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Reachability Guidance: A novel concept to improve mid-course guidance

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Abstract— **Reachability Guidance is a new concept developed from a combination of robot and missile guidance using shortest path techniques. It has been developed to improve mid-course guidance in medium range tactical, missile to missile, engagements. This paper considers the limitations of a range of missile guidance genres in solving challenging missile to missile engagements and identifies that the ability of the missile to reach the target is a common area of uncertainty. Using the SAM versus supersonic sea-skimming ASM problem as an illustrative example, a novel technique which employs the earliest intercept line (EIL) to facilitate reachability ('R') guidance is introduced.**

I. INTRODUCTION

Once launched the onus is on the guidance system to ensure a missile will reach its target. If the target trajectory is relatively benign, successful intercept is quite probable. However when the target is a modern, highly maneuverable missile the likelihood of reaching the target greatly diminishes due to the unpredictability of the target's future trajectory and the missile's ability to respond. The difficulties are exacerbated by the limited time available to prosecute the target. Future maneuver is especially difficult to predict if the missile is attacking a target rich environment. Despite these challenges, sophisticated Command and Control systems can quickly develop an optimum launch solution based on current target kinematics. Nevertheless, once launched the guidance system response and missile maneuverability may themselves limit the chance of success and it may fail to intercept the target. These problems are not new and in dealing with them over the years a range of missile guidance genres have been developed. These genres have well published strengths and weaknesses. Yet there is little guarantee that any modern guidance system can successfully intercept such a challenging target. To provide some certainty that reachability can be achieved, the current solution may be to launch a number of defensive missiles to improve the kill probability. This paper focuses specifically on the reachability issue using a challenging modern missile to missile scenario to illustrate the problems. It considers medium range surface to air (SAM), missile to missile engagements in the maritime environment, referred to here as the SAM versus ASM (anti-ship missile) problem (SvA). The key to solving the SvA problem is considered to be reachability. A review of the principal guidance genres is employed to identify where current guidance methods fail to address this problem. A number of concepts are then integrated using a novel geometric reachability approach, loosely based on Dubins [1] 'curves of minimum length', to develop earliest intercept curves from a bounded set of reachability points. Then it is shown that by comparing the features of the curves over discrete time steps, guidance commands can be developed to maintain reachability and enable interception.

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A. The SAM versus ASM (SvA) problem

A modern anti-air warfare (AAW) ship is likely to be part of a wider layered air-defence strategy [2], [3]. By stripping away some of these layers, the basic AAW process, from detection to kill assessment, can be considered. Then by focussing on the missiles which ultimately deliver successful defence or attack, a basic maritime scenario can be presented as the SvA problem. The problem is solved if the SAMs intercept successfully before the ASMs reach their targets. To limit the scope addressed in this paper, the range of problems will be summarized and the paper will concentrate on the mid-course guidance problem posed by a highly maneuverable supersonic sea-skimming ASM in the horizontal plane. In the simplified scenario, Figure 1, a single AAW ship defends itself and other ships against an ASM.

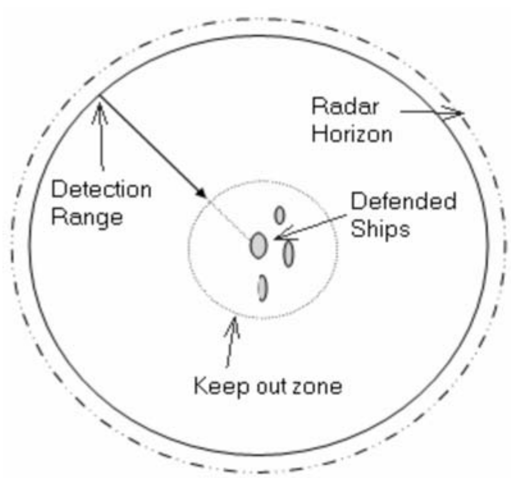

Fig. 1. The simplified AAW Scenario

The key AAW process elements [4] are target detection, track formation, firing solution creation, launch, mid-course, terminal-phase, kill assessment, and if necessary launch second salvo. For a ship employing its own sensors, an ASM can approach from beyond the radar horizon, employing passive transit at sea-skimming heights of 5 to 10 meters. This will limit the ships detection range to 20 to 30Km depending on the ships radar antenna height. Once within detection range the ASM may employ a wide range of maneuvers, using combinations of crossing rate, high-G turns and weaves to complicate the SAM firing solution and reduce the likelihood of successful interception. Once launched the SAM guidance system will need to cope with the uncertainties caused by such maneuvers during the mid-course and terminal phases. Unlike the almost continuous update rate during the terminal phase, the mid-course phase data update rate from the ships sensors and C2 systems is unlikely to be higher than once or twice a second. The defended ships complicate the problem and the AAW ship will be uncertain as to which ship is being targeted. The problem will be exacerbated if the defended ships are well separated. A significant problem is that the SAM is likely to be a boost-coast missile, thus drag will steadily reduce its speed [5]. Thus the SAM must maneuver expeditiously to ensure it can intercept the ASM as the SAM's energy is limited. Holder and Sylvester [6] employed the integral of the acceleration during the flight to determine efficiency; such simple approaches are useful for comparative studies.

The SAM switches from mid-course guidance to terminal guidance a few kilometers before intercept [7]. It is at this handover point that the sum of all the errors accumulated in solving the SvA problem become apparent. The key questions at this time are: Can the SAM seeker lock onto the ASM? Can the SAM guidance system avoid excessive saturation during the terminal phase? Can the SAM reach the ASM before it reaches the ships? Solving the SvA problem requires a 'yes' to each of these. Modern gimbaled missile seekers [7] are capable of wide-angled searches, so unless the mid-course guidance was woefully inadequate most seekers are capable of locking onto the target after a brief search. So whilst not simplistic, seeker lock-on may be considered solvable. Guidance system saturation occurs if excessive demands exist for a significant period of the terminal phase then the outcome is likely to be increased miss-distance. Zarchan [8] and Ben-Asher [9] address a wide range of miss-distance factors across a number of guidance systems. Optimal guidance with acceleration constraints was considered by Rusnak and Meir [10] in developing their OG system. Reachability is clearly implicit in most-guidance systems; the guidance law and the speed-advantage suggest eventual reachability (not withstanding the saturation and limited energy issues!). In the SvA problem, reachability is a more explicit issue and the reachability time frame is constrained by earliest time at which the ASM can strike one of the ship targets.

II. CONTRIBUTION

This paper makes 3 specific contributions. An extensive review, summarized here, identifies a common weakness in most missile guidance genres from a target reachability perspective. Equi-time rotating involutes are employed to resolve the worst case reachability solution and the use of the Earliest Intercept Line (EIL) is introduced as a reachability guidance mechanism.

III. REVIEW OF GUIDANCE TECHNIQUES

The following section summarizes an extensive review of common missile genres from a realistic maneuver and reachability perspective. The key genres considered were PN, APN, OG, DGT and MTG (Minimum time Guidance). PN's simplicity is its strength and the capabilities and limitations of PN have been well researched [11], [12], [13], [14], [15]. Simply maneuvering to reduce the angular rate of change of the LOS enables interception of a non-maneuvering target. Much of the early literature deals with the dilemma of the PPN intractability [11], compared to the relatively easily derived closed form solution for TPNG. The most important conclusion, regarding PN when considering the SvA problem, is that PN is efficient and effective against non-maneuvering targets, but much less effective against maneuvering targets. The PN process only requires the rate of change of LOS, thus autonomous missiles under simple PN control cannot determine reachability (whether they will reach the target early enough to defend the ships) or whether they will saturate and miss in the terminal phase.

ASMs may maneuver significantly and a number of guidance techniques have been developed to deal with maneuver. Augmented PN, which compensates for target acceleration in its guidance law, enables maneuvering targets to be intercepted. Zarchan's comparison of simple PN and APN systems shows that APN's early responsiveness generally reduces the acceleration demands in the terminal phase [10]. Nevertheless, even with perfect APN, if terminal phase acceleration demands are beyond the capability of the SAM then the missile is likely to miss. To respond to ASM maneuver, the SAM system must measure the ASM acceleration. Whilst this is relatively straightforward during the high data rate terminal phase it is much more difficult during the low data rate mid-course phase. Biased PNG may help in this situation as such systems only require maneuver detection, not measurement, and then add a bias to the guidance law to improve response. The same effect could be achieved by employing a dynamic navigation 'constant'. Kim et al [16]successfully employed biased PNG to determine impact angle between a missile and a target. As well as maneuverability one must consider reachability. To understand this it is necessary to consider the ASM's maneuver capability and the SAM's potential to respond to any maneuver. Though more capable than PN, APN remains a relatively simple action/reaction guidance system which considers only its own and the ASM's kinematics in developing its solution. Thus terminal phase lateral acceleration saturation or late maneuver may still contribute to its failure.

Much work has been undertaken in modern guidance using optimal control techniques which enable responses according to a predetermined cost function to enable the missile better to counteract missile maneuver. Such guidance systems may employ a simple cost-function or more complex forms which include a combination of cost-functions [17]. Kreindler's OGL/PN comparison [18] confirmed that PN is an optimal solution for the non-maneuvering target case. This important result may be used as a base-line test when assessing the range of effectiveness of sophisticated guidance laws. Yang's analysis of optimal mid-course guidance [19] concentrated on maximizing the final speed of the SAM by minimizing energy expenditure. Imado proposed that mid-course guidance should either maximize the final speed or minimize the flight time [20]. These papers are highly relevant to the SvA problem. Most OG systems rely heavily upon the determination of an accurate intercept time. Implicitly this time provides a single point reachability solution which is highly likely to change over time. Tahk and Ryoo addressed this problem using 'recursive time-to-go estimation' [21]. Despite this inclusion of an implicit variable reachability point, the weaknesses with respect to the SvA problem remain. There is no knowledge of the more complete reachability set and maneuver induced lateral acceleration saturation is still highly probable in the terminal phase.

Differential Game Theory (DGT) does explicitly address the maneuver problem. Rufus Isaacs [22] is cited in many DGT missile guidance papers. For at least the last 15 years DGT seems to have been the principal province of Shinar [23], [24], [25], [26], [27]. His key innovative approach is not to engage the current trajectory, but to employ the worst case evasive trajectory to predict a suitable engagement strategy [27]. This approach explicitly addresses a specific reachability case which is highly likely in missile/aircraft scenarios where evasion will be a primary concern for the pilot. However it is less likely in missile/missile scenarios. ASMs will be programmed to maneuver to improve survivability against the general AAW scenario but would not normally maneuver to evade a specific SAM. Nevertheless modified applications based on Shinar's concepts, but addressing a wider range of possible trajectories, could be employed to address the SvA problem.

Minimum Time Guidance (MTG) has been widely researched under various titles. The concept of achieving a minimum time intercept has been addressed using an OGL model by Guelman and Shinar [27], Siouris and Leros [28] and Hull et al [29]. The concept is simple. For a non-maneuvering or constant maneuver target, MTG will generate the earliest intercept point (EIP) by using a minimum radius (maximum G) turn followed by a straight line trajectory to the predicted intercept point, Figure 2. Using this simple MTG technique and considering not only the worst case evasive trajectory, but the whole range of possible minimum time trajectories, then a set of EIPs could be generated showing the overall reachability situation at any time during the engagement.

From this brief review of the common missile genres there are several consistent themes. All missile guidance systems develop dynamic (action/reaction) intercept trajectories using one of a range of techniques. Rapid response to counteract maneuver (APN, OGL and MTG) generally reduces the terminal phase acceleration demands. A time-variant reachability point is determined by OGL, DGT and MTG systems. As they stand none of these guidance systems can give the confidence required that the SvA reachability problem will be solved as they do not consider the ASM's target time constraint. However as suggested above, development of the MTG concept shows promise in presenting a more comprehensive appreciation of reachability during the engagement.

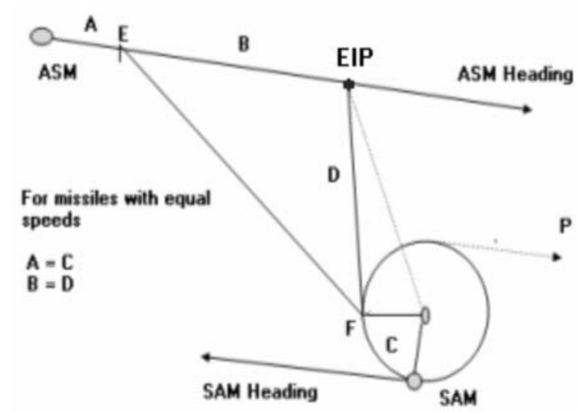

Fig. 2. The Minimum Time Guidance (MTG) Concept

This paper develops a guidance concept based on the current ASM trajectory and a large set of other possible trajectories. The intention is that the guidance law can then be based on the characteristics of the resultant predicted intercept curve and its relationship to the ASM's possible targets. As will be seen the set of earliest intercepts for a wide range of possible ASM maneuvers generates an earliest intercept line (EIL). Determination of each intercept point can employ a Siouris [28] like trajectory; a minimum radius turn followed by a straight line. But the paper introduces rotated equi-time involutes to provide a simpler and novel solution. It is re-emphasized at this point that the mid-course phase, with the missiles well separated, is the key thrust of this research.

IV. DEVELOPING THE REACHABILITY CONCEPT

Dubins [1] groundbreaking paper on minimum path lengths is extensively cited in minimum path robotic 'car' solutions [30], [31], [32] but less frequently in dealing with airborne problems [33], [34]. Yet minimum paths are highly relevant in missile guidance. In the mid-course phase, especially with the missiles well separated, the missile guidance problem can be restated using a suitably adapted interpretation of Dubins' paths. The 6 generic Dubins trajectories which provide the shortest paths for a single maneuverable object with initial position coordinates and a desired final position and heading are RSL, RSR, LSR, LSL, LRL and RLR. In a simple missile guidance case, consider a missile M flying towards a static point T in the minimum time. By equating the missile to the Dubins start conditions and T to the required final position and omitting the terminal direction constraint, it can be seen, as shown in Figure 3 that all 6 shortest paths require CS (Curve/Straight) trajectories (RS or LS). Thus a missile with constrained curvature may reach a static point in the plane in the minimum time via a sub-string of the Dubins path set; as long as the aim point is not inside the missile's turning circle. Other viable substrings not included in Figure 3 include the single element sub-strings S, L or R (Straight, Left, Right). As will be shown, presenting the guidance problem this way shows a key path which is highly relevant to reachability.

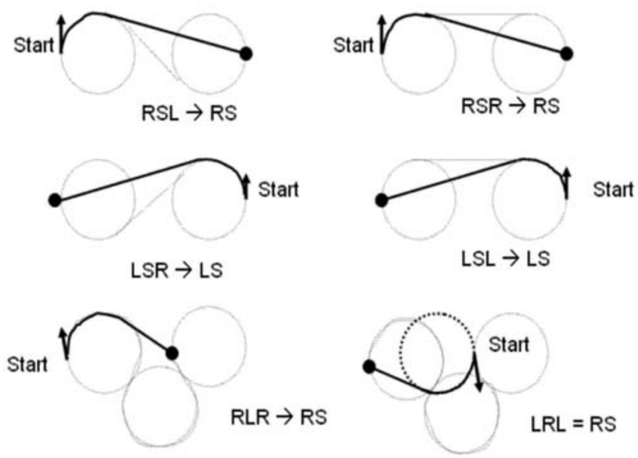

Fig. 3. Removing terminal point direction constraint

Considering a flat plane and constant velocity missiles, each of which have declared initial position, minimum turn radii and headings, a mid-course predicted intercept may be represented as shown in Figure 4 .

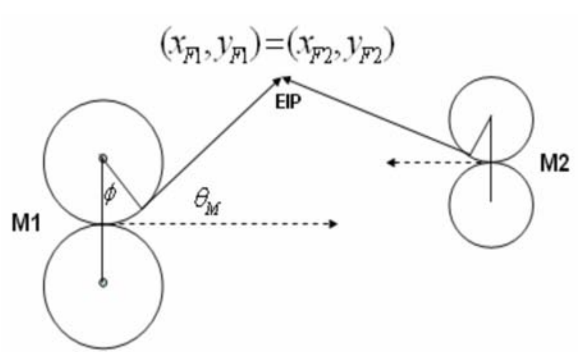

Fig. 4. The missiles, turning circles and a single member (EIP) from the earliest intercept set

Using simple coordinate geometry and minimum radius turns, (through angles of ϕ_1, ϕ_2) the future possible earliest intercept positions (x_{F_1}, y_{F_1} and x_{F_2}, y_{F_2}) are easily determined. At any instant, either missile may maneuver left or right, up to its minimum turn radius, or continue to fly straight; there are no other options. In the case shown $M1$ makes a left turn and M2 generates an intercept trajectory using a right turn.

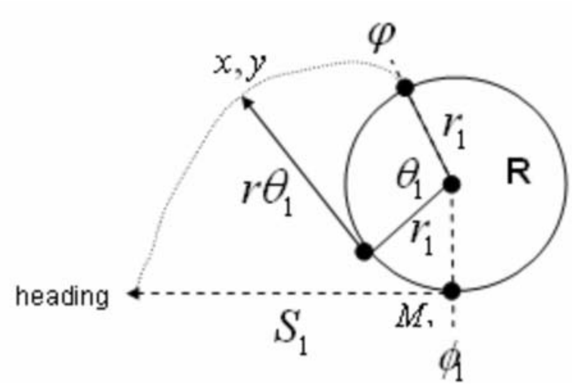

Fig. 5. Geometry for a right turn circle

Considering the locus of the furthest on point :

$$\phi = \theta_{heading} + \frac{\pi}{2} \tag{1}$$

$$0 \leq \theta \leq \frac{S_1}{r_1} \tag{2}$$

where $S_1 = V_m t$

$$\varphi = \phi - \frac{S}{r} \tag{3}$$

$$\begin{aligned} x &= x_c + r\cos(\varphi+\theta) + r\theta\cos\left(\varphi+\theta-\frac{\pi}{2}\right) \\ y &= y_c + r\sin(\varphi+\theta) + r\theta\sin\left(\varphi+\theta-\frac{\pi}{2}\right) \end{aligned} \tag{4}$$

$$\begin{aligned} x &= x_c + r\left(\cos(\varphi+\theta) + \theta\sin(\varphi+\theta)\right) \\ y &= y_c + r\left(\sin(\varphi+\theta) - \theta\cos(\varphi+\theta)\right) \end{aligned} \tag{5}$$

Equation (5) is a modified equation for the involute of a circle where φ is the involute start point and $0 \leq \theta \leq \frac{S}{r}$ where $S = V_M t$.

In a similar way it can be shown that a left turn circle furthest on point for any time 't' can be determined using the involute with reversed direction.

$$\begin{aligned} x &= x_c + r\left(\cos(\varphi-\theta) - \theta\sin(\varphi+\theta)\right) \\ y &= y_c + r\left(\sin(\varphi-\theta) + \theta\cos(\varphi-\theta)\right) \end{aligned} \tag{6}$$

For midcourse guidance with the missile minimum turn circles well separated, the minimum time engagement path always lies on the common internal or external tangent between 2 of the turn circles as shown in Figure 6. (For this case the path is formed by 2 sets of Dubins sub-strings RS and RS. i.e. both missiles turn right for a time then fly straight).

$$S = S_1 + S_2 = r_1\phi_1 + r_2\phi_2 + S_{Tangent} \tag{7}$$

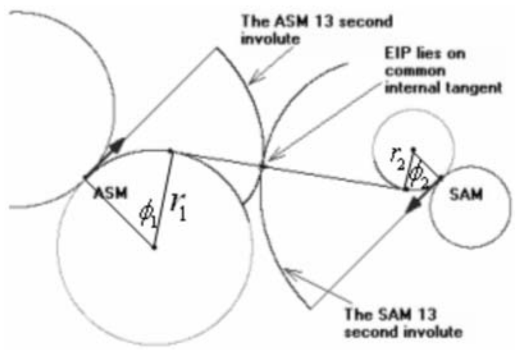

Fig. 6. The key EIP lies along the common tangent

Then this EIP can be found from the velocity ratios

$$\frac{V_{M2}}{V_{M1}+V_{M2}}S \tag{8}$$

or

$$\frac{V_{M1}}{V_{M1}+V_{M2}}S \tag{9}$$

Whilst such a trajectory may seem absurd, as its realization would require mutual cooperation between the missiles, it is a highly significant singular point indicating the earliest possible intercept time. It provides start time for the set of reachability points. For any mid-course configuration, reachability is only achievable from this time and as will be shown, the reachability curve is continuous for times greater than the smallest EIP up until the reachability thresholds. Figure 6 shows this earliest intercept point (EIP) point determined using time-coincident involutes for the turn circles where $EIP \leq t \leq t_{max}$

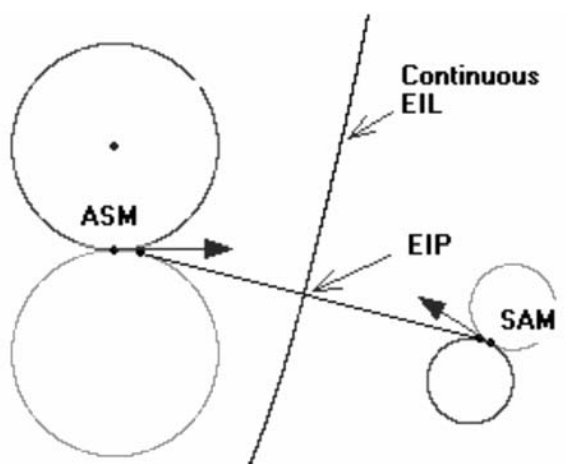

Fig. 7. EIL developed from the set of EIPs

Using equations (5) and (6) for time $t \geq EIP$, the intersection of the equi-time involutes provides the earliest intercept solution for 't'. For each $t \geq EIP$ there will be one or 2 solutions until an upper threshold is reached.

Awareness that the reachability is continuous between the EIP and the upper threshold values enables any closely

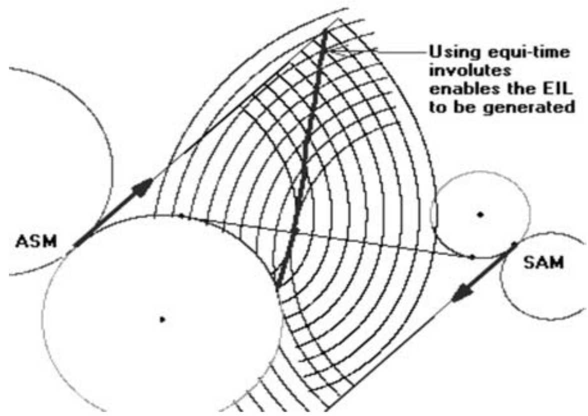

Fig. 8. The earliest intercept line (EIL)

spaced resulting discrete set of intercept points to be joined to form the continuous earliest intercept line (EIL) as shown in Figure 8. The EIL is a reachable set of solutions to the ASM's possible furthest on trajectories. As such any other more benign trajectories or more complex maneuvers will result in solutions to the left of the EIL; thus the EIL is wholly reachable and offers a credible defense line which can be determined if both missiles capabilities and maneuver limitations are known. How this EIL may be incorporated into a guidance law will now be considered.

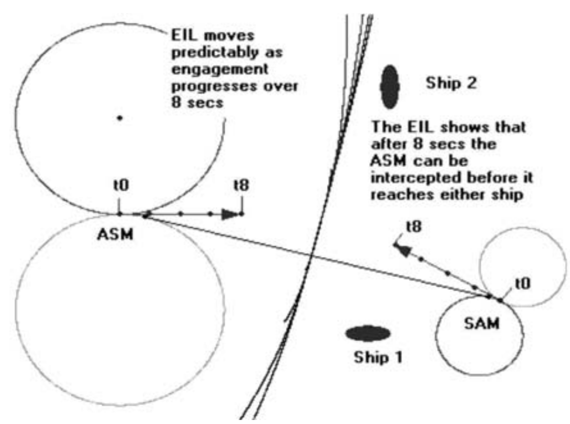

Fig. 9. The EIL continues to indicate earliest reachability throughout the engagement

An interesting fact about the EIL in the mid-course guidance case is that once predicted, if the missiles are on a collision course and don't maneuver, the EIL coordinates remain relatively static, although the EIL decreases in length until the EIL becomes a single EIP as intercept is achieved. However when the missiles are not on a collision course, the EIL will rotate over time with the rotation rate increasing with time and range. As shown in figure 10 the SAM doesn't maneuver and after a few seconds the ASM is able to reach a ship before the SAM can intercept it.

The EIL concept outlined above uses the same kinematic information as the majority of guidance systems; target position, heading and velocity. In addition it will require an

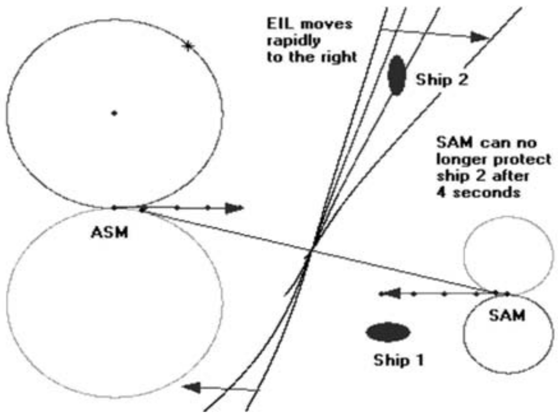

Fig. 10. Here the SAM does not maneuver and after a few seconds it can be seen ship 2 cannot be defended.

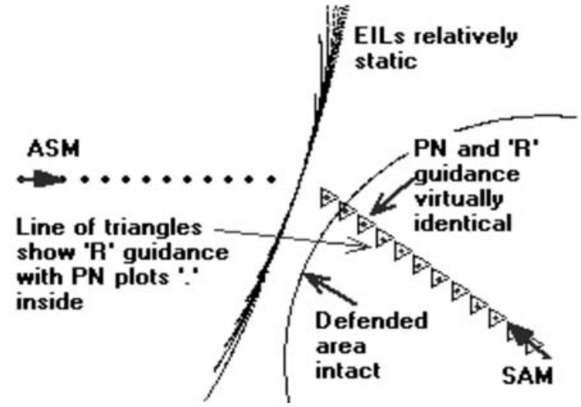

Fig. 11. PN and ’R’ guidance virtually identical against a benign threat

estimate of the ASM turn capability (but not a measurement of its rate of turn) as well as the positions of the defended targets or defended area. The question here might well be; so what was new? Quite simply, whilst most other guidance laws generate almost immediate responses to counteract target maneuver, EIL (or ’R’) guidance doesn’t need to. Based on the EIL parameters, ’R’ guidance may choose not to respond immediately whilst ensuring that the ship targets are defended by containing the movement of the EIL to a desired limit. If successfully implemented such a law would enable a different approach to a whole range of engagements. For instance most current guidance systems tackle a weaving target by employing a response induced time-lagged weave. In the ’R’ guidance case the proposition is that there may be no need to maneuver significantly until the defended asset is threatened or the ASM approaches unreachability; thus the resultant SAM trajectory may be quite different. Expanding this concept may enable the protection of multiple separated assets using a single missile.

V. Initial Simulation Results

To employ the EIL successfully as a guidance mechanism the challenge was to maintain the EIL relative position as the engagement progressed or to allow the EIL to move where there was no threatened asset. Investigation using 2 well separated lag-free missiles (approximately 20Km apart) with identical speeds and characteristics but with data update rates in the order of 0.01s (unrealistic for mid-course guidance) showed that the EIL remained virtually stationary when the SAM followed a MTG trajectory. Returning to Figure 9, this seems intuitive. If the ASM in Figure 9 was to maintain its current heading and the SAM followed a MTG the intercept would occur at the (non-maneuver trajectory) EIP which is on the EIL. This finding enabled the development of an elementary ’R’ guidance law.

Initial results were promising. When compared against PN using a benign non-maneuvering target, the tracks are very similar as shown; indicating the ’R’ Guidance also displays an optimal trajectory [18]. When an artificial constraint restricted the SAM launch bearing, the ’R’ guidance system was able to maintain the integrity of the defended area against a weave whilst the PN solution failed. Suggestions that APN should have been employed in the second trial would be fair. APN would probably achieve an intercept (ignoring the limited mid-course data update rate); however there would be no appreciation whether the defended area remained intact. Even if the EILs were plotted for APN, the APN guidance law would remain reactive only to target maneuver. The strength of the ’R’ guidance solution is that the guidance law can determine whether to react or not and by how much by assessing the impact on the EIL of predicted action or inaction.

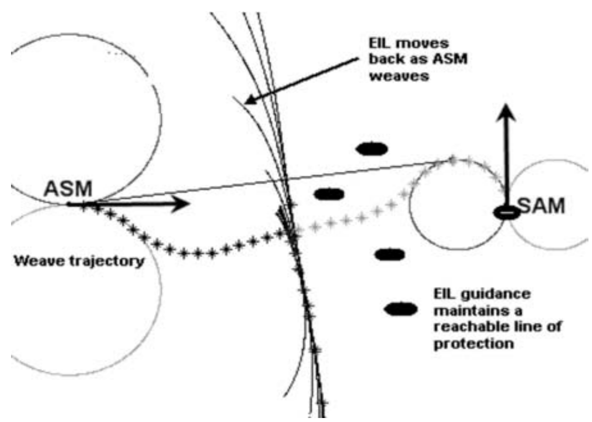

Fig. 12. ’R’ guidance successfully defends area against random weave despite imposed launch direction

Figure 12 also serves to demonstrate what happens when the ASM does not follow an MTG trajectory. As the EIL is a furthest on prediction, the subsequent EILs will move away from the defended area. The SAM then attacks the new EIL. Whilst the development of ’R’ guidance using the EIL is in its infancy, the knowledge of reachability and associated lateral acceleration demands for the SAM mean that ’R’ guidance helps solve the SvA problem posed here.

VI. DISCUSSION AND CONCLUSION

This paper has illustrated the concept of 'R' guidance which determines reachability using minimum time guidance (MTG) estimation processes to develop earliest intercept lines (EILs). These EILs can then be incorporated into a guidance law in conjunction with the coordinates of defended areas and the 'R' guidance will maneuver the SAM to best protect the defended areas. As the EIL may be developed using a SAM lateral acceleration value below saturation, use of EILs in the guidance can enable the SAM to reach the ASM with a low likelihood of lateral acceleration saturation in the terminal phase. It was demonstrated that the 'R' Guidance concept shows promise in addressing the key SvA problems discussed in the introduction. This early work has also indicated that 'R' guidance may be useful in a number of wider areas including the following non-exhaustive group:

- C2 engagement planning.
- Mixed strategy missile salvo launches.
- Target capturability determination
- Path planning for air vehicles.

Research continues apace developing, optimising and testing the viability of 'R' Guidance against a more comprehensive range of possible threat trajectories. A rigorous, documented comparison against a range of guidance techniques is being prepared.

VII. ACKNOWLEDGEMENT

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