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An adaptive GLR estimator for state estimation of a maneuvering target

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***Abstract*— This paper presents a novel adaptive generalized likelihood ratio (A-GLR) state estimator applied to rapidly maneuvering targets in pursuit-evasion scenarios. The A-GLR estimator employs a bank of adaptive models which is constructed and updated on-line. The state estimate is a probabilistic mixture of the model-matched estimates. The adaptation of the models, the model-matched estimates, and the a-posteriori probabilities of the models are calculated recursively by employing a previously developed adaptive-$\mathscr{H}_0$ GLR algorithm. Numerical simulations of a maneuvering target (a ballistic missile) show that the A-GLR estimator delivers state estimates characterized by a smaller average error and a smaller covariance as compared with those obtained using the interacting multiple model (IMM) estimator.**

I. INTRODUCTION

Continuous-time dynamical systems are often subject to structural changes occurring at discrete points in time. Examples include systems such as processes subject to failures/repairs and maneuvering targets. The latter are conveniently represented as hybrid systems which are combinations of continuous-time systems and discrete-event systems. The major challenge in state estimation for hybrid systems arises from the presence of two types of uncertainties: the measurement uncertainty and the uncertainty about the current structure of the system. The last type of uncertainty occurs when a continuous-time system is subject to an abrupt structural change with only partially known characteristics. The optimal state estimation problem for stochastic linear hybrid systems, i.e., hybrid systems subject to *random* discrete events, is, in general, computationally intractable as it often fails to translate into a finite recursive state estimation scheme, cf. [1]. Several suboptimal estimators have thus been proposed, most notably adaptive multiple model estimators, cf. [2], and the IMM estimator, cf. [3] and [4]. For the specific problem of tracking a maneuvering target, the IMM estimator is recognized as one of the best practical estimators, cf. [5] and [6]. Another class of suboptimal estimators employs generalized likelihood ratios (GLR) of hypotheses to yield a state estimate, cf. [7]. Previous applications of the GLR approach to target tracking demonstrated good estimation performance, cf. [8], [9] and [10].

The novel estimator presented here pertains to the GLR approach and is an extension of the estimator presented in [8]. In [8], a bank of specific input realizations (the reference realizations) is provided a-priori to the estimator. The new estimator, referred to as the adaptive GLR (A-GLR) estimator, replaces the bank of a-priori realizations by a parametric family of reference realizations. The actual reference realization is adapted *on-line* as a member of this family. Hence, the single, adaptive, reference realization requires less information about the hybrid stochastic input process than a bank of pre-specified reference realizations and is more efficient from a numerical point of view. The adaptation of the reference realization employs a sequential probability ratio test (SPRT) and is based on the adaptive-$\mathscr{H}_0$ GLR algorithm presented in [11]. The adaptive-$\mathscr{H}_0$ GLR algorithm is an extension of the GLR algorithm of [12] to parametric family of reference realizations.

The performance of the resulting A-GLR estimator is compared with that of several implementations of the IMM estimator. For the comparison, the example system is a target tracking problem in a pursuit-evasion scenario against a maneuvering ballistic missile.

II. PROBLEM STATEMENT

Consider a discrete-time stochastic linear hybrid system, with a continuous-time valued based state x sampled at equal intervals Δ:

$$\begin{aligned} x(k+1) &= \mathbf{F}(k)x(k) + \mathbf{G}_1(k)u(k) + \mathbf{G}_2(k)z(k) + \omega(k) \\ y(k) &= \mathbf{H}(k)x(k) + \eta(k) \end{aligned} \tag{1}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^q$, $z \in \mathbb{R}^p$, $y \in \mathbb{R}^m$. The process and measurement noises, $\omega \sim \mathcal{N}(0, \mathbf{Q}_\omega)$, and $\eta \sim \mathcal{N}(0, \mathbf{Q}_\eta)$, are assumed to be normally distributed and independent. The state and measurement variables, x and y, are random time series which are solutions of the above linear stochastic system, and u is assumed to be a known external input. The process z has the meaning of an unknown input whose behavior is subject to additive abrupt changes and is not directly observed. The realizations of the process z before and after an abrupt change are restricted to belong to parametric families of functions. Moreover, the time interval between the abrupt changes occurring in z exceeds a value $w^\star$ such that $w^\star >> \Delta$ and the ratio between the largest time constant in the system and $w^{\star}$ is small. The last assumption renders the system slowly-varying in structure. For the filtering problem to be meaningful, the system is assumed to be observable.

The objective is to develop a fast and statistically reliable state estimator for the system.

For further use, let $\mathcal{Y}_{k_0}^k$ denote the σ-algebra generated by the measurements:

$$\mathcal{Y}_{k_0}^k = \sigma\{y(s) : k_0 < s \leq k\} \tag{2}$$

III. Description of the A-GLR Estimator

The optimal Bayesian estimator for the hybrid system in Eq. (1) is a NP-complete problem involving an exponentially growing tree of models and, as such, cannot be implemented in real time [13]. Practical Bayesian based estimators are suboptimal in that they have to rely on certain model management techniques to keep the number of models limited, thus allowing the computational scheme to be finite.

The A-GLR estimator limits the number of models by pruning on-line the unlikely models from the full model tree. The pruning of the models is carried out in two steps. First, parametric families of models are selected. Each parametric family describes the unknown input either before or after an abrupt change. Next, the bank of models is constructed on-line by selecting at each time instant k the most likely models from within the parametric families. The selection of the most likely models, along with the calculation of their a posteriori probabilities and model-matched estimates, is achieved by employing an adaptive-$\mathscr{H}_0$ GLR algorithm [11]. The resulting A-GLR estimator is recursive and the computational effort involved increases only linearly with the number of models.

Contrary to most multiple model estimators, the A-GLR estimator does not explicitly employ a bank of model-matched filters computed in parallel. Instead, a *single model-matched Kalman filter* is employed; the last is referred to as the *reference Kalman filter*. The adaptive-$\mathscr{H}_0$ GLR algorithm calculates all the model-matched estimates from the outputs of the reference Kalman filter. The calculation of the model-matched estimates by a GLR algorithm is more numerically efficient than running a bank of filters in parallel [14] and naturally permits for on-line adaptation of the models. However, as a trade-off, the GLR algorithm is applicable to models which differ by their inputs only, the transition matrix must be the same in all the models considered.

The flowchart of the A-GLR estimator is shown in Figure 1 which can be summarized in six repetitive steps: (1) formation of a set of hypotheses, (2) GLR computations, (3) formation of the model bank, (4) model-matched estimation, (5) calculation of the a-posteriori probabilities of the models, and (6) estimate fusion.

A. The Formation of the Set of Hypotheses

The GLR algorithm employs multiple hypotheses. The set of hypotheses describes the parametric families of which the input z is a member. This set of hypotheses, $S_{\mathscr{H}}^k$, contains one reference hypothesis characterized by no additive abrupt change, $\mathscr{H}_0$, and w hypotheses that assume different onset time instants of the abrupt change as well as different functional realizations, $\mathscr{H}_i^k$, $i \in \{1, \cdots, w\}$. Hence, $S_{\mathscr{H}}^k = \{\mathscr{H}_0, \mathscr{H}_1^k, \cdots, \mathscr{H}_w^k\}$.

The reference hypothesis defines a parametric family of functions characterized by a dynamic profile, $f_{\mathscr{H}} \in \mathscr{H}_0$, of the process z before an abrupt change. The actual realization for z before an abrupt change is referred to as the reference realization and is given by $z = \nu_{\mathscr{H}} f_{\mathscr{H}}$, where the scaling factor $\nu_{\mathscr{H}}$ has to be estimated on-line. Each abrupt change hypothesis represents a parametric family of functions characterized by two parameters: a time instant for the onset of the abrupt change, $k_i^{\star}$, and a dynamic profile of the abrupt change, f_i. The realization for z after an abrupt change associated with hypothesis $\mathscr{H}_i^k$ is given by $z = \nu_{\mathscr{H}} f_{\mathscr{H}} + \nu_i f_i$ where both scaling factors $\nu_{\mathscr{H}}$ and ν_i are estimated on-line by the adaptive-$\mathscr{H}_0$ GLR algorithm.

B. The Adaptive-$\mathscr{H}_0$ GLR Algorithm

The adaptive-$\mathscr{H}_0$ GLR algorithm is an improved version of the GLR algorithm of [12] and was first introduced in [11] for the purpose of fault detection in systems with one unknown input. The adaptive-$\mathscr{H}_0$ GLR algorithm provides an estimate of the reference realization, $\hat{z}_{\mathscr{H}}$, and a maximum likelihood estimate, $\hat{z}_i^{\mathrm{ML}}$, for the realization of each hypothesis $\mathscr{H}_i^k$. It also calculates the likelihood ratios, $L(\mathscr{H}_i^k, \hat{z}_i^{\mathrm{ML}})$, between the estimated realizations of the abrupt change hypotheses and the estimated realization of the reference hypothesis:

$$\hat{z}_i^{\mathrm{ML}} \triangleq \hat{z}_{\mathscr{H}} + \arg\max_{\tilde{z}} \mathrm{p}\left(\mathcal{Y}_{k_0}^k \,\middle|\, \mathscr{H}_i^k, \tilde{z}\right) \tag{3a}$$

$$= \hat{z}_{\mathscr{H}} + \frac{d(k,i)}{J(k,i)} f_i \tag{3b}$$

$$L(\mathscr{H}_i^k, \hat{z}_i^{\mathrm{ML}}) \triangleq \frac{\mathrm{p}\left(\mathcal{Y}_{k_0}^k \,\middle|\, \mathscr{H}_i^k, \hat{z}_i^{\mathrm{ML}}, \hat{z}_{\mathscr{H}}\right)}{\mathrm{p}\left(\mathcal{Y}_{k_0}^k \,\middle|\, \mathscr{H}_0, \hat{z}_{\mathscr{H}}\right)} = e^{\frac{1}{2}\frac{d^2(k,i)}{J(k,i)}} \tag{4}$$

In the above, $\hat{z}_{\mathscr{H}}$ is the estimated reference realization and $\tilde{z} \triangleq \nu_i f_i$ is a realization of the abrupt change. The symbol d is the signature correlation and the symbol J is

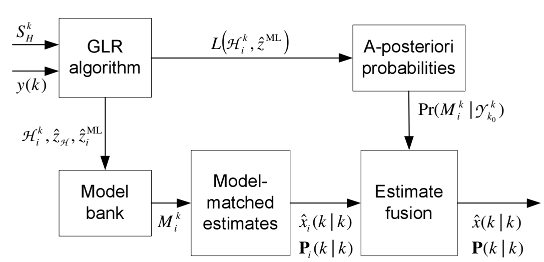

Fig. 1. A-GLR estimator.

the normalized Kullback-Leibler divergence (an information theoretic norm) of hypothesis $\mathscr{H}_i^k$, see [15]. The value of d and J is calculated based on the system matrices of a reference Kalman filter and on its residuals, γ, as follows

$$d(k,i) \triangleq d(k-1,i) + \rho^T(k,i)V^{-1}(k)\gamma(k) \quad (5a)$$
$$J(k,i) \triangleq J(k-1,i) + \rho^T(k,i)V^{-1}(k)\rho(k,i) \quad (5b)$$
$$\rho(k,i) \triangleq H(k)\Gamma(k,i) \quad (5c)$$
$$\Gamma(k,i) \triangleq G_2(k)f_i(k) + \mathcal{F}(k-1)\Gamma(k-1,i) \quad (5d)$$
$$\mathcal{F}(k-1) \triangleq F(k)[I - K(k-1)H(k)] \quad (5e)$$

where V is the residual covariance and K is the Kalman gain. The recursions for d, J and Γ are initiated by $d(k_i^\star,i)=0$, $J(k_i^\star,i)=0$, and $\Gamma(k_i^\star,i)=0$. The reference Kalman filter requires a realization for the unknown process z; the employed realization is the estimated reference realization $\hat{z}_{\mathscr{H}} \in \mathscr{H}_0$. The adaptive-$\mathscr{H}_0$ algorithm adapts the reference realization $\hat{z}_{\mathscr{H}}$ judging by the likelihood ratio between the maximum likelihood estimate of the reference at time instant k, see Eq. (3), as compared with the estimated reference realization at time instant $k-1$; the threshold of the decision function is conditioned by the desired Type I error probability, α. See Ref. [11] for details.

C. The Formation of the Model Bank

The bank of models describes several realizations for the process z over the time interval $[k_0, k]$. At each time instant k, a model, M_i^k, is associated with the reference hypothesis $\mathscr{H}_0$ and with an abrupt change hypothesis $\mathscr{H}_i^k \in S_j^k$ as follows

$$M_0^k : \ \hat{z}_{\mathscr{H}}(l) \quad (6a)$$
$$M_i^k : \ \hat{z}_{\mathscr{H}}(l)\left[\mathbf{1}(k_0) - \mathbf{1}(k_i^\star)\right] + \hat{z}_i^{\mathrm{ML}}(l)\mathbf{1}(k_i^\star) \quad (6b)$$

where $l \in \{k_0, \cdots, k\}$, $i \in \{1, \cdots, w\}$, the symbol $\mathbf{1}(l)$ denotes the unit step function at time instant l, and $k_i^\star$ is the onset time of the abrupt change assumed by $\mathscr{H}_i^k$.

D. The Model-Matched State Estimates

Each model-matched state estimate, $\hat{x}_i(k|k)$, is calculated assuming that model M_i^k is true

$$\hat{x}_i(k|k) \triangleq E\left(x \left| M_i^k, \mathcal{Y}_{k_0}^k \right.\right) \quad (7)$$

The model-matched estimate $\hat{x}_0(k|k)$ and its covariance $\mathbf{P}_0(k|k)$ are calculated by the reference Kalman filter employed by the GLR algorithm. The remaining $\hat{x}_i(k|k)$ and $\mathbf{P}_i(k|k)$ are calculated using outputs of the GLR algorithm

$$\hat{x}_i(k|k) = \hat{x}_0(k|k) + \frac{d(k,i)}{J(k,i)}\Upsilon_i^k \quad (8a)$$

$$\mathbf{P}_i(k|k) = \mathbf{P}_0(k|k) + \frac{\Upsilon_i^k {\Upsilon_i^k}^T}{J(k,i)} \quad (8b)$$

where $\Upsilon_i^k \triangleq (I - K(k)H(k))\Gamma(k,i)$.

E. The A-Posteriori Probabilities of the Models

The a-posteriori probability of each model, $\Pr\left(M_i^k \left| \mathcal{Y}_{k_0}^k \right.\right)$, is calculated from the likelihood ratios of the models using the Bayes' rule

$$\Pr\left(M_i^k \left| \mathcal{Y}_{k_0}^k \right.\right) = \frac{L\left(M_i^k\right)\Pr\left(M_i^k\right)}{\sum_{j=0}^{w} L\left(M_j^k\right)\Pr\left(M_j^k\right)} \quad (9)$$

where $\Pr(M_i)$ is the total probability of model M_i^k and $L(M_i^k)$ is a likelihood ratio defined as:

$$L(M_i^k) \triangleq \frac{\mathrm{p}\left(\mathcal{Y}_{k_0}^k \middle| M_i^k\right)}{\mathrm{p}\left(\mathcal{Y}_{k_0}^k \middle| M_0^k\right)} \quad (10)$$

Since the model M_i^k is chosen to be the maximum likelihood realization of hypothesis $\mathscr{H}_i^k$, the value of $L(M_i^k)$ is

$$L(M_i^k) = L(\mathscr{H}_i^k, \hat{z}_i^{\mathrm{ML}}) \quad (11)$$

F. The Estimate Fusion

The minimum mean square state estimate, $\hat{x}(k|k)$, is expressed as a probabilistic mixture using the law of total probability:

$$\hat{x}(k|k) \triangleq E\left(x \left| \mathcal{Y}_{k_0}^k \right.\right) = \sum_{i=0}^{w} \hat{x}_i(k|k)\Pr\left(M_i^k \left| \mathcal{Y}_{k_0}^k \right.\right) \quad (12a)$$

$$\mathbf{P}(k|k) = \sum_{i=0}^{w} \Pr\left(M_i^k \left| \mathcal{Y}_{k_0}^k \right.\right) \times \Big\{\mathbf{P}_i(k|k) + \left[\hat{x}_i(k|k) - \hat{x}(k|k)\right]\left[\hat{x}_i(k|k) - \hat{x}(k|k)\right]^T\Big\} \quad (12b)$$

IV. Comparison of the IMM and A-GLR Estimators

For the sake of comparison, a specific example is considered involving the terminal engagement between an interceptor (the pursuer) and a maneuverable ballistic missile (the evader). During the terminal engagement, the speed of the opponents is assumed constant and the dynamics of the pursuer and the evader are linearized along the initial line of sight. The maneuvering dynamics of the pursuer and evader is approximated by first-order transfer functions with time constants τ_P and τ_E, respectively. The pursuer is equipped with a single on-board sensor which measures the lateral separation between the target and the interceptor. These assumptions result in a fourth-order model of the form of Eq. (1) whose matrices are found in Ref. [2]. In the model, the known input, u, is the pursuer's command acceleration and the unknown input, z, subject to abrupt changes is the target's command acceleration. The value of the pursuer's command acceleration is selected using the DGL/1 guidance law [2]. The engagement lasts 4 seconds and the target performs a bang-bang maneuver at $t = 2.0$ s when the target's command acceleration, z, changes from $+z^{\max}$ to $-z^{\max}$. Both the onset time of the bang-bang maneuver and the value of $z^{\max}$ are unknown to the pursuer. The initial target's acceleration is 0 [g]. The parameters of the simulations are provided in Table I where σ is the standard deviation of the *angular* measurement noise. The covariance of the linearized measurement noise, $\mathbf{Q}_\eta$, is calculated from σ assuming the range, r, is known:

$$\mathbf{Q}_\eta(k) = (r(k)\sigma)^2 \tag{13}$$

For comparison, the IMM estimator is selected since the latter is recognized to have good performance in tracking problems of highly maneuvering targets [5]. Four different IMM estimators, denoted IMM1, ..., IMM4, are compared with two implementations of the A-GLR estimator, denoted AGLR1 and AGLR2. The estimators are compared using the statistics obtained through Monte Carlo simulation which involves 100 different noise realizations. The noise realizations are the same for all the estimators.

A. Selection of Parameters for the Estimators

The AGLR1 and AGLR2 estimators employ a Type I error probability, α, set to $\alpha = 0.001$, and dynamic profiles $f_{\mathscr{H}}$ and f_i set to a constant non-zero values. The hypotheses differ only by the onset time of the abrupt change, $k_i^\star$. At any current time instant k, the hypotheses are selected such that the value of $k_i^\star$ is uniformly distributed in the time interval $k_i^\star \in [k-70, k]$. The (unnormalized) total probability, $\Pr(M_i^k)$, is calculated as follows:

$$\Pr(M_i^k) = \begin{cases} e^{-\frac{1}{2}\left(\hat{z}_0^{\mathrm{ML}}(k) - \hat{z}_{\mathscr{H}}(k)\right)^2/\sigma_a^2}, & i = 0 \\ e^{-\frac{1}{2}\left(|\hat{z}_i^{\mathrm{ML}}(k)| - |\hat{z}_{\mathscr{H}}(k)|\right)^2/\sigma_b^2}, & i \neq 0 \end{cases} \tag{14}$$

where $i \in \{0, \cdots, w\}$, $\sigma_a = 10$ [g] and $\sigma_b = 2$ [g]. The total probability can be unnormalized since it is only used within a ratio, see Eq. (9). Contrary to the IMM estimators, the A-GLR estimator does not employ Markovian transition probabilities between the models. The two A-GLR estimators differ by the number of hypotheses, w. The AGLR1 estimator employs $w = 10$ while the AGLR2 estimator employs $w = 70$.

The estimators IMM1, IMM2, and IMM3 employ a bank of three Kalman filters with shaping filters (SF). The SF approximates the unknown target's command acceleration by a Wiener process acceleration model (WPAM) [13]. The three Kalman filter differs only by the covariance, $\mathbf{Q}_w$, of the WPAM; the values employed are $\mathbf{Q}_w \in \{0, 9, 225\}$ [g^2] for IMM1, and $\mathbf{Q}_w \in \{0, 25, 2500\}$ [g^2] for IMM2 and IMM3. The estimators IMM2 and IMM3 differ only by their Markovian transition probability matrices, see below.

TABLE I

SIMULATION PARAMETERS

Pursuer velocity	$V_P = 2\,300$ m/s
Evader velocity	$V_E = 2\,700$ m/s
Pursuer max. acc.	$u^{\max} = 30$ g
Evader max. acc.	$z^{\max} = 15$ g
Pursuer time constant	$\tau_P = 0.2$ s
Evader time constant	$\tau_E = 0.2$ s
Initial range	$r(0) = 20\,000$ m
Measurement freq.	$f = 100$ Hz
Std. dev. ang. noise	$\sigma = 0.1$ mrad

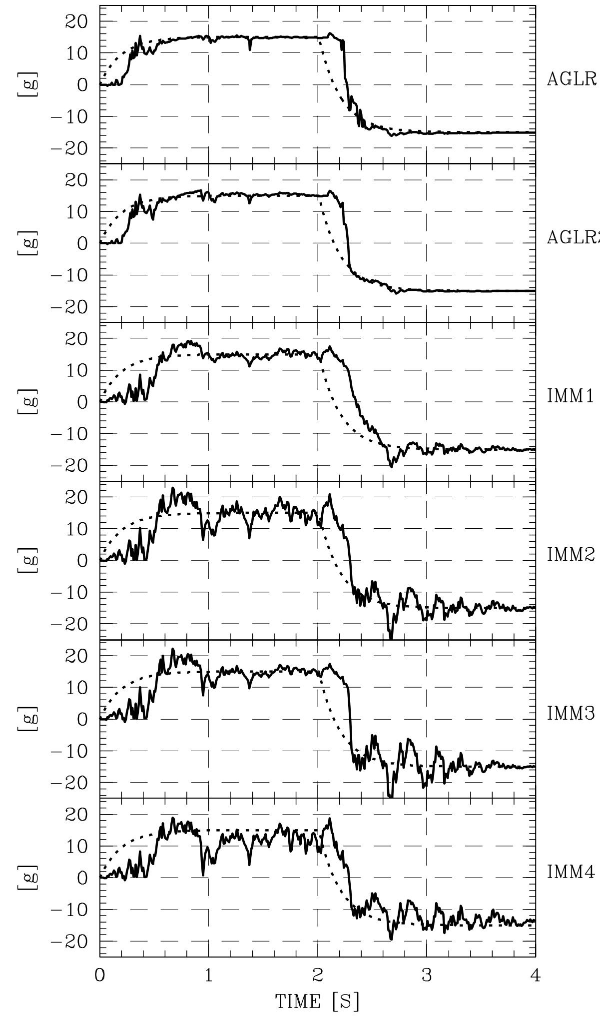

Fig. 2. Estimation of the target's acceleration. Solid line: estimated value, dashed line: true value.

The estimator IMM4 incorporates 9 Kalman filters in its bank. Each filter assumes a constant acceleration level for z. The selected levels are $\{-30, -20, -10, -5, 0, 5, 10, 20, 30\}$ [g]. None of the filters match at any time the true target's command acceleration; the last is realistic since $\hat{z}^{\max}$ is unknown to the pursuer.

The elements, $\Pr(M_i^k | M_j^{k-1})$, of the Markovian transition probability matrix are set to:

$$\left.\begin{matrix}\text{IMM1} \\ \text{IMM2}\end{matrix}\right\} \Longrightarrow \begin{cases} \Pr(M_i^k | M_i^{k-1}) = & 0.98 \\ \Pr(M_i^k | M_j^{k-1}) = & 0.01 \quad i \neq j \end{cases} \tag{15}$$

$$\text{IMM3} \Longrightarrow \begin{cases} \Pr(M_i^k | M_i^{k-1}) = & 0.995 \\ \Pr(M_i^k | M_j^{k-1}) = & 0.0025 \quad i \neq j \end{cases} \tag{16}$$

$$\text{IMM4} \Longrightarrow \begin{cases} \Pr(M_i^k | M_i^{k-1}) = & 0.98 \\ \Pr(M_i^k | M_j^{k-1}) = & 0.0025 \quad i \neq j \end{cases} \tag{17}$$

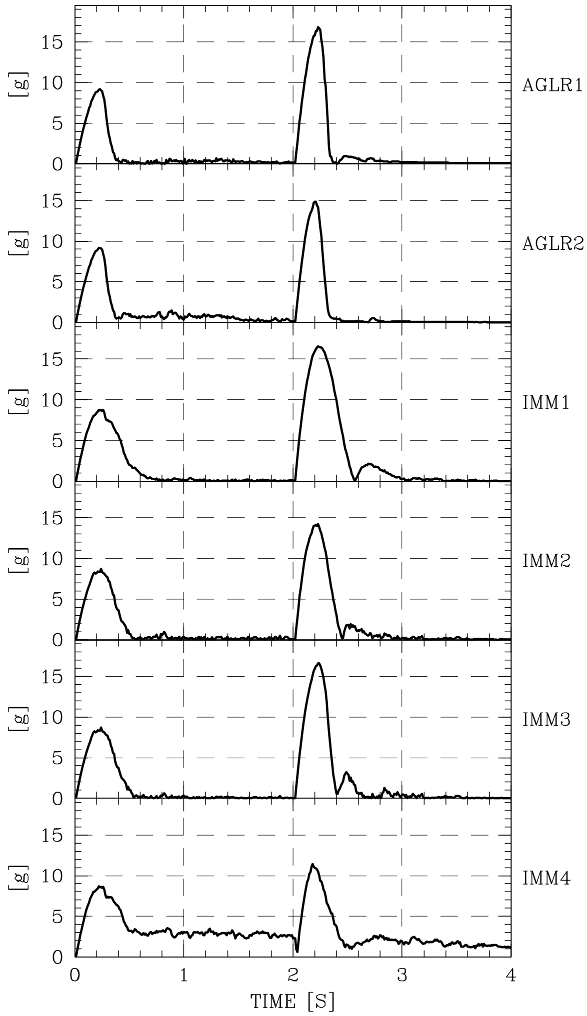

Fig. 3. Magnitude of the average estimation error. The estimate is the target's acceleration.

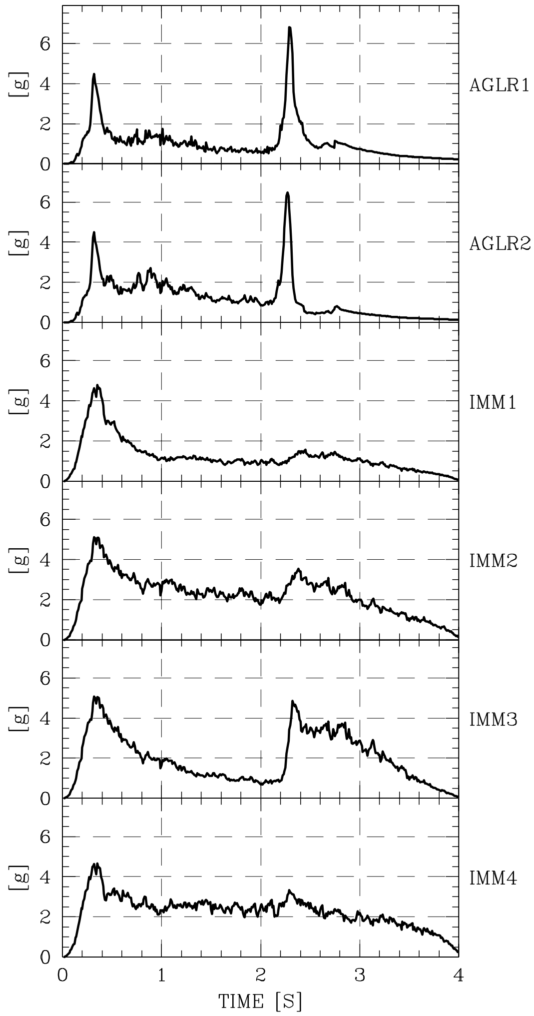

Fig. 4. Standard deviation of the estimation error. The estimate is the target's acceleration.

B. Simulation results

The estimated target's acceleration is depicted in Figure 2 for a sample noise realization; all the estimators employ the same sample noise realization. As compared to the estimates from the IMM estimators, the estimates from the A-GLR estimators are characterized by a better noise rejection and a faster convergence after an abrupt change.

The magnitude of the average error in the estimate of the target's acceleration is depicted in Figure 3. As compared to the estimates from the IMM estimators, the estimates from the A-GLR estimators converge faster after an abrupt change. The IMM1 estimator exhibits both a large average error and a slow convergence of the estimate. The IMM2 shows that increasing the covariance of the SF improves the convergence of the IMM estimate after an abrupt change, but not sufficiently to reach the rate of convergence of the A-GLR estimator. Further increasing the covariance of the SF at a level higher than IMM2 (not shown here) does not yield further improvements in the convergence of the estimate. The IMM3 estimator converges faster than the IMM2 estimator despite using the same bank of filters because it employs lower Markovian transition probabilities. However, the IMM3 estimator also exhibits the largest worse-case average error. The IMM4 estimator is characterized by a biased estimate. Additional simulations, not shown here, employing the IMM4 estimator with different values for its Markovian transition probabilities did not significantly improve performance.

The standard deviation (SD) in the estimate of the target's acceleration is depicted in Figure 4. The estimates from the A-GLR estimators exhibit a peak at $t \sim 2.4$ s, the last occurs in reaction to the abrupt change in the target command acceleration. Before the peak, the A-GLR estimators and the IMM1 estimator yield estimates with similar SD. After the peak, the lowest SD is achieved by the A-GLR estimators. The IMM1 estimator exhibits a SD lower than the IMM2 estimator because the covariance of its SF is the lowest. The IMM3 estimator demonstrates a large SD of its estimate at the beginning of the engagement and after the peak; its low Markovian transition probabilities render the estimate

TABLE II

COMPUTATIONAL REQUIREMENTS

Estimator	time [10^{-2}s]	factor
IMM with 3 filters	31	1×
IMM with 9 filters	129	4.2×
A-GLR with 10 hypotheses	26	0.8×
A-GLR with 70 hypotheses	115	3.7×

sensitive to noise initially and after an abrupt change. The IMM4 estimator is similar to the IMM2 estimator in terms of the requirements for the SD.

To summarize the results in Figures 3 and 4, the estimates from the A-GLR estimators demonstrate simultaneously fast convergence after an abrupt change and a low standard deviation. In the same situation, the IMM estimator can yield either an estimate with a fast convergence, or an estimate with a low standard deviation, *but not both simultaneously*.

The computational load for the A-GLR estimator increases linearly with the number of hypotheses [14]. By comparison, the computational load for the IMM estimator increases faster than linearly with the number of filters; this happens because the IMM procedure for mixing initial conditions is a quadratic operation with respect to the number of filters. The actual computational effort as required by the implemented estimators is displayed in Table II; as a baseline, the factor parameter is assumed to be equal to one for the IMM estimator with 3 filters. In terms of computational effort, the A-GLR estimator with 10 hypotheses is similar to an IMM estimator with 3 filters while the A-GLR estimator with 70 hypotheses is similar to an IMM estimator with 9 filters. The IMM estimator with 9 filters requires five times more computational time than that with 3 filters. The computational effort needed by the A-GLR estimator increases almost linearly with the number of hypotheses.

V. CONCLUDING REMARKS

A novel state estimator is presented for linear systems with unknown inputs subject to additive abrupt changes. The new algorithm is recursive and employs banks of parametric families of input functions in conjunction with a GLR algorithm to yield a state estimate. The estimator is implemented in the difficult case of tracking a randomly maneuvering ballistic missile and its performance is compared to several implementations of the IMM estimator. Extensive simulation results show that the A-GLR estimator delivers a better trade-off between the estimation reliability as expressed by the standard deviation of the estimation error and speed of convergence after an abrupt change. None of the implemented IMM estimators is capable of delivering estimates characterized by a similar standard deviation error while simultaneously exhibiting comparable rate of convergence.

The overall superior performance of the novel A-GLR estimator is attributed to more accurate modeling of the unknown input which is realized by introducing banks of parametric families of input functions describing explicitly the admissible shapes for the unknown input (such as the type of maneuvers performed by a target) and on-line model-matching of the input to members of theses families. The above benefits of the A-GLR estimator are achievable at relatively modest computational expense as, unlike the IMM estimator, *the A-GLR estimator employs only a single Kalman filter*.

Based on the demonstrated superiority of the A-GLR estimator, it is concluded that the new scheme is a powerful tool for state estimation of rapidly maneuvering targets and also may be useful in other estimation problems of similar mathematical description. Current research involves the development of a yet more sophisticated GLR multiple model estimator by allowing interactions between the models.

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