{rr}}{I_{eff}} - \frac{C_x v_{x_s}^2}{m} + v_{y_s}\dot{\psi}_s \\ f_1 = \frac{-2\,c_f}{m}\frac{v_{y_s}+l_f\dot{\psi}_s}{v_{x_s}} - \frac{2\,c_r}{m}\frac{v_{y_s}-l_r\dot{\psi}_s}{v_{x_s}} - \frac{C_y v_{y_s}^2}{m} - v_{x_s}\dot{\psi}_s \\ g_1 = \frac{2c_f}{m} \\ f_2 = \frac{-l_f}{I_z}2c_f\frac{v_{y_s}+l_f\dot{\psi}_s}{v_{x_s}} + \frac{l_r}{I_z}2c_r\left(\frac{v_{y_s}-l_r\dot{\psi}_s}{v_{x_s}}\right) \\ g_2 = \frac{l_f}{I_z}2c_f \end{cases} \tag{4}$$

III. ABSOLUTE AND RELATIVE VEHICLE POSITIONING

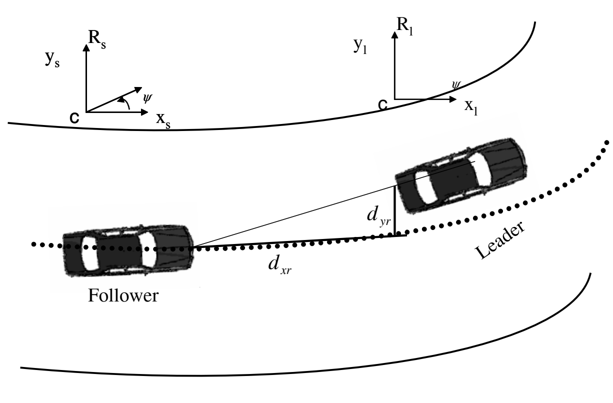

Fig. 1. Relative positioning of vehicles

In order to make the design and the analysis of the control strategies easier, we have to rewrite our system of equations using the lateral deviation and the spacing between the follower vehicle and the preceding vehicle. According to the figure 1, the absolute positions and the yaw angle of the follower vehicle are given by :

$$\begin{cases} \psi_s = \int_0^t \dot{\psi}_s d\tau + \psi_s(0) \\ x_s(t) = \int_0^t (v_{x_s}\cos\psi_s - v_{y_s}\sin\psi_s)\,d\tau \\ y_s(t) = \int_0^t (v_{x_s}\sin\psi_s + v_{y_s}\cos\psi_s)\,d\tau \end{cases}$$

where $\psi_s(0)$ represents the initial yaw angle.

For the leader vehicle, we obtain the triplet (x_l, y_l, ψ_l) by writing the relative positions d_{xr} and d_{yr} :

$$\begin{bmatrix} d_{xr} \\ d_{yr} \end{bmatrix} = R(\psi_l)\left(\begin{bmatrix} x_s + l_f\cos\psi_s \\ y_s + l_f\sin\psi_s \end{bmatrix} - \begin{bmatrix} x_l - l_r\cos\psi_l \\ y_l - l_r\sin\psi_l \end{bmatrix}\right)$$

where $R(\psi_l)$ is the rotation matrix with angle ψ_l.The derivative of the previous equation is given by :

$$\begin{cases} \dot{d}_{xr} = -v_{x_l} + d_{yr}\dot{\psi}_l + v_{x_s}\cos\psi_r + \left(v_{y_s} + \dot{\psi}_s l_f\right)\sin\psi_r \\ \dot{d}_{yr} = -v_{y_l} + (l_r - d_{xr})\,\dot{\psi}_l - v_{x_s}\sin\psi_r + \\ (v_{y_s} + \dot{\psi}_s l_f)\cos\psi_r \end{cases} \tag{5}$$

IV. CONTROL LAW CONCEPTS

Backstepping is a control method based on a recursive procedure approach. It is used for linear or nonlinear systems [8]. It is a sequential and systematic way to build a stabilizing Lyapunov function.

Let us give the following system :

$$\dot{x} = f(x) + g(x)u \tag{6}$$

where x is the state of the system, u is the control input, f and g are smooth vector fields of appropriate dimension.

It is supposed that there exists for the system (6) a control law $u = \alpha(x)$ and a positive definite unbounded function $V_1 : R^n \to R$ such that :

$$\frac{\partial V_1}{\partial x}(x)[f(x) + g(x)u(x)] \le -W(x) \le 0, \quad \forall x \in R^n \tag{7}$$

where W $R^n \to R$ is a positive semidefinite

Backstepping approach [9] consists in extending the system with an integrator which leads to :

$$\dot{x} = f(x) + g(x)\zeta \tag{8a}$$

$$\dot{\zeta} = u \tag{8b}$$

where ζ is regarded as a virtual input, and the virtual control law $\alpha(x)$ is conceived (6).with the Lyapunov function $V_1(x)$. At this stage, one has defined a new variable which is the error between the actual input and the desired one : $z = \zeta - \alpha(x)$. We can obtain a new system of equations :

$$\dot{x} = f(x) + g(x)(\alpha(x) + z) \tag{9}$$

$$\dot{z} = \dot{\zeta} - \dot{\alpha} = u - \frac{\partial\alpha}{\partial x}(x)[f(x) + g(x)(\alpha(x) + z)] \tag{10}$$

Lemma 1: [6] Let the system (8) satisfies(7) with $\zeta \in \mathbb{R}$ as its control.

If $W(x)$is positive definite, then

$$V_a(x,\zeta) = V(x) + \frac{1}{2}[\zeta - \alpha(x)]^2$$

is a control lyapunov function for the full system (8), that is, there exists a feedback control $u = (x, \zeta)$ which renders $x = 0, \zeta = 0$ the Global Asymptotic Stablity equilibrium of (8).
One such control is:

$$u = -k(\zeta - \alpha(x)) + \frac{\partial \alpha}{\partial x}(x)[f(x) + g(x)] - \frac{\partial V_1}{\partial x}(x)g(x), k > 0$$

V.

VI. Synthesis procedure

The goal is to control the lateral deviation by using the steering angle of the front wheels in order to make the vehicle follow the leader [10]. The lateral variation has to tend to the desired trajectory noted d_{yrdes}.

A. Step1

At the beginning, the first error that has to be cancelled is :

$$z = d_{yr} - d_{yrdes} \tag{11}$$

From (5), the time derivative of the lateral deviation can be written as :

$$\dot{d}_{yr} = f_3 + g_3 v_{ys} \tag{12}$$

with

$$\begin{cases} f_3 = -v_{y_l} + (l_r - d_{xr})\,\dot{\psi}_l - v_{x_s}\sin\psi_r + \dot{\psi}_s l_f \cos\psi_r \\ g_3 = \cos\psi_r \end{cases}$$

For that, one chooses a Lyapunov function V_1 as follows :

$$V_1 = \frac{1}{2}z^2 \tag{13}$$

The derivative of this function leads to :

$$\dot{V}_1 = \dot{z}z = (f_3 + g_3 v_{ys} - \dot{d}_{yrdes})z \tag{14}$$

One poses :

$$\zeta = v_{ys} \tag{15}$$

The condition for which z tends towards zero is that $\dot{V}_1$ must be negative definite as presented below :

$$\dot{V}_1 = -k_1 z^2 \leq 0 \tag{16}$$

with k_1, a positive constant. The control input does not appear in $\dot{V}_1$ yet, and in order to check (16), one defines another virtual input α that takes the following form :

$$\alpha = \frac{-f_3 + \dot{d}_{yrdes} - k_1 z}{g_3} \tag{17}$$

B. Step2

Now, the second step consists in redefining another error to be cancelled :

$$z_1 = (\zeta - \alpha)$$

One proceeds as previously and builds a Lyapunov function :

$$V_2 = \frac{1}{2}z_1^2 \tag{18}$$

Its derivative is :

$$\dot{V}_2 = \dot{z}_1 z_1$$

$$\dot{V}_2 = (\dot{\zeta} - \dot{\alpha})z_1$$

As refering to the equation (17), the derivative of α is given by :

$$\dot{\alpha} = \frac{(-\dot{f}_3 - k_1\dot{z} + \ddot{d}_{yrdes})g_3 - \dot{g}_3(-f_3 + \dot{d}_{yrdes} - k_1 z)}{(g_3)^2}$$

In the following, d_{yrdes} will be considered as constant. It is necessary to express $\dot{\alpha}$ such that :

$$\dot{\alpha} = C + D\delta_f$$

Where

$$\begin{cases} C = \frac{(\dot{v}_{y_l} + \dot{d}_{xr}\dot{\psi}_l + (l_r - d_{xr})\ddot{\psi}_l + f_0 \sin\psi_r + v_{x_s}\dot{\psi}_r\cos\psi_r)g_3}{(g_3)^2} \\ + \frac{(-f_2 l_f \cos\psi_r + \dot{\psi}_s\dot{\psi}_r l_f \sin\psi_r)g_3 - k_1\dot{z}g_3 - (-f_3 + d_{yrdes} - k_1 z)\dot{g}_3}{(g_3)^2} \\ D = \frac{g_2 l_f \cos\psi_r}{g_3} \end{cases} \tag{19}$$

The derivative of the Lyapunov function :

$$\dot{V}_2 = (\dot{\zeta} - \dot{\alpha})z_1 \tag{20}$$

leads to :

$$\dot{V}_2 = -k_2 z_1^2 \leq 0$$

with $k_2 > 0$, and where we have replaced the expressions of (2) and (19) in (20), the real control input δ_f takes then the following form :

$$\delta_f = -\frac{1}{(g_1 - D)}(f_1 - C + k_2 z_1) \tag{21}$$

Constants k_1 and k_2 are adjusted in order to achieve a good compromise between performances in terms of speed and comfort of the vehicle passengers.

VII. Simulation results

It is important to remind that the considered car-following problem is dedicated to low speed automation in congested sub-urban highways. The developed longitudinal and lateral control laws have to cover all operational speeds including stop-and-go traffic conditions. The speed is however limited to 60 km/h for safety reasons. The maneuvers of the leader include acceleration, braking and lane change.

A. Lane change maneuver

The design of the control law approach previously presented in (21) is tested for the following scenario with these initial conditions and goals :

- The leader vehicle is located on the right lane and evolves with a speed of 60 km/h. Its initial longitudinal position is at $x_l(0) = 20m$ and goes straight ahead.
- The follower has the same speed than the leader and is located on the adjacent left lane at a distance of 3 m. Initial longitudinal position is $x_s(0) = 0m$.
- The goal of the follower is to autonomously execute a lane change maneuver in order to follow the trajectory of the preceeding vehicle.

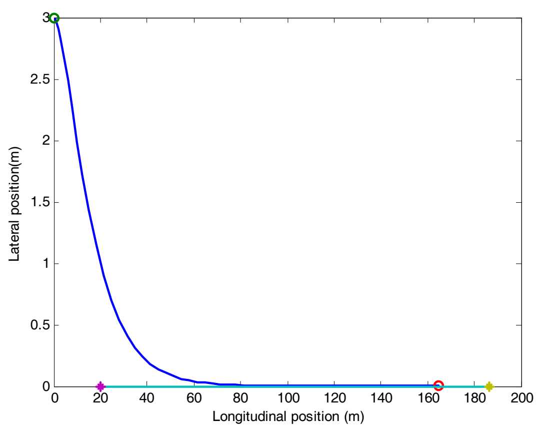

Fig. 2. Cut in maneuver of the follower lateral and longitudinal positions of the leader (*,-.) and the follower (o,-)

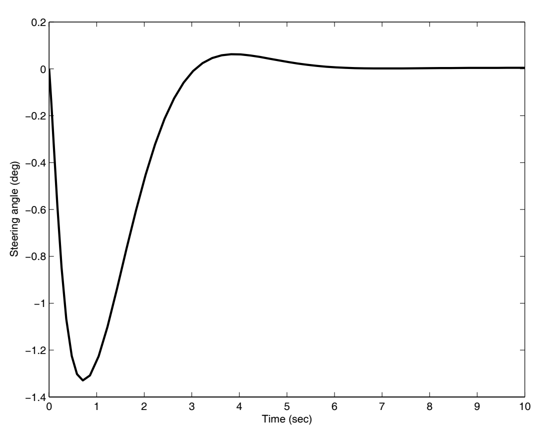

Fig. 3. Lane change maneuver : follower vehicle steering angle

Figure 2 shows the evolution of both vehicles in the (x,y) plane. It can be noticed that the follower performs the lane change maneuver and then follows the trace of the leader with neglected steady state error. One wanted to show, by referring to the figures 4 and 5, which provide the lateral speed and the lateral acceleration, one can notice that the lane change maneuver is carried out with success

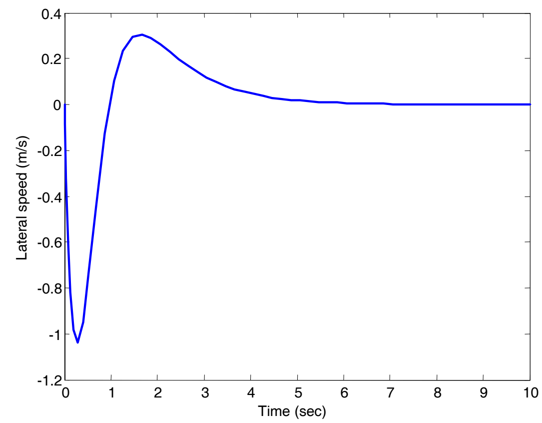

Fig. 4. Lane change maneuver : follower vehicle lateral speed

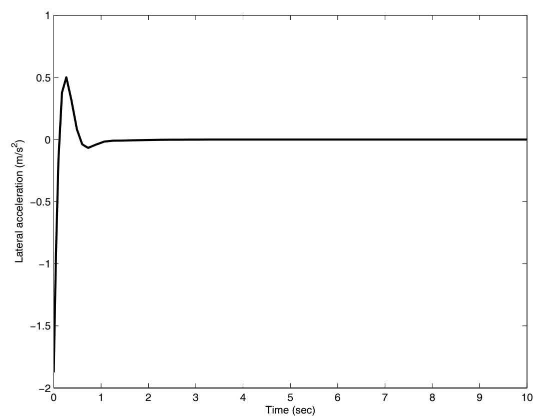

Fig. 5. Lane change maneuver : follower vehicle lateral acceleration

with respect to the passengers comfort where the reasonable boundaries on acceleration range are from 0 to $0.3g$.

B. Heading variation

This scenario is summarized below :

- The leader vehicle evolves with same speed as previously and is located on the right lane. It starts its trajectory at $x_l(0) = 20m$ and begins a heading change at distance $x_l = 60m$ with a yaw angle $\psi_l = 0.1rad$.
- The follower vehicle is initially on the same lane at a longitudinal position $x_s(0) = 0m$.

Figure 6 also shows that the follower vehicle reacts with effectiveness but with a transient error reaching 20 cm and that is quickly cancelled.

C. Combined maneuver

One now gives another scenario which combines both previous scenarios. This combined maneuver is subdivided into two maneuvers where at the beginning of the scenario, the follower vehicle makes a lane change and then follows the leader heading variation. Once again, the follower vehicle carried out a very good reaction as it is illustrated

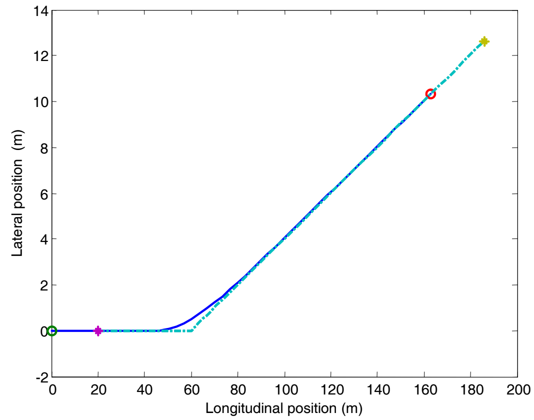

Fig. 6. Leader vehicle heading change : Positions of leader (*,-.) and the follower (o,-)

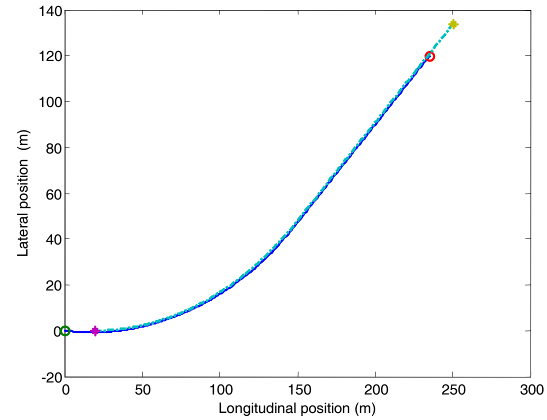

Fig. 8. Trajectory of the head and following vehicles (*,-.), (o,-) in nominal conditions

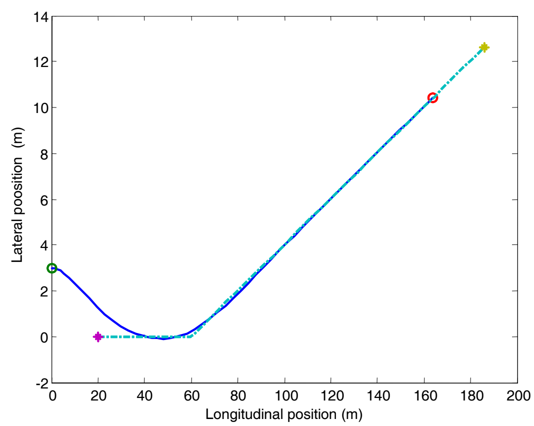

Fig. 7. Lane change maneuver combined with leader vehicle heading (*,-.) and the follower (o,-)

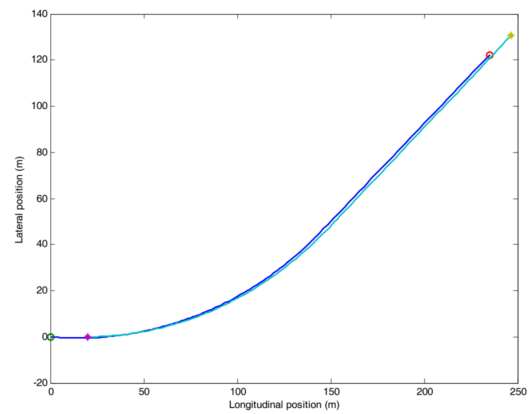

Fig. 9. Trajectory of the head and following vehicles (*,-.), (o,-) under road effect

in the figure 7. It is also noted that the steady state error is near zero.

VIII. Robustness

The results previously obtained in simulation are very promising. However following analysis deals with the evaluation of the robustness of the control law with respect to parametric uncertainties such as adhesion reduction, leader behavior. The analysis of the robustness of the control law is thus essential to guarantee a satisfying behavior of the vehicle under different conditions. The test is as follows : The leader vehicle performs a trajectory characterized by the succession of circular arc of radius $200m$ and a straight line.

A. Road adhesion variation

In practice, the dynamic model used to determine the control law does not correspond exactly to the real dynamics of the system. The main source of errors is related to the fact that tire/road adhesion is unknown. However, we consider a degradation of adhesion from 70% to 20% of the nominal value, which corresponds physically to a wet and very slipping road.

Tests are carried out taking into account the adhesion variation. The results appear in the figures 8 and 9 and still show good vehicle tracking. However, as the road adhesion is reduced, the performance is gradually deteriorated. It is thus necessary to have an estimate of the road adhesion in order to cover all weather conditions. A simulation study revealed that a rough estimation of about 20% of accuracy is sufficient.

B. Longitudinal speed variation

In order to check the robustness of the control, one voluntarily chooses the variations of the leader vehicle longitudinal speed in order to observe the reaction and the evolution of the follower vehicle. The maneuver shows on the figure 10 a combination of three movements : the first one is a uniformly decelerated movement while decreasing the speed from 60 km/h to 20 km/h, the second one is

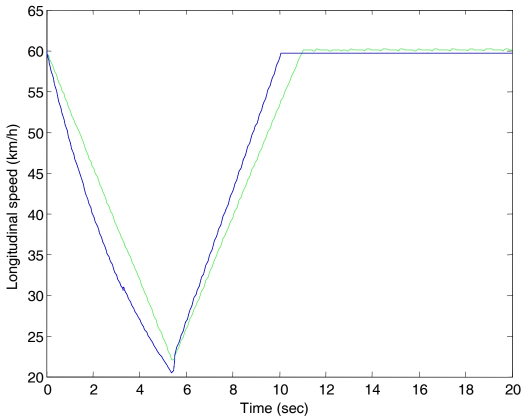

Fig. 10. Longitudinal speed variations: Leader(–,.), Follower(-)

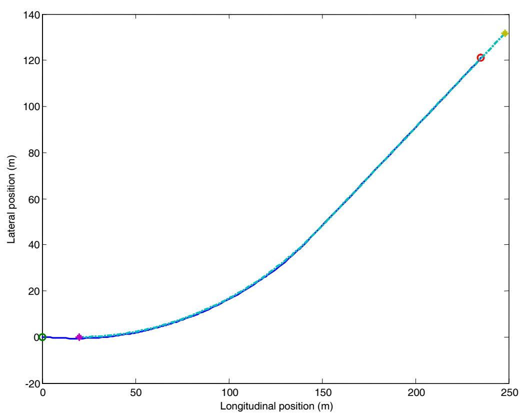

Fig. 11. Following vehicle profile with longitudinal variation: Leader (*,-.) and Follower (o,-)

a uniformly accelerated movement in order to reach 60 km/h; finally, the trajectory is finished with a constant speed of 60 km/h. One can note that longitudinal control is not carried out but one acts on the composite torque such as the follower moves with a profile similar to the leader vehicle.

The obtained results are satisfactory in response to this type of maneuver. One can note in figure 11 that the performances obtained in this case are similar to those obtained with a constant speed of 60 km/h. This proves the robustness of this control law face to the speed variations.

IX. CONCLUSION

In this paper, a car-following including lateral control is presented. We showed that it is possible to develop a non linear controller based on a backstepping procedure. Simulations have been performed to investigate the performance of the proposed control approach. Moreover we evaluated the robustness of this control law where the results seem very promising. The next step includes the implementation and tests in our prototype vehicle.

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Numerical values

$T_{rr} = 300N.m,$

$c_f = 57.5KN/rad, \quad c_r = 57.5KN/rad,$

$C_x = 0.35N.s^2m^{-2}, \quad C_y = 0.45N.s^2m^{-2}$

$l_f = 1m, \quad l_r = 1.5m$

$m = 1500kg, \quad I_z = 2500kg.m^2$

$I_{eff} = 0.3 \times m,$

$k_1 = 2, \quad k_2 = 5$