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L_2 Stability and Performance Analysis of Missile Systems with Linear Autopilots and PN Guidance Laws

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Abstract— **This note presents the stability analysis for missile guidance systems with linear time-invariant autopilot systems. The concept of L_2 stability is employed in order to analyze missile systems. The well-known PN guidance law is considered as a missile guidance law and the nonlinear planar pursuit situation is fully considered in the analysis. It is shown first that the missile systems can be L_2 stable for target acceleration inputs which belong to L_2 space. A sufficient condition for a missile autopilot transfer function to guarantee L_2 stability of the missile systems is also provided as a function of total miss distance, which offers a criterion for missile autopilot design.**

I. INTRODUCTION

In this paper, we attempt to analyze missile systems considering autopilot dynamics and nonlinear dynamics of pursuit geometry.

In fact, missile subsystems such as autopilot and guidance systems are usually designed separately. When they are combined, the performance analysis is very difficult because of their own nonlinearity and coupling effects. Thus, the analysis of whole missile systems still poses severe challenges to control engineers [1]-[4].

In case of planar pursuit situation and linear autopilot, the performance of PN guidance systems were analyzed under the assumptions for the linearization of nonlinear pursuit dynamics [5]. In that analysis, the target acceleration was assumed to be zero and the dynamics including linear autopilot and nonlinear planar pursuit dynamics were linearized and combined under the assumptions that the closing velocity is constant.

In this note, we propose an L_2-space approach to performance analysis of missile systems. In the analysis, target acceleration functions are assumed to belong to L_2 space and nonlinear planar pursuit dynamics are fully considered. This note presents a sufficient condition that guarantees L_2 stability of missile guidance systems with linear autopilot and objective performances such as a miss distance. This sufficient condition is a guideline to the design of linear autopilot which guarantees the objective miss distance.

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II. PRELIMINARIES

In this note, a planar pursuit situation is considered as in [2] and [5]. The planar pursuit equation is described as follows.

$$\begin{aligned} \frac{d}{dt}(r\dot{\lambda}) &= -\dot{r}\dot{\lambda} - a_M cos(\theta_M) + a_T cos(\theta_T) \\ \dot{r} &= -V_M cos(\theta_M) + V_T cos(\theta_T) \\ r\dot{\lambda} &= -V_M sin(\theta_M) + V_T sin(\theta_T) \end{aligned} \tag{1}$$

where $r, \dot{r}$ are the relative distance between target and missile, its derivative, $\dot{\lambda}$ is Line of sight (LOS) angular rate, a_M, a_T are missile and target acceleration, $V_M, V_T \stackrel{\triangle}{=} \rho V_M$ are missile and target acceleration, θ_M is the angle between missile velocity vector and LOS, and θ_T is the angle between target velocity vector and LOS.

The missile guidance system with linear autopilot can be written by the following equations similar to [5].

$$\begin{aligned} \dot{x} &= Ax + bu, u = NV_M\dot{\lambda} \\ a_M &= cx + du \end{aligned} \tag{2}$$

where A is stable, (A, b) is controllable, (c, A) is observable, V_M is a missile velocity, and N is a navigation constant.

Using the result of lemma 2 in [2], the following lemma can be easily obtained.

Lemma 1: The following assumptions are satisfied.

(A2.1) $0 \leq \rho < 1.$

(A2.2) The target acceleration a_T is a function in L_2 space and For some $\alpha > 0$,

$$\|a_T(t)\| \leq \alpha, \quad \forall\ t \geq 0. \tag{3}$$

(A2.3)

$$|\ \theta_m(t)\ | \ < \ \frac{\pi}{2}, \ \ \forall\ t \geq 0. \tag{4}$$

(A2.4) The navigation constant N satisfies the following inequality for $\beta \leq 1 - \rho$.

$$N \ > \ 1 \ + \ \frac{\rho \ + \ \alpha\ r(0)/(\beta|V_M|^2)}{\sqrt{1 - |\rho + \beta|^2}}. \tag{5}$$

Then, the following inequalities holds.

$$\frac{d}{dt}|r\dot{\lambda}| \leq -\frac{\alpha}{\beta V_M}|r\dot{\lambda}| + \{|a_T| + |a_M - NV_M\dot{\lambda}|\},\ \forall\ t\ \geq\ 0, \tag{6}$$

□

The inequality (6) implies by the triangle inequality that

$$\|r\dot{\lambda}\|_2 \leq \frac{\beta V_M}{\alpha}\{\|a_T\|_2 + \|a_M - NV_M\dot{\lambda}\|_2\}. \qquad (7)$$

Thus, if we redefine the equation (1) by

$$\begin{aligned} \frac{d}{dt}(r\dot{\lambda}) &= -\{\dot{r} + NV_M sin(\theta_M)\}\dot{\lambda} + v(t) \\ \dot{r} &= -V_M cos(\theta_M) + V_T cos(\theta_T) \end{aligned} \qquad (8)$$

and the input to this dynamics is defined as $v(t) \triangleq a_T cos(\theta_T) - \{a_M - NV_M\dot{\lambda}\}cos(\theta_M)$, then the system defined by (8) is L_2 stable.

Lemma 2: Suppose that the assumptions in Lemma 1 are satisfied. Then, the system described by (8) with an input $v(t)$ and an output $y \triangleq r\dot{\lambda}$ is L_2 stable. That is,

$$\|r\dot{\lambda}\|_2 \leq \gamma_1\{\|a_T\|_2 + \|a_M - NV_M\dot{\lambda}\|_2\} \qquad (9)$$

where $\gamma_1 \triangleq \frac{\beta V_M}{\alpha}$. □

III. Main Results

The systems considered in this note can be considered as an interconnected systems composed of the following subsystems.

$$\begin{aligned} \Sigma_1: \quad & \dot{x}_1 = -\frac{\dot{r} - NV_M}{r}x_1 + bu_1 \\ & y_1 = x_1 \end{aligned} \qquad (10)$$

$$\begin{aligned} \Sigma_2: \quad & \dot{x}_2 = Ax_2 + bu_2 \\ & y_2 = cx_2 + (d-1)u_2 \end{aligned} \qquad (11)$$

where

$$\begin{aligned} u_1 &\triangleq a_T cos(\theta_T) - y_2 cos(\theta_M), y_1 \triangleq r\dot{\lambda} \\ u_2 &\triangleq NV_M\dot{\lambda} = \frac{NV_M}{r}y_1 \end{aligned} \qquad (12)$$

Define the transfer function of the linear autopilot by

$$G(s) \triangleq c(sI - A)^{-1}b + d. \qquad (13)$$

Since the system matrix A is stable, note that the system Σ_2 is L_2 stable for any $u_2 \in L_2$. Thus, the following inequality holds from the well-known input-output properties[6].

$$\|y_2\|_2 \leq \gamma_2\|u_2\|_2 \qquad (14)$$

where $\gamma_2 \triangleq \sup\{|G(jw) - 1| : w \in [0, \infty)\}$.

In order to prove the main result, we need the following lemma.

Lemma 3: Assume that (A2.1)-(A2.4) in Lemma 1 are satisfied. If

$$|r\dot{\lambda}| \leq \beta V_M, \forall\, t > t_0 \qquad (15)$$

, then

$$\dot{r} < -V_M\sqrt{(1-\rho)^2 - \beta^2}, \forall\, t > t_0. \qquad (16)$$

□

Note that Lemma 3 can be obtained from Lemma 1 in [2]. Now, the main result is given in Theorem 1.

Theorem 1: Suppose that the assumptions in Lemma 1 are satisfied and the transfer function $G(s)$ of linear autopilot satisfies the following inequality for some $r_M > 0$.

$$\gamma_1\gamma_2\frac{NV_M}{r_M} < 1 \qquad (17)$$

Then, for any $a_T \in L_2$, the missile system is L_2 stable until the final miss distance is achieved and the final miss distance $r_m \triangleq min\{r(t)\}$ is less than r_M. □

Proof) We prove by contradiction. Suppose $r_m > r_M$. Then, $r(t) > r_M, \forall t$. From (14) and the definition of u_2, it follows that

$$\|y_2\|_2 \leq \gamma_2\frac{NV_M}{r_M}\|r\dot{\lambda}\|_2. \qquad (18)$$

The condition (17) implies that the feedback system (10) and (10) is L_2 stable by the small-gain theorem and the following inequality holds.

$$\|r\dot{\lambda}\|_2 \leq \frac{\beta V_M}{\alpha(1 - \gamma_1\gamma_2\frac{NV_M}{r_M})}\|a_T\|_2 \qquad (19)$$

Thus, it follows that there exists a $T > 0$ such that for a given positive constant $\epsilon < \beta V_M$

$$|r\dot{\lambda}| < \epsilon, \forall t > T. \qquad (20)$$

From Lemma 3, it follows that

$$\dot{r} < -V_M\{(1-\rho)^2 - \beta^2\}^{1/2}, \forall t > T \qquad (21)$$

Therefore, there exists a $T_1 > T$ such that $r(T_1) \leq r_M$. This contradicts to the assumption $r_m > r_M$. Q.E.D

IV. CONCLUSIONS

In this note, stability and performance analysis of missile systems has been considered in terms of L_2 stability. This approach facilitates the performance analysis of missile systems composed of complicate subsystems. A sufficient condition was provided which could offer a criterion for missile systems to guarantee a guidance performance such as miss distance. The analysis has taken into account the exact nonlinear pursuit geometry without any assumptions for linearization. This approach can be easily extended to 3-dimensional pursuit situations.

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