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Yaw Rate Estimation Using Two 1-Axis Accelerometers

Wonshik Chee, PhD.

Abstract— **This paper presents a new way of estimating yaw rate of a vehicle using two 1-axis accelerometers. Measurement of yaw rate is inevitable for vehicle stability control systems adopted in contemporary high performance sedans and for the automatic steering control systems for intelligent vehicles. Compared with accelerometers, the cost of rate gyros used to measure the yaw rate is significantly higher. As an economical software solution to replace the high-cost rate gyros with minimal additional hardware, an algorithm to estimate yaw rate using two 1-axis accelerometers is proposed in this paper.**

I. Introduction

YAW rate is one of the key information for vehicle directional stability control systems as well as other vehicle control applications. Especially, yaw rate measurement can significantly improve the performance of the lateral motion control system for intelligent vehicles.

Typically, rate gyros are used to measure the yaw rate. However, the cost of the rate gyros is more expensive compared with other sensors such as accelerometers. As a low cost alternative to the yaw rate gyro, an algorithm to estimate the yaw rate using two 1-axis accelerometers is presented here.

There have been two attempts to estimate yaw rate using several accelerometers ([2],[3]). Shimada et al. [2], and other attempts listed in [3] proposed algorithms based on vehicle kinematics. Therefore, these methods are sensitive measurement noise. An algorithm based on linear vehicle model was presented in [3]. However, due to the sensor locations, the algorithm presented in [3] is sensitive to tilt of the vehicle.

The algorithm proposed here overcomes the effect of the tilts by installing the accelerometers longitudinally. Using the two lateral acceleration measurements, rate change of the yaw rate can be found. Then, the estimation of the yaw rate and the slip velocity can be obtained by designing a state estimation algorithm.

Organization of this paper is as follows. Following the introduction, Description of the Vehicle Model to be used for the estimation design is presented. Then, the main idea of the proposed algorithm is discussed. Analytical analysis of the effect of the tilt of sprung mass of the vehicle generated either by rolling motion or by super elevation follows the main idea description. Finally, simulation results and conclusions will follow.

Wonshik Chee, PhD is an assistant professor of The University of Texas, San Antonio, 6900 N. Loop 1604 W., San Antonio, TX 78249. Phone: 210-458-6478; Fax: 210-458-6504; e-mail: wonshik.chee@utsa.edu.

II. Description of the Vehicle Model

For the algorithm development, the bicycle model [4] is used. The bicycle model represents vehicle motions in a horizontal plane. The model is obtained by lumping the two front wheels into one imaginary front wheel and the two rear wheels into one imaginary rear wheel. In addition, suspension dynamics are neglected. Thus, roll, pitch, and heave motions are not included.

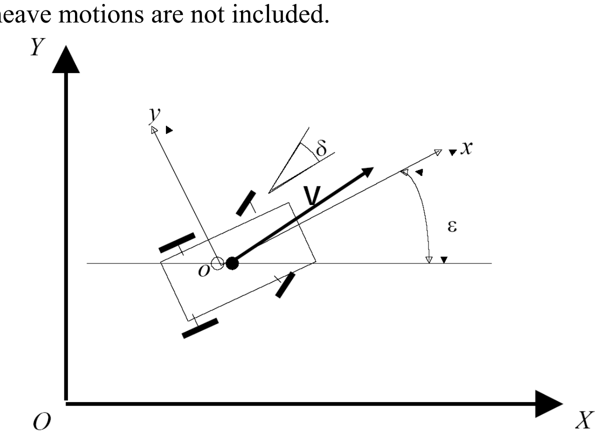

Fig. 1. Variables and Coordinates for Bicycle Model

Figure 1 presents key variables to describe the dynamics of the bicycle model, and the coordinates used for derivation of the equation of motion.

If we assume that the longitudinal speed (V_x) is constant, the equations of motion using the axes fixed to the vehicle (oxy) can be written as follows.

$$m\left(\dot{V}_y + V_x\dot{\varepsilon}\right) = f_f \cos\delta + f_r \tag{1}$$

$$I_z\ddot{\varepsilon} = l_1 f_f \cos\delta - l_2 f_r \tag{2}$$

In these equations, V_x denotes the speed in the direction of *ox* axis and V_y the one in the direction of *oy* axis. The velocity vector of a vehicle, **V**, is obtained from V_x and V_y. ε denotes the yaw angle with respect to the absolute coordinates, *OXY,* and δ represents the steering angle of the vehicle. f_f and f_r are the side forces for the front tire and the rear tire, respectively. Other symbols and their nominal values are listed in Table 1.

TABLE 1: VARIABLE AND PARAMETERS OF BICYCLE MODEL

δ	Front steering wheel angle
ε	Yaw angle of the vehicle
V_x	Longitudinal velocity of the vehicle (31.1*m*/sec)
V_y	Lateral velocity of the vehicle
C_s	Cornering stiffness (40000 *N/rad*)
m	Mass of the vehicle (1720 *kg*)
I_z	Moment of inertia of the vehicle (3250 kgm^2)
l_1	Distance from c.g. to front axle (1.137 *m*)
l_2	Distance from c.g. to rear axle (1.530 *m*)

The side force generated by each tire is a function of tire sideslip angle. The slip angle is defined as the angle between the direction of an object and the velocity vector of the object. If the object is a tire, we can obtain the tire sideslip angle as follows.

$$\alpha_f = \delta - \frac{l_1\dot{\varepsilon} + V_y}{V_x} \tag{3}$$

$$\alpha_r = \frac{l_2\dot{\varepsilon} - V_y}{V_x} \tag{4}$$

Here, α_f and α_r represent the front and the rear tire sideslip angle, respectively. If we assume the tire sideslip angles are small, the side forces are written as

$$f_f = 2C_s\alpha_f \tag{5}$$

$$f_r = 2C_s\alpha_r \tag{6}$$

Using the simplified tire model, the linearized version of the bicycle model can be written as follows.

$$\dot{V}_y = a_1V_y + a_2r + b_1\delta \tag{7}$$

$$\dot{r} = a_3V_y + a_4r + b_2\delta \tag{8}$$

Here, r denotes the yaw rate of the vehicle, i.e. $r = \dot{\varepsilon}$. The coefficients of these two equations are defined as follows.

$$a_1 = -\frac{4C_s}{mV_x}, a_2 = -2C_s\frac{l_1 - l_2}{mV_x} - V_x,$$
$$a_3 = -2C_s\frac{l_1 - l_2}{I_zV_x},\ a_4 = -2C_s\frac{l_1^2 + l_2^2}{I_zV_x} \tag{9}$$

$$b_1 = 2\frac{C_s}{m},\ b_2 = 2l_1\frac{C_s}{I_z} \tag{10}$$

III. THE ALGORITHM

The proposed algorithm has been developed based on the configuration of the two 1-axis accelerometers as shown in Figure 2. Here, the two 1-axis accelerometers are assumed to be installed on the longitudinal centerline of the vehicle. One (Front Accelerometer) is installed in front of the C.G., and the other (Rear Accelerometer) is in rear of the C.G. as shown in Figure 2.

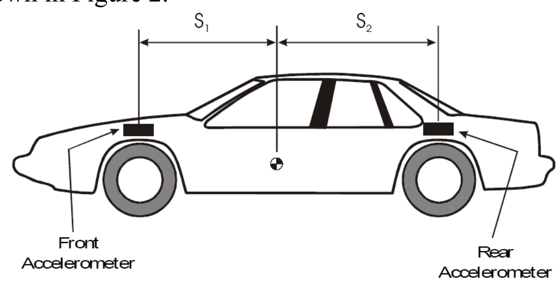

Fig. 2. Variables and Coordinates for Bicycle Model

The distance between Accelerometer 1 and CG is denoted as S_1, and the one between Accelerometer 2 and CG as S_2. Note that the choice of S_1 and S_2 can be arbitrary, but should not be zero.

The acceleration measured by Accelerometer 1 (a_{yf}) can be modeled as follows considering the kinematics.

$$a_{yf} = \dot{V}_y + V_xr + S_1\dot{r} \tag{11}$$

Also, the acceleration measured by Accelerometer 2 (a_{yr}) can be modeled as follows.

$$a_{yr} = \dot{V}_y + V_xr - S_2\dot{r} \tag{12}$$

Then, by subtracting equation (12) from equation (11), the following equation can be obtained.

$$\dot{r} = \frac{a_{yf} - a_{yr}}{S_1 + S_2} \tag{13}$$

Now, by taking equation (13) as the output of the system representing the lateral dynamics (equations (7) and (8)), one can design an observer as follows.

Also, note that even though S_1 and S_2 can be varied due to variation of the location of C.G., $S_1 + S_2$ is always constant unless the accelerometers are replaced.

THEOREM 1: Suppose that the output of the system represented by equations (7) and (8) is chosen as

$$Y = \frac{a_{yf} - a_{yr}}{S_1 + S_2} \tag{14}$$

Then, the system is observable for any nonzero vehicle speed and for any cornering stiffness if the weight distribution ratio is not 50:50.

(Proof) Recalling the equation (13) and (8), equation (14) becomes as follows.

$$Y = a_3 V_y + a_4 r + b_2 \delta \tag{15}$$

Then, observability matrix can be found as

$$\mathrm{O} = \begin{bmatrix} a_3 & a_4 \\ a_3 a_1 + a_4 a_3 & a_3 a_2 + a_4^2 \end{bmatrix}$$

Now, for the range of the velocity of the vehicle and the range of the cornering stiffness value, determinant of the observability matrix becomes nonzero since

$$\det(\mathrm{O}) = a_3^2 a_2 + a_3 a_4^2 - a_3 a_4 a_1 - a_3 a_4^2$$
$$= a_3 (a_2 a_3 - a_4 a_1) \neq 0$$

if the weight distribution ratio is not 50:50. Note that, if the weight distribution ratio is 50:50, a_3 becomes 0. Therefore, the system is observable. ■

THEOREM 2: Suppose that a state observer for the system represented by equations (7) and (8) using the output obtained as equation (14) is constructed as

$$\dot{\hat{V}}_y = a_1 \hat{V}_y + a_2 \hat{r} + b_1 \delta + L_1 \left(Y - \hat{Y}\right) \tag{16}$$

$$\dot{\hat{r}} = a_3 \hat{V}_y + a_4 \hat{r} + b_2 \delta + L_2 \left(Y - \hat{Y}\right) \tag{17}$$

where

$$\hat{Y} = a_3 \hat{V}_y + a_4 \hat{r} + b_2 \delta \tag{18}$$

Then, estimation errors, $V_y - \hat{V}_y$ and $r - \hat{r}$, converge to zero with proper choice of L_1 and L_2.

(Proof) Omitted.

IV. Effect of Roll and Superelevation

The proposed algorithm was developed by ignoring roll and super-elevation. However, the effect of the roll and the super-elevation can be analytically predicted.

Effectively, the roll angle and the super-elevation angle can be represented as lateral tilting angle of the sprung mass of the vehicle as shown in Figure 3 and Figure 4.

Suppose that the vehicle shown in Figure 3 experiences rolling motion whose magnitude is γ. Then, the lateral acceleration makes the same angle with the direction of the accelerometer measurement.

Now consider the situation shown in Figure 4. Here, the angle of the super-elevation is denoted as γ. Again, the direction of the lateral acceleration and the direction of the accelerometer make the same angle with the super-elevation angle. Therefore, for both cases, the acceleration measurement can be presented as follows.

$$\left(a_y\right)_{measured} = a_y \cos\gamma + g \sin\gamma \tag{19}$$

Here, a_y is the lateral acceleration of the vehicle, which is presented as either equation (11) or (12), and γ is the angle of the tilt which generated by either roll motion or Super-Elevation. $\left(a_y\right)_{measured}$ is the measured acceleration by the accelerometer.

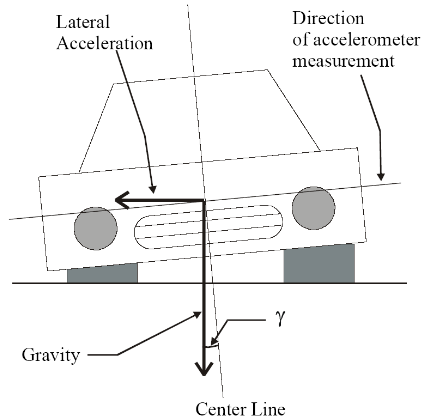

Fig. 3. Tilt due to roll motion

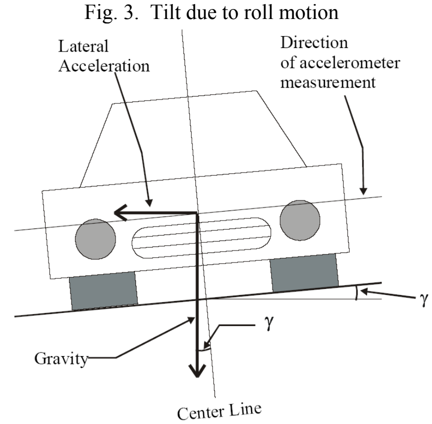

Fig. 4. Tilt due to Super-Elevation

THEOREM 3: If the vehicle tilts by γ (radians) and $\gamma = o(1)$, the measured rate change of yaw rate by equation (13) (or (14)) will have relative error in the order of γ^2.

(Proof) When the vehicle tilts, the measured accelerations presented as equations (11) and (12) become as follows.

$$\left(a_{yf}\right)_{measured} = \left(\dot{V}_y + V_x r + S_1 \dot{r}\right)\cos\gamma + g\sin\gamma$$
$$\left(a_{yr}\right)_{measured} = \left(\dot{V}_y + V_x r - S_2 \dot{r}\right)\cos\gamma + g\sin\gamma$$

Using these two equations, (14) becomes

$$\left(Y\right)_{measured} = Y\cos\gamma$$

Now, define the error of Y due to the tilt as

$$\left(Y\right)_{error} = Y - \left(Y\right)_{measured}$$

Then, the error becomes

$$(Y)_{error} = Y(1-\cos\gamma)$$

Since $\gamma = o(1)$, Taylor series of $1-\cos\gamma$ can be obtained as follow.

$$1-\cos\gamma = \frac{1}{2}\gamma^2 - \frac{1}{24}\gamma^4 + O(\gamma^6)$$

Therefore, the relative error can be

$$\frac{(Y)_{error}}{Y} = 1-\cos\gamma = \frac{1}{2}\gamma^2 - \frac{1}{24}\gamma^4 + O(\gamma^6)$$

or

$$\frac{(Y)_{error}}{Y} = O(\gamma^2) \quad \blacksquare$$

Note: For 10^o of the tilt, the relative error is $\frac{1}{2}\left(\frac{10\pi}{180}\right)^2 = 0.0152 = 1.52\%$. For 20^o of the tilt, the relative error is 0.0603 or 6.03%. Therefore, the error due to the tilt can be negligible for reasonable driving conditions.

V. Simulation Results

Simulation model has been developed using MATLAB/SIMULINK. For the simulation, discrete time version of the observer was derived by assuming zero order hold [1]. The sampling time was selected 20 msec.

The speed of the vehicle is selected as $31.1 m/\sec$, which is 70 MPH. The bandwidth of the observer is chosen as 1.5 Hz, and the damping ratio as 0.8. Sensor noise is modeled as a uniformed distributed random variable ranged over $[-0.2 m/s^2, 0.2 m/s^2]$.

The following steering angle, shown in Figure 5, is selected to validate the proposed algorithm.

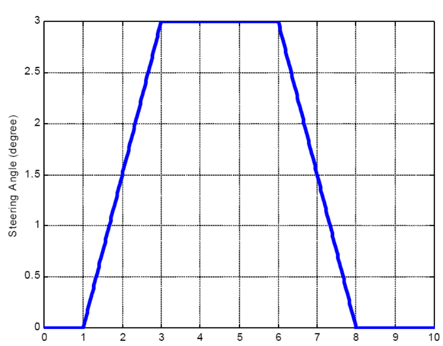

Fig 5: Steering Angle for Simulation

IDEAL CONDITION SIMULATION – Now, consider the case when there are no tilt and no sensor noise. Simulations of this case are shown in Figure 6, Figure 7 and Figure 8. Figure 6 shows the performance of yaw rate estimation. The upper plot shows the comparison between actual yaw rate and the estimated yaw rate. The lower plot shows the estimation error. From the lower plot, the maximum error is $1.1961\times10^{-3} rad/\sec$. The relative error is 0.4008%.

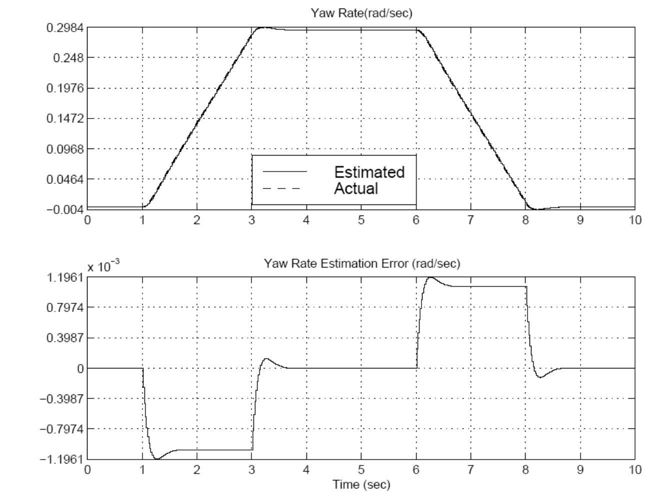

Fig 6: Yaw Rate Estimation when there is no tilt and no sensor noise.

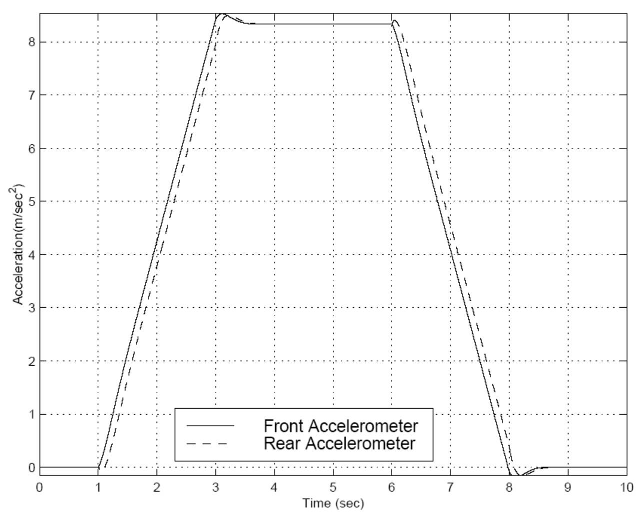

Fig 7: Acceleration of the accelerometers when there is no tilt and no sensor noise.

Figure 7 shows the measured accelerations from the accelerometers. The solid line represents the acceleration measured by the front accelerometer shown in Figure 6. The dashed line represents the acceleration measured by the rear accelerometer. Note that the difference between the two accelerometers is correlated with the slope of the yaw rate shown in Figure 6. The behavior confirms equation (13). In other words, the difference between the front accelerometer and the rear accelerometer is the slope of the yaw rate.

Figure 8 shows the estimation of the slip velocity. Note that slip velocity cannot be measured by any means. Since the proposed algorithm is a state estimation algorithm for the states of the bicycle model, the slip velocity is also obtained. The maximum estimation error for the slip velocity is $2.9753\times10^{-3} m/\sec$, or the maximum relative error 0.3649%.

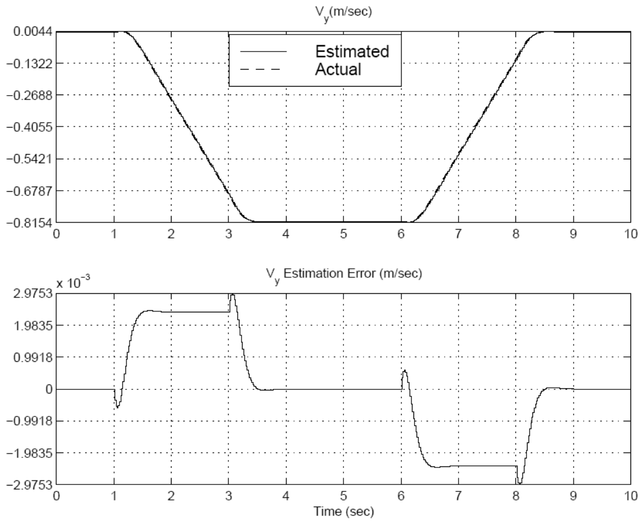

Fig 8: Slip Velocity Estimation when there is no tilt and no sensor noise.

Note that the estimation errors for both the yaw rate and the slip velocity are less than 0.5%. Therefore, the proposed algorithm can provide sufficient accuracy for other control applications such as vehicle stability control or lateral guidance.

EFFECT OF TILT – Now, investigate the effect of tilting of the sprung mass of the vehicle, generated either by rolling motion or super elevation. In order to simplify the simulation, it is assumed that the vehicle is maneuvered with specified tilting angle. Thus, the effect of the tilt can be modeled as presented in (19).

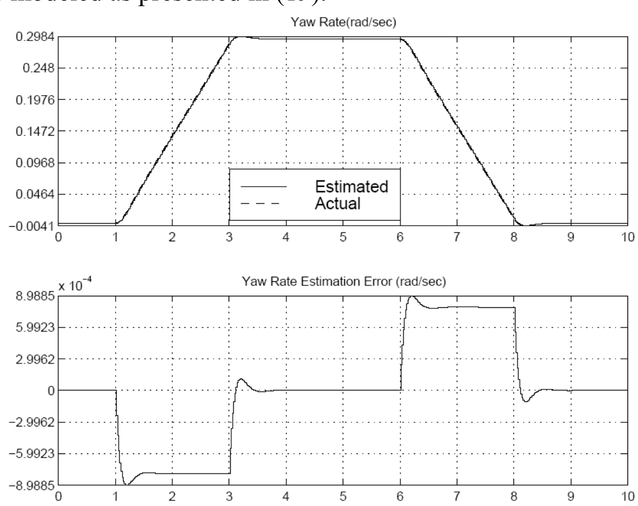

Fig 9: Yaw Rate Estimation when there is tilt and no sensor noise.

Figure 9 shows the yaw rate estimation when the tilting angle is 20^o. Note that 20^o of tilt angle is extremely large magnitude, and, usually, tilt due to rolling motion and/or the super-elevation is less than 10^o. Now, the maximum error is $8.9885\times10^{-4} rad/\sec$, or, relative error is 0.3012%. Note that the estimation error for this case is smaller than the ideal case. This confirms that the effect of the tilt on the proposed algorithm can be negligible for most cases.

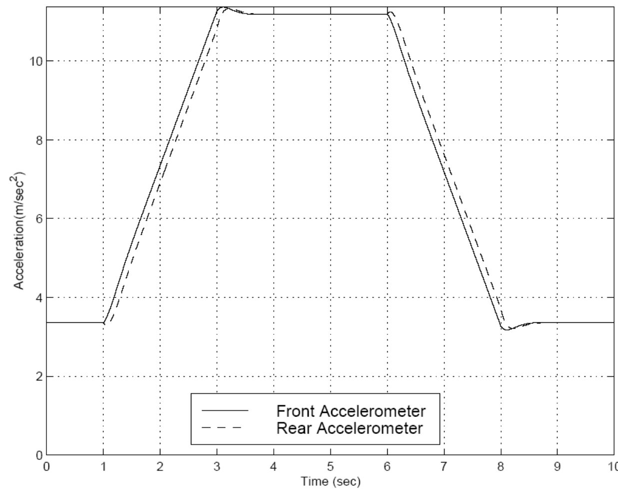

Fig 10: Acceleration of the accelerometers when there is tilt and no sensor noise.

Figure 10 shows the accelerations measured by the accelerometers. Note that, due to the tilt, the base line of the accelerations are not zero. The base line acceleration is $3.552 m/\sec^2$, which is equal to $9.81\times\sin 20^o$.

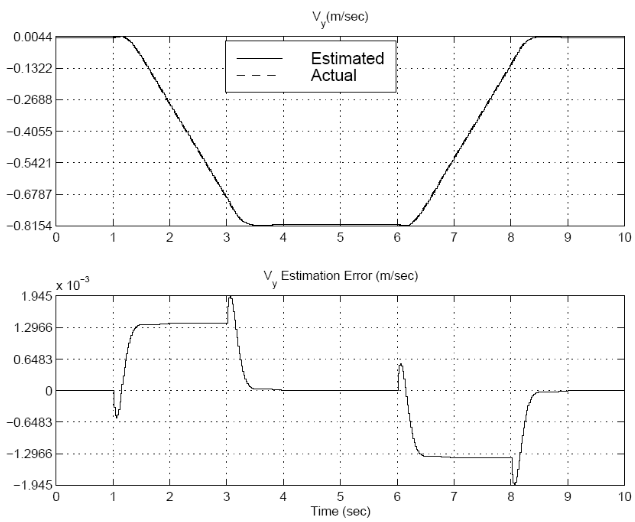

Fig 11: Slip Velocity Estimation when there is tilt and no sensor noise.

Figure 11 shows the slip velocity estimation and its estimation error. The maximum error is $1.945\times10^{-3} m/\sec$, or the relative error is 0.2385%.

From Figure 9, Figure 10 and Figure 11, the maximum relative estimation error is still less than 0.5%. Therefore, the proposed algorithm can be effective in spite of the tilt of the vehicle.

EFFECT OF TILT AND SENSOR NOISE – Now, examine the effect of the sensor noise on the performance of the estimation when the vehicle is tilted. Again, the tilt angle is chosen as 20^o. The sensor noise is modeled as uniform random variable whose range is $[-0.2m/s^2, 0.2m/s^2]$.

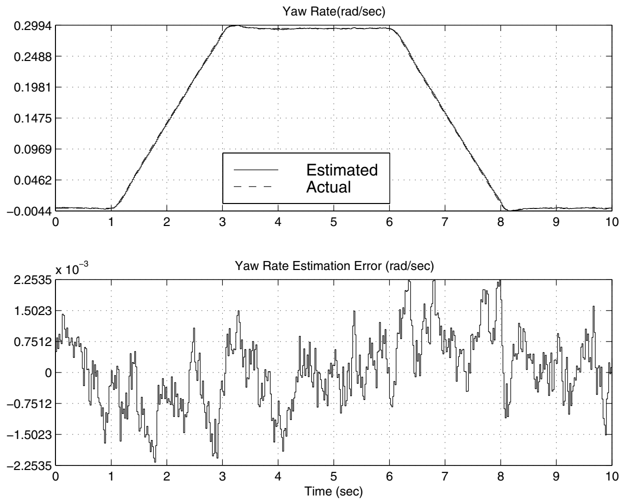

Fig. 12: Yaw Rate Estimation when there is tilt and sensor noise.

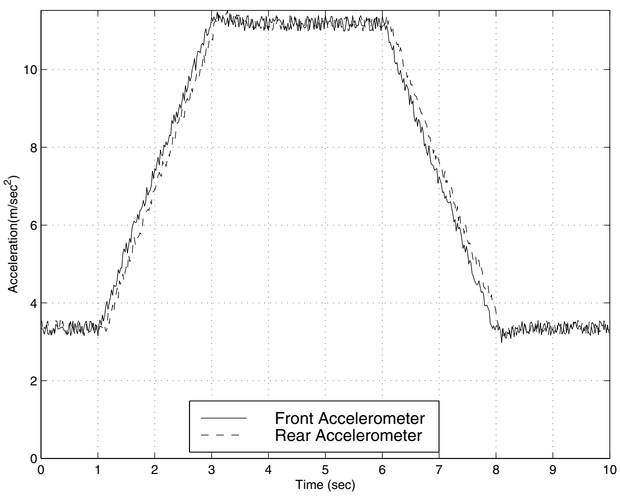

Fig. 13: Acceleration of the accelerometers when there is tilt and sensor noise.

Figure 12 shows the yaw rate estimation when the sensor noise is implemented. From this figure, the maximum estimation error can be found as $2.2535 \times 10^{-3} rad/\sec$, or relatively 0.7552%. Note that this quantity amounts to almost 2.5 times the case without sensor noise. Effect of the sensor noise can be reduced by lowering the bandwidth of the observer. If the lower bandwidth is attempted, the estimation will show significant phase lag and significant error when there is discrepancy between the actual system parameters and the model parameters used for the design of the estimation algorithm. Therefore, one cannot arbitrarily reduce the bandwidth. The measured accelerations are shown in Figure 13.

Figure 14 shows the estimation of the slip velocity. In this case, the maximum estimation error is $5.2104 \times 10^{-3} m/\sec$, or, relatively 0.6390%. Compared with the case without the sensor noise, the relative estimation error grows about 2.5 times. Based on the findings from Figure 12 and Figure 14, sensor noise is the dominant disturbance source of the estimation error.

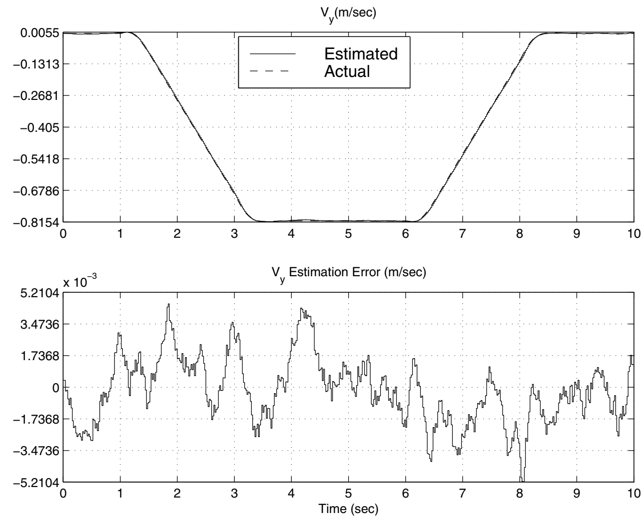

Fig. 14: Slip Velocity Estimation when there is tilt and is sensor noise.

VI. Conclusion

A new estimation algorithm to estimate the yaw rate of the vehicle using 2 accelerometers that are installed along the centerline of the vehicle was presented. According to the analysis, the effect of the tilt, generated either by rolling motion or by super-elevation, on the proposed algorithm is negligible. From the simulation studies, it was found that the sensor noise is the dominant source of the estimation error. Even though, under the adverse effect of the noise, the estimation error was bounded by 1%. Therefore, it can be concluded that the proposed algorithm provides accurate and smooth estimation of the yaw rate.

In the future, study of robustness to the model parameter uncertainties needs to be performed since the proposed algorithm is a model-based estimation algorithm. Also, experimental validation needs to be performed.

References

[1] G. F. Franklin, J. D. Powel and M. Workman, *Digital Control of Dynamic Systems*, 3rd Ed., Addison Wesley Longman, Inc., 1998.

[2] K. Shimada, Y. Nakamura, S. Horikoshi, H. Sugawara and T. Monji, “Angular Rate Detection Apparatus, Acceleration Detection Apparatus and Movement Control Apparatus of Moving Body,” US Patent 5,247,466, 1993.

[3] N. Sivashankar and A. G. Ulsoy, “Yaw Rate Estimation for Vehicle Control Applications,” *Journal of Dynamic Systems, Measurement, and Control*, Vol. 120, pp. 267 ~ 274, 1998

[4] J. Y Wong, *Theory of Ground Vehicles*, 2nd Ed., John Wiley & Sons, Inc., 1993.