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Adaptive Learning Control for A Class of Nonlinearly Parameterized Uncertain Systems[1]

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Abstract

An adaptive learning control law is developed in this paper to attack a class of nonlinearly parameterized uncertain systems. Specifically, by assuming that the system uncertainty can be separated into an unknown parameter vector multiplying a periodic, yet unknown nonlinear function, we construct an adaptive learning controller to ensure global asymptotic tracking of the system state while compensate for the uncertainty associated with the system parameters and the unknown periodic functions simultaneously. Simulation results are included to demonstrate the efficacy of the proposed adaptive learning control law.

1 Introduction

In many applications, the system dynamics of the plants are either completely or partially unknown. Therefore, the problem of designing controllers for uncertain nonlinear systems has been a topic challenging the control field over decades. As a result, various control methods have been proposed to address specific uncertainty associated with system dynamics. In general, if the structure of the dynamics is known while the system parameters are assumed to be unknown and enter the state equations linearly, in other words, if the system uncertainty satisfies the so-called Linear Parameterization (LP) condition, then adaptive control strategy can be utilized to achieve an asymptotically tracking result [14]. On the other hand, if the system uncertainty can be bounded by some known bounding function, then a sliding mode controller or a robust controller can often be constructed through size dominating in a Lyapunov argument (see [12, 11, 17] and the references within for details). Further, if the system uncertainty is periodic with the period known, a learning controller can then be adopted in which an on-line estimation for the system uncertainty is usually combined with a standard feedback component to achieve the desired control results [3, 4].

Among the aforementioned control strategies, adaptive control often exhibits superb performance as it is not so conservative as the other controllers [10]. Besides, since the adaptive controller is constructed via an on-line estimation for the unknown parameters, the system identification task can then be completed simultaneously if the so-called Persistent Excitation (PE) condition is guaranteed. Unfortunately, the LP condition, as stated previously, is a necessary and strong requirment on the system dynamics to enable the design of a standard adaptive controller, which subsequently limits the application of adaptive control. Based on this reason, many researchers in the control field are investigating the control of nonlinearly parameterized uncertain systems by novel adaptive control strategies [1, 6, 8, 10, 13, 16, 18]. Specifically, various control methodologies have been successfully combined with the standard adaptive control to address these systems. In [6], Ge *etc.* proposed a robust adaptive control approach for a class of time-varying uncertain nonlinear systems in the strict feedback form with completely unknown time-varying virtual control coefficients, uncertain time-varying parameters and unknown time-varying bounded disturbances; unfortunately, the controller can only guarantee a global uniform ultimate boundedness(GUUB) stability. In [10], Qu *etc.* designed a class of adaptive controller based on robust observer for a fractionally parameterized uncertain system to achieve a semiglobal uniform ultimate boundedness stability. Cao *etc.* studied the parameters identification problem for a NPS system and they proposed a polynomial adaptive estimator algorithm in [2] which guarantees parameter convergence, provided that the nonlinear persistent excitation condition is guaranteed. Based on some strong periodic assumptions, [15] then designed an adaptive repetitive controller to asymptotically track a periodic trajectory in which an intergral Lyapunov function is utilized to remove the controller singular-

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ity. More recently, Xu. proposed to attack the nonlinear parameterization systems by fusing the standard adaptive control law with iterative learning control strategy to utilize the advantage of adaptive control, and to enable the closed-loop system to deal with time-varying parametric uncertainties successfully. Specifically, in [16], a periodic type adaptation law is introduced for continuous time systems which updates the parameters and the control signal periodically in a pointwise manner over one entire period and the system achieves asymptotic tracking convergence. Unfortunately, all the controllers presented above have to impose various assumptions on the plant dynamics and thus are only suitable for some specific systems. Therefore, the control of uncertain nonlinear parameterization system is still an open problem challenging the control field.

In this paper, we propose an adaptive learning controller to attack a class of nonlinearly parameterized uncertain systems. Specifically, by assuming that the system uncertainty can be separated into an unknown parameter vector multiplying a periodic, yet unknown nonlinear function, we construct an adaptive learning control law consisting of: (i) a standard feedback term, (ii) an on-line update law for the system parameters, and (iii) a learning-based estimation for the unknown periodic functions. The designed control law ensures global asymptotic tracking of the system state while compensates for the uncertainty associated with the system parameters and the unknown periodic functions. Further, an on-line frequency estimation strategy is fused with the proposed control law to remove the requirement of having the period as a *priori* to implement the controller. The result is based on a stability analysis that involves the application of Lyapunov techniques and some simulation results are included to demonstrate the efficacy of the designed adaptive learning control law.

The remainder of the paper is organized as follows. The dynamics of the uncertain nonlinear system is introduced in Section II. The adaptive learning controller is introduced in Section III, which includes error system development, adaptive learning controller construction and Lyanpunov-based stability analysis for the closed-loop system. The extension of the controller design for the case of unknown periods is described in Section IV. Simulation results illustrating the performance of the proposed control law are presented in Section V and final conclusions are made in Section VI.

2 System Dynamics

In the subsequent analysis, we will attack the control problem of the following nonlinear system

$$\dot{x} = f(x,t) - u \tag{1}$$

where $f(x,t) \in \Re^1$ denotes an unknown function which is nonlinearly parameterized as follows

$$f(x,t) = W(x,\theta_2,t) \cdot \theta_1 \tag{2}$$

where $\theta_1 \in \Re^{p\times 1}$, $\theta_2 \in \Re^{q\times 1}$ denote unknown constant system parameter vectors, $W(x,\theta_2,t) \in \Re^{1\times p}$ represents some nonlinear function whose component can be further rewritten as follows

$$W_i(x,\theta_2,t) = [Y_i(x,t) \cdot g_i(x,\theta_2,t)]\,,\ i = 1,2,...p \tag{3}$$

where $Y_i(x,t) \in \Re$ is measurable and, if $x(t), \dot{x}(t) \in \mathcal{L}_\infty$ then $Y_i(x,t), \dot{Y}_i(x,\dot{x},t) \in \mathcal{L}_\infty$, for $i = 1,2,...p$, and $g_i(x,\theta_2,t) \in \Re^1$ denotes the following unknown periodic function[1]

$$g_i(t) = g_i(t - T_i)\,, \qquad |g_i(t)| \le g_{oi} \tag{4}$$

with $T_i \in \Re^1$ being the known period and $g_{oi} \in \Re^1$ representing a positive bounding constant. Substituting the relationship of (2) and (3) into the system dynamics of (1) yields

$$\dot{x} = \sum_{i=1}^{p} [\theta_{1i} \cdot Y_i(x,t) \cdot g_i(t)] - u. \tag{5}$$

Remark 1 *In many applications, the sign of the system parameters can often be decided as a priori; hence, without loss of generosity, we assume in (2) that the sign of the system parameters is known which then enables us to move the sign into $W(x,\theta_2,t)$ to guarantee that*

$$\theta_{1i} > 0,\ i = 1,2,...p, \tag{6}$$

this fact will be utilized in the subsequent controller design and stability analysis.

Remark 2 *Different from [16], we have separated the system parameters θ_{1i} from the unknown periodic function $g_i(t)$. As shown in the subsuquent controller design, this manipulation provides flexibility of estimating the parameters θ_{1i} by an on-line update law. Further, if the Persistent Excitation (PE) condition is satisfied for the system, then the exact asymptotic identification for the system parameters can be guaranteed.*

[1] Hereon, for simplicity, the arguments x, θ_2 are left out of the function $g_i(\cdot)$.

3 Adaptive Learning Controller

3.1 Error System Development

The control objective is to make the system state track some desired trajectory $x_d \in \Re^1$ with the assumption of $x_d, \dot{x}_d \in \mathcal{L}_\infty$. To aid the subsequent controller construction and stability analysis, the tracking error, denoted as $e(t) \in \Re^1$, is define as follows

$$e = x_d - x. \tag{7}$$

After taking the time derivative of (7) and then substituting (5) into the resulting expression for $\dot{x}(t)$, we obtain the following open-loop error system

$$\dot{e} = \dot{x}_d - \sum_{i=1}^{p} \left[\theta_{1i} \cdot Y_i(x,t) \cdot g_i(t)\right] + u. \tag{8}$$

3.2 Adaptive Learning Controller Design

Based on the open loop dynamics of (8) and the subsequent stability analysis, we design the following adaptive learning control law

$$u = \sum_{i=1}^{p} \left[\hat{\theta}_{1i} \cdot Y_i(x,t) \cdot \hat{g}_i(t)\right] - ke - \dot{x}_d \tag{9}$$

where $k \in \Re^1$ is a positive control gain, $\hat{g}_i(t) \in \Re^1$ represents a learning-based estimate for $g(t)$ that is generated on-line via the following expression

$$\hat{g}_i(t) = \begin{cases} sat_{g_{oi}}\left(\hat{g}_i(t - T_i)\right) - k_{2i} e Y_i, & t \geq T_i \\ -k_{2i}\left(\frac{t}{T_i}\right) e Y_i, & 0 \leq t \leq T_i \end{cases} \quad i = 1, 2, \ldots p \tag{10}$$

where $k_{2i} \in \Re^1$ denotes a positive control gain, and the saturation function $sat_{g_{oi}}(\cdot)$ is defined in the following manner

$$sat_{g_{oi}}(x) = \begin{cases} x, & \text{for } |x| \leq g_{oi} \\ g_{oi}, & \text{for } x > g_{oi} \\ -g_{oi}, & \text{for } x < -g_{oi} \end{cases} \quad i = 1, 2, \ldots p. \tag{11}$$

In (9), $\hat{\theta}_{1i}(t)$ represents the on-line estimation of θ_{1i} updated as follows

$$\dot{\hat{\theta}}_{1i} = -\Gamma_i Y_i(x,t) \cdot \hat{g}_i(t) \cdot e, \ i = 1, 2, \ldots p \tag{12}$$

where $\Gamma_i \in \Re$ denotes a positive parameter update gain. From the definition of $sat_{g_{oi}}(\cdot)$ given in (11), the following inequality can be proven [5]

$$\left[g_i(t) - sat_{g_{oi}}\left(\hat{g}_i(t)\right)\right]^2 \leq \left[g_i(t) - \hat{g}_i(t)\right]^2. \tag{13}$$

Substituting the adaptive learning controller $u(t)$ into the system dynamics of (8) yields

$$\begin{aligned} \dot{e} &= \sum_{i=1}^{p} \left[\hat{\theta}_{1i} \cdot Y_i(x,t) \cdot \hat{g}_i(t)\right] \\ &\quad - \sum_{i=1}^{p} \left[\theta_{1i} \cdot Y_i(x,t) \cdot g_i(t)\right] - ke. \end{aligned} \tag{14}$$

After adding and subtracting $\sum_{i=1}^{p} \left[Y_i \theta_{1i} \hat{g}_i\right]$ from (14) and performing some mathematical manipulation for the resulting expression, we obtain

$$\dot{e} = -ke - \sum_{i=1}^{p} \left(Y_i \tilde{\theta}_{1i} \hat{g}_i\right) - \sum_{i=1}^{p} \left(Y_i \theta_{1i} \tilde{g}_i\right) \tag{15}$$

where $\tilde{\theta}_{1i}(t) \in \Re^1$ denotes the following parameter estimation error

$$\tilde{\theta}_{1i} = \theta_{1i} - \hat{\theta}_{1i}, \ i = 1, 2, \ldots p \tag{16}$$

then

$$\dot{\tilde{\theta}}_{1i} = -\dot{\hat{\theta}}_{1i}, \ i = 1, 2, \ldots p, \tag{17}$$

and $\tilde{g}_i(t) \in \Re^1$ representes the learning estimation error of $g_i(t)$

$$\tilde{g}_i(t) = g_i(t) - \hat{g}_i(t), \ i = 1, 2, \ldots p. \tag{18}$$

After substituting (10) into (18) for $\hat{g}_i(t)$ and performing some mathematical manipulation, we can show the fact that for $\forall t \geq T_i$, $i = 1, 2, \ldots p$

$$\begin{aligned} \tilde{g}_i(t) &= g_i(t) - \hat{g}_i(t) \\ &= g_i(t - T_i) - sat_{g_{oi}}\left(\hat{g}_i(t - T_i)\right) + k_{2i} e Y_i \end{aligned} \tag{19}$$

where (4) has been utilized.

Remark 3 *The introduction of the saturation function (11) in the learning law of (10) aims to guarantee the boundedness of the learning based estimate $\hat{g}_i(t)$ and the adaptive learning controller of (9).*

Remark 4 *It can be shown that the learning law designed in (10) is continuous, as a result, the constructed adaptive learning controller of (9) is subsequently continuous and thus physically realizable.*

3.3 Stability Analysis[2]

Theorem 1 *The proposed adaptive learning control law of (9) ensures global asymptotic tracking of the system state in the sense that*

$$\lim_{t \to \infty} e(t) = 0. \tag{20}$$

Proof: In order to prove Theorem 1, the following non-negative, scalar function $V(t) \in \Re^1$ is defined

$$\begin{aligned} V &= \frac{1}{2} e^2 + \sum_{i=1}^{p} \frac{1}{2\Gamma_i} \tilde{\theta}_{1i}^2 + \\ &\quad \sum_{i=1}^{p} \left\{ \frac{\theta_{1i}}{2k_{2i}} \int_{t-T_i}^{t} \left[g_i(\sigma) - sat_{g_{oi}}\left(\hat{g}_i(\sigma)\right)\right]^2 d\sigma \right\} \end{aligned} \tag{21}$$

[2] As the learning law of (10) is piecewise continuous, the strict stability ananlysis should be separated into two steps. That is, the first step aims to show that all the signals remain bounded in the first period, and the second step then proves that the tracking error converges afterwards; However, as the first step is pretty straightforward, we will focus our analysis on the second step to show the convergence of the tracking error.

After taking the time derivative of (21), the following expression for $\dot{V}(t)$ can be obtained

$$\begin{aligned} \dot{V} &= e\left[-ke-\sum_{i=1}^{p}\left(Y_i\tilde{\theta}_{1i}\hat{g}_i\right)-\sum_{i=1}^{p}\left(Y_i\theta_{1i}\tilde{g}_i\right)\right] \qquad (22) \\ &\quad -\sum_{i=1}^{p}\frac{1}{\Gamma_i}\tilde{\theta}_{1i}\,\dot{\hat{\theta}}_{1i}+\sum_{i=1}^{p}\frac{\theta_{1i}}{2k_{2i}}\left[g_i(t)-sat_{g_{oi}}\left(\hat{g}_i(t)\right)\right]^2 \\ &\quad -\sum_{i=1}^{p}\frac{\theta_{1i}}{2k_{2i}}\left[g_i(t-T_i)-sat_{g_{oi}}\left(\hat{g}_i(t-T_i)\right)\right]^2 \end{aligned}$$

where (15) and (17) have been utilized. After substituting the update law of (12) into (22) for $\dot{\hat{\theta}}_{1i}(t)$ and cancelling common terms, we obtain

$$\begin{aligned} \dot{V} &= -ke^2-\sum_{i=1}^{p}\left(eY_i\theta_{1i}\tilde{g}_i\right) \qquad (23) \\ &\quad +\sum_{i=1}^{p}\frac{\theta_{1i}}{2k_{2i}}\left[g_i(t)-sat_{g_{oi}}\left(\hat{g}_i(t)\right)\right]^2 \\ &\quad -\sum_{i=1}^{p}\frac{\theta_{1i}}{2k_{2i}}\left[g_i(t-T_i)-sat_{g_{oi}}\left(\hat{g}_i(t-T_i)\right)\right]^2. \end{aligned}$$

After applying the relationship of (19) to the last term of (23) and then simplifying the resulting expression, $\dot{V}(t)$ can be further rewritten into the following manner

$$\begin{aligned} \dot{V} &= -ke^2-\sum_{i=1}^{p}\frac{k_{2i}}{2}\theta_{1i}\left[eY_i\right]^2 \qquad (24) \\ &\quad +\sum_{i=1}^{p}\frac{\theta_{1i}}{2k_{2i}}\left\{\left[g_i(t)-sat_{g_{oi}}\left(\hat{g}_i(t)\right)\right]^2-\left[\tilde{g}_i(t)\right]^2\right\}. \end{aligned}$$

We can then utilize (13) to upper bound (24) in the following manner

$$\dot{V}\leq -ke^2 \qquad (25)$$

where (6) has been utilized.

From (21), (25), (7) and (16), we can prove that $V(t)$, $e(t)$, $x(t)$, $Y_i(x,t)$, $\tilde{\theta}_{1i}(t)$, $\hat{\theta}_{1i}(t)\in\mathcal{L}_\infty$ and that $e(t)\in\mathcal{L}_2$. Based on the previous facts, (10), (9) and (8) can then be utilized to show that $\hat{g}_i(t)$, $u(t)$, $\dot{e}(t)\in\mathcal{L}_\infty$. Based on the facts of $e(t)\in\mathcal{L}_\infty\cap\mathcal{L}_2$ and $\dot{e}(t)\in\mathcal{L}_\infty$, Barbalat's Lemma [14] can then be employed to illustrate the result presented in (20). We can now apply standard signal chasing arguments to conclude that all signals in the control and system remain bounded during closed-loop operation.

4 Extension to the Case of Unknown Periods

In the previous section of adaptive learning control law development, the peroids T_i has been assumed as a *priori;* unfortunately, the period of an unknown signal is usually difficult to determine in practice. Therefore, it is imperative to design controllers to address the nonlinearly parameterized uncertain systems of (5) with unknown periods T_i. Based on this thought, we apply the adaptive notch filter presented in [9] to on-line estimate the unknown frequency, and then combine it with the proposed adaptive learning control law (9) to attack the nonlinear problem of (5) with unknown periods. Specifically, as described in [9], the frequency of the unknown periodic signal $g_i(t)$ can be on-line estimated via the following adaptive notch filter

$$\begin{cases} \ddot{x}+2\xi\hat{\omega}\dot{x}+\hat{\omega}^2x=2\xi\hat{\omega}^2g_i(t) \\ \dot{\hat{\omega}}=-\gamma x\left[2\xi\hat{\omega}^2g_i(t)-2\xi\hat{\omega}\dot{x}\right] \end{cases} \qquad (26)$$

where $\hat{\omega}(t)\in\Re^1$ represents the estimated frequency, $\xi\in\Re^1$ is the the damping ratio and $\gamma\in\Re^1$ denotes an adaptation gain. It can then be shown that the estimation $\hat{\omega}(t)$ approaches to the unknown frequency ω asymptotically in the sense that [9]

$$\lim_{t\to\infty}\hat{\omega}(t)=\omega. \qquad (27)$$

Based on that fact, the periods can then be calculated by the estimation of the frequency as follows

$$\hat{T}=\frac{2\pi}{\hat{\omega}} \qquad (28)$$

which can then be passed to the learning law of (10) to learn the unknown periodic signals $g_i(t)$.

Remark 5 *Though we cannot theoretically prove that the control strategy consisting of the adaptive learning control law (9) and the adaptive notch filter (26) achieves convergence for a nonlinearly parameterized uncertain system with unknown frequency, simulation results presented in the next section demonstrate the efficacy of this overall control scheme.*

5 Simulation Results

To illustrate the performance of the proposed adaptive learning control law (9), the following system were simulated via Matlab's Simulink

$$\dot{x}=a_1\sin(\omega_1t+\phi_1)(1+x)+a_2\cos(\omega_2t+\phi_2)x^2-u \qquad (29)$$

where the system parameters were choses as

$$\begin{matrix} a_1=2, & \omega_1=2, & \phi_1=1; \\ a_2=4, & \omega_2=1, & \phi_2=1 \end{matrix} \qquad (30)$$

then

$$\begin{matrix} \theta_{11}=a_1=2, & Y_1(t)=1+x, & g_1(t)=\sin(2t+1); \\ \theta_{12}=a_2=4, & Y_2(t)=x^2, & g_2(t)=\cos(t+1). \end{matrix}$$

The desired trajectory $x_d(t)$ was selected in the following manner

$$x_d(t) = \sin(5t). \tag{31}$$

To illustrate the convergence abilities of the proposed control law, $x(t)$ is initialized as

$$x(0) = -1, \tag{32}$$

and the parameter estimate $\hat{\theta}_{1i}$ were all set to zero. In the simulation, the frequencies of the unknown periodic signals $g_1(t)$ and $g_2(t)$ were estimated on-line via the strategy of (26) with the initial frequencies chosen as

$$\hat{\omega}_1(0) = 20, \qquad \hat{\omega}_2(0) = 20. \tag{33}$$

The control and adaptation gains were tuned by trial-and-error method until a good tracking performance was achieved. This resulted in the following set of control and adaptation gains

$$k = 10, \qquad k_{21} = 20, \qquad k_{22} = 10 \tag{34}$$

$$\Gamma_1 = 5, \qquad \Gamma_2 = 100, \tag{35}$$

and the following gains for the frequency estimation

$$\begin{aligned} \xi_1 &= 0.7, & \gamma_1 &= 0.3; \\ \xi_2 &= 0.8, & \gamma_2 &= 0.3. \end{aligned}$$

Figure 1 illustrates the tracking error performance while the control input is shown in Figure 2. The frequency estimation and the system parameter estimation are presented in Figures 3 and 4, respectively.

6 Conclusions

In this paper, an adaptive learning control law is proposed to attack a class of nonlinearly parameterized uncertain systems which ensures global asymptotic tracking for the system state while compensates for the uncertainty associated with the system parameters and the unknown periodic functions. Further, the control law is extended by fusing it with an online frequency estimation strategy to remove the requirement of having the periods as a *priori* to implement the controller. Simulation results are included to demonstrate the efficacy of the proposed control law. Future work will focus on testing the proposed control law by experimental results. Future work will also target on designing nonlinear controllers for more general uncertain nonlinear systems.

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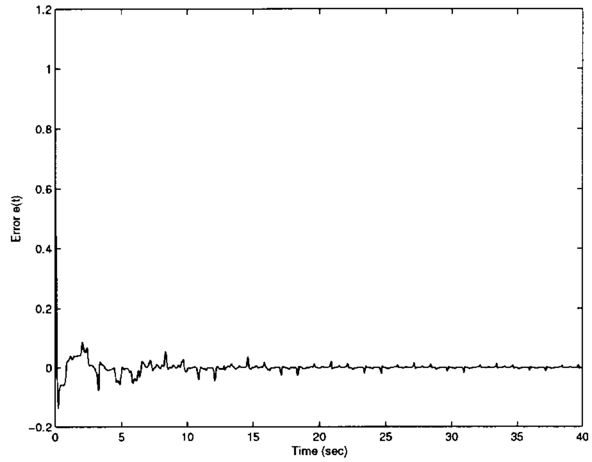

Figure 1: Tracking Error $e(t)$

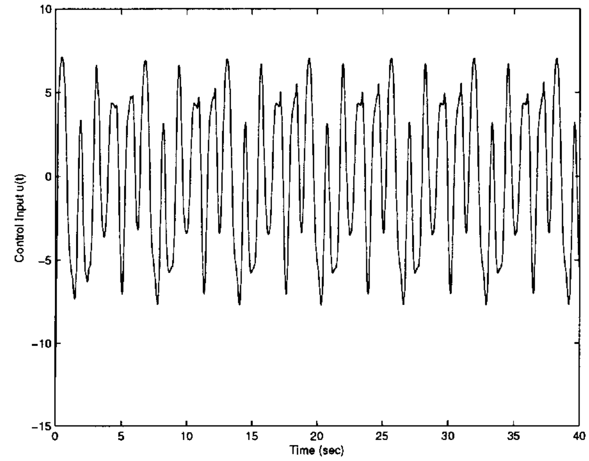

Figure 2: Control Input $u(t)$

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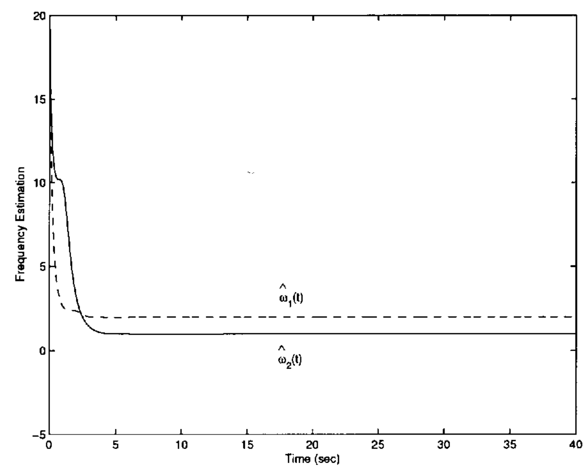

Figure 3: Frequency Estimation

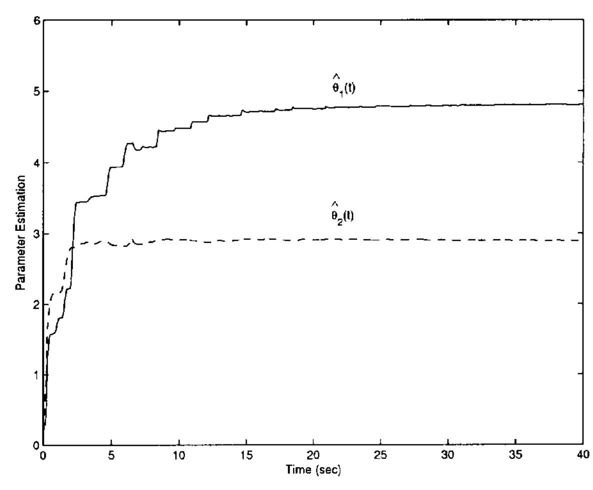

Figure 4: Parameter Estimation