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Convergence Analysis of Terminal ILC in the z Domain

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Abstract—**This paper shows how we can apply z-transform theory to analyze the convergence of a terminal ILC algorithm. This approach uses an equivalent system viewed in the cycle domain and analyzes it with a z-transform. Then, conventional discrete time control is applied to the equivalent system. This control is viewed by the real system as a cycle-to-cycle control. Therefore, the stability analysis of the controlled equivalent system corresponds to convergence analysis used in ILC. Furthermore, a "dead beat" convergence is feasible and corresponds to the fastest convergence rate of the ILC algorithm.**

I. Introduction

The idea of iterative learning control (ILC) is to use the knowledge of previous output error measurement to update the input so as to reduce the error. Many papers have been written about ILC; see the survey paper by Moore [1]. The terminal ILC control (also called point-to-point ILC) is an approach whose goal is to reduce the error at the end of the cycle. In rapid thermal processing, terminal ILC helps to reduce thickness error [2-6]. In our project we want to apply terminal ILC to the reheat phase of the thermoforming process. To know more about thermoforming, refer to [7].

In most works [2-5] the behavior of the terminal ILC is analyzed via a classic convergence analysis, in the sense of the evolution of the norm of the error.

In this paper, we will use a new approach based on an equivalent system built in the cycle domain, from the system in the time domain. A closed-loop analysis is done in the z-domain with a controller connected to the equivalent system. This control appears for the system in the time domain as a cycle-to-cycle control. The stability analysis done on the closed-loop equivalent system corresponds to a convergence analysis done with the corresponding cycle-to-cycle control.

In Section II, we define the system analyzed with our approach. Section III considers terminal ILC control for the SISO version of the system defined in Section II. Section IV does the same for the MIMO case. Section V gives simulation results obtained with terminal ILC designed using the analysis done in Sections III and IV. Finally, Section V concludes and suggests some directions for future work with this approach.

II. Defining System to Control with Terminal ILC

In this paper, we apply terminal ILC control to a linear discretized system represented by:

$$\begin{aligned} x_k(t+h) &= Ax_k(t) + Bu_k(t) \\ y_k(t) &= Cx_k(t) \end{aligned} \tag{1}$$

where $t \in \{0, h, 2h, \cdots, Nh\}$, h is the sampling period, N+1 is the number of samples per cycle, and the subscript $k \in \mathbb{Z}_+$ is the cycle number. Matrices A, B, and C are time invariant. The state vector is $x_k(t) \in \mathbb{R}^n$, the input vector is $u_k(t) \in \mathbb{R}^m$, and the output vector is $y_k(t) \in \mathbb{R}^p$. Here, we assume that the number of inputs is the same as the number of outputs, so $p = m$.

The control task is to update the control input $u_k(t)$ after cycle k-1 such that the terminal output $y_k(Nh)$ converges to a given terminal value $y_d \in \mathbb{R}^p$ at time Nh. From linear system theory, one can write the solution of (1) at $t = Nh$:

$$x_k(Nh) = A^N x_k(0) + \sum_{j=0}^{N-1} A^{N-j-1} Bu_k(j) \tag{2}$$

From this terminal state we calculate the corresponding terminal output as:

$$y_k(Nh) = CA^N x_k(0) + C\sum_{j=0}^{N-1} A^{N-j-1} Bu_k(j) . \tag{3}$$

For the particular case of a thermoforming process, we keep the control input (heater temperature setpoint) constant during a cycle. So in this case, we can define

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$u_k(\tau) = u_k(t) =: u(k)$, $\forall \tau \in [0, Nh]$ and rewrite (3) as:

$$y_T(k) = \Gamma x_0(k) + \Psi u(k), \tag{4}$$

where $y_T(k) = y_k(Nh)$, $x_0(k) = x_k(0)$ and the constant matrices Γ and Ψ are defined as:

$$\Gamma := CA^N \tag{5}$$

and:

$$\Psi := C\sum_{j=0}^{N-1} A^{N-j-1}B. \tag{6}$$

The change in notation is to emphasize the fact that for the cycle-to-cycle control cycle *k*, can be looked at as a discrete step so that the system (4) is equivalent to discrete time system. Then, we will apply in cycle control on the terminal output (4) that will appear like cycle-to-cycle control to the system (1).

The following assumptions are made for this paper:

A1) Repetition of the initial state is satisfied. Then $x_k(0) = x_0(k)$ must be at the same value for all cycles.

A2) There exists a unique input $u_k(t) = u(k)$ such that the system exhibits the output $y_T(k) = y_k(Nh)$. This forces the matrix Ψ to be of full rank. Hence, the system must be completely observable and controllable.

III. ILC CONTROL OF AN SISO SYSTEM

The terminal output (4) can be controlled by any in cycle control. In this section, we assume that the corresponding system (1) is SISO. We analyze this SISO system controlled by proportional cycle-to-cycle control and by integral cycle-to-cycle control.

A. Proportional cycle-to-cycle control of an SISO system

The proportional cycle-to-cycle controller for (1) is defined as:

$$u(k) = k_p e(k-1) = k_p\left(y_d - y_T(k-1)\right), \tag{7}$$

where $k_p > 0$ is the proportional gain of the controller.

We can use z-transform theory to analyze the behavior of the controlled system. Then, the z-transform of (4) can be written as:

$$\hat{y}_T(z) = \Psi\hat{u}(z) + \Gamma\hat{x}_0(z) \tag{8}$$

and the z-transform of (7) is:

$$\hat{u}(z) = z^{-1}k_p\left(\hat{y}_d(z) - \hat{y}_T(z)\right) \tag{9}$$

where $\hat{u}(z), \hat{y}_T(z), \hat{y}_d(z)$ and $\hat{x}_0(z)$ are the z-transforms of $u(k), y_T(k), y_d(k)$ and $x_0(k)$, respectively.

The closed-loop transfer function is obtained next:

$$\hat{y}_T(k) = z^{-1}\Psi k_p\left(\hat{y}_d(z) - \hat{y}_T(z)\right) + \Gamma\hat{x}_0(z). \tag{10}$$

Rearranging the terms in (10), one can write:

$$\hat{y}_T(z) = \frac{z^{-1}\Psi k_p}{1 + z^{-1}\Psi k_p}\hat{y}_d(z) + \frac{\Gamma}{1 + z^{-1}\Psi k_p}\hat{x}_0(z). \tag{11}$$

Proposition 1: Assume a constant initial state vector $x_0(k) = \chi_0$ and let the desired terminal value be $y_d = \gamma$. With the constraint that $k_p > 0$, the proportional cycle-to-cycle control will converge to a terminal value of:

$$y_T(\infty) = \frac{\Psi k_p}{1 + \Psi k_p}\gamma + \frac{\Gamma}{1 + \Psi k_p}\chi_0 \tag{12}$$

as $k \to \infty$ iff:

$$0 < k_p < 1/\Psi. \tag{13}$$

Proof: The z-transform of $x_0(k) = \chi_0$ and $y_d = \gamma$ are:

$$\hat{x}_0(z) = \chi_0 \big/ \left(1 - z^{-1}\right), \tag{14}$$

and:

$$\hat{y}_d(z) = \gamma \big/ \left(1 - z^{-1}\right). \tag{15}$$

By applying the final value theorem, one can write:

$$\begin{aligned} y_T(\infty) &= \lim_{k\to\infty} y_T(k) = \lim_{z\to 1}\left[\left(1 - z^{-1}\right)\hat{y}_T(z)\right] \\ &= \frac{\Psi k_p}{1 + \Psi k_p}\gamma + \frac{\Gamma}{1 + \Psi k_p}\chi_0. \end{aligned} \tag{16}$$

The stability of the closed-loop system (corresponding to the convergence of the cycle-to-cycle control as given in (12)) depends on the root of the characteristic equation:

$$z + \Psi k_p = 0. \tag{17}$$

From z-transform theory, we know that the closed-loop system is stable iff the root $z_1 := -\Psi k_P$ is strictly inside the unit circle. Since $k_p > 0$, we find that for $|z_1| < 1$, we must have $0 < k_p < 1/\Psi$.

□

B. Integral cycle-to-cycle control of an SISO system

Suppose we try to control the terminal output (4) with an integral control law expressed in the z-domain as:

$$\hat{u}(z) = \frac{z^{-1}}{1 - z^{-1}}k_I\left(\hat{y}_d(z) - \hat{y}_T(z)\right), \tag{18}$$

where $k_I > 0$ is the integral gain of the controller.

If we express this control law in the cycle domain, we can write it as:

$$\begin{aligned} u(k) &= u(k-1) + k_I\left(y_d - y_T(k-1)\right) \\ &= u(k-1) + k_I e(k-1). \end{aligned} \tag{19}$$

One can see in (19), the usual ILC control law, named "integral type ILC" (I-ILC). On some papers about terminal ILC [2-5], a convergence analysis is performed. Here, we analyze the stability in the z-domain. The two analyses are equivalent (when we consider ILC control of order one in [2-5]), therefore stability in the z-domain implies convergence of the terminal I-ILC algorithm.

Combining (8) and (18), we can get the closed-loop transfer function:

$$\hat{y}_T(k) = \frac{z^{-1}\Psi k_I}{1-z^{-1}}\left(\hat{y}_d(z) - \hat{y}_T(z)\right) + \Gamma\hat{x}_0(z). \tag{20}$$

Rearranging all the terms, we have:

$$\begin{aligned} \hat{y}_T(z) &= \frac{z^{-1}\Psi k_I}{1+\left(\Psi k_I - 1\right)z^{-1}}\hat{y}_d(z) \\ &+ \frac{\left(1-z^{-1}\right)\Gamma}{1+\left(\Psi k_I - 1\right)z^{-1}}\hat{x}_0(z). \end{aligned} \tag{21}$$

Proposition 2: Assume a constant initial state vector $x_0(k) = \chi_0$ and the desired terminal value $y_d = \gamma$. The terminal I-ILC control will converge to the desired terminal value:

$$y_T(\infty) = \gamma \tag{22}$$

as $k \to \infty$ iff:

$$0 < k_I < 2/\Psi. \tag{23}$$

Proof: With the z-transform of $x_0(k) = \chi_0$ and $y_d = \gamma$ defined in (14) and (15) and by applying the final value theorem, we can write:

$$y_T(\infty) = \lim_{k\to\infty} y_T(k) = \lim_{z\to 1}\left[\left(1-z^{-1}\right)\hat{y}_T(z)\right] = \gamma. \tag{24}$$

Again, the stability of the closed-loop system depends on the root of this characteristic equation:

$$z + \left(\Psi k_I - 1\right) = 0. \tag{25}$$

Then, the closed-loop root $z_1 := 1 - \Psi k_I$ must be strictly inside the unit circle to ensure stability (convergence). So to have $|z_1| = |1 - \Psi k_I| < 1$, the gain k_I must satisfy $0 < k_I < 2/\Psi$.

□

As one can see, we have convergence of the terminal output to the desired one if the integral gain is selected properly.

When the gain is set to $k_I = 1/\Psi$, we have the root of the characteristic equation equal to 0. This is known as the "dead beat response" in discrete time control. For that particular gain, the convergence of the terminal I-ILC algorithm is obtained in only one cycle. That is the fastest rate of convergence that the I-ILC algorithm can achieve.

Proposition 3: Assume a constant initial state vector $x_0(k) = \chi_0$ and the desired terminal value $y_d = \gamma$. Assume also a gain $k_I = 1/\Psi$. The terminal I-ILC control will converge to the desired terminal value in one cycle.

Proof: With the selected gain we can rewrite (21) as:

$$\hat{y}_T(z) = z^{-1}\hat{y}_d(z) + \left(1-z^{-1}\right)\Gamma\hat{x}_0(z) \tag{26}$$

Using the inverse z-transform, we can write:

$$y_T(k) = y_d(k-1) + \Gamma\left(x_0(k) - x_0(k-1)\right) \tag{27}$$

So at the second cycle, we have directly the desired terminal output as $y_T(1) = y_d(0) = \gamma$ and we stay on it for all subsequent cycle.

□

In practice, the knowledge of Ψ is approximate and then, using an ILC algorithm is useful. Therefore, it is difficult to obtain the dead beat response. If knowledge of Ψ is perfect (and for a known constant initial state vector $x_0(k) = \chi_0$), one can calculate directly the input without using ILC, so it becomes useless.

IV. EXTENSION TO MIMO SYSTEMS

In the previous section, we analyzed the closed-loop behavior of (4) using proportional and integral control. Since the integral control was shown as the more effective approach in the SISO case, we will use only this control approach on MIMO systems. Here, we assume that the number of inputs is equal to the number of outputs.

The integral control law for MIMO systems can be defined as:

$$\begin{aligned} \hat{y}(z) &= G_{I-ILC}(z)\left(\hat{y}_d(z) - \hat{y}_T(z)\right) \\ &= \frac{z^{-1}}{1-z^{-1}}K_{I-ILC}\left(\hat{y}_d(z) - \hat{y}_T(z)\right) \end{aligned} \tag{28}$$

where K_{I-ILC} is a positive definite diagonal matrix and:

$$G_{I-ILC}(z)=\frac{z^{-1}}{1-z^{-1}}K_{I-ILC}\,. \quad (29)$$

From (8) and (28), one can write this MIMO closed-loop equation:

$$\hat{y}_T(z)=\frac{z^{-1}}{1-z^{-1}}\Psi K_{I-ILC}\left(\hat{y}_d(z)-\hat{y}_T(z)\right)+\Gamma\hat{x}_0(z) \quad (30)$$

which we can simplify to:

$$\begin{aligned}\hat{y}_T(z)&=z^{-1}\left\{I+\left(\Psi K_{I-ILC}-I\right)z^{-1}\right\}^{-1}\Psi K_{I-ILC}\hat{y}_d(z)\\&+\left\{I+\left(\Psi K_{I-ILC}-I\right)z^{-1}\right\}^{-1}\left(1-z^{-1}\right)\Gamma\hat{x}_0(z)\,.\end{aligned} \quad (31)$$

Proposition 4: Assume a constant initial state vector $x_0(k)=\chi_0$ and the desired terminal output vector $y_d=\gamma$. The MIMO terminal I-ILC control will converge to a terminal value of:

$$y_T(\infty)=\gamma \quad (32)$$

as $k\to\infty$ if all roots of $\det\left[Iz+\left(\Psi K_{I-ILC}-I\right)\right]=0$ are such that:

$$\left|z_j\right|<1,\forall j\in\left[1,\cdots,m\right] \quad (33)$$

Proof: Following the proof of Proposition 2, by applying the final value theorem, we can write:

$$y_T(\infty)=\lim_{k\to\infty}y_T(k)=\lim_{z\to1}\left[\left(1-z^{-1}\right)\hat{y}_T(z)\right]=\gamma\,. \quad (34)$$

Now, the stability of the closed-loop system depends on the root of the characteristic equation obtained by calculating:

$$\det\left[Iz+\left(\Psi K_{I-ILC}-I\right)\right]=0 \quad (35)$$

Because we assume that the number of inputs equals the number of outputs, the order of the characteristic equation will be equal to the number of inputs and outputs. For stability (convergence), the roots $z_1,z_2,\cdots,z_m$ must lie strictly inside the unit circle, then $\left|z_j\right|<1,\forall j\in\left[1,\cdots,m\right]$.

□

Case 1: Ψ diagonal. That implies the complete decoupling of each input/output dynamic. In that case, we have:

$$\det\left[Iz+\left(\Psi K_{I-ILC}-I\right)\right]=\prod_{j=1}^{m}\left(z+\left(\Psi_{jj}k_{Ij}-1\right)\right)=0 \quad (36)$$

where $k_{Ij},\forall j\in\left\{1,\cdots,m\right\}$ are the elements of the diagonal of K_{I-ILC}. Then finally each gain k_{Ij} must be such that:

$$0<k_{Ij}<2/\Psi_{jj} \quad (37)$$

to ensure stability (convergence).

Case 2: Ψ triangular. In that case, we have the same result as case 1, since the determinant of a diagonal matrix is the product of all element of the diagonal. So we can find the gains using (37).

Case 3: Ψ is neither diagonal nor triangular. In that case no simplification can be made and we must calculate the roots of:

$$\det\left[Iz+\left(\Psi K_{I-ILC}-I\right)\right]=0 \quad (38)$$

From (38) we can find gains that ensure that all $\left|z_j\right|<1,\forall j\in\left[1,\cdots,m\right]$.

Another way to state the roots condition of (35) is to say that the eigenvalues of ΨK_{I-ILC} are strictly inside a unit circle centered at (1,0). And that is equivalent to say that:

$$\left\|\Psi K_{I-ILC}-I\right\|<1 \quad (39)$$

This kind of norm inequality appears in the ILC literature [1-5].

The "dead beat" convergence for MIMO systems is equivalent to the "dead beat" convergence in the SISO case if the Ψ matrix is diagonal when the gain matrix K_{I-ILC} is also diagonal. For the case of non-diagonal Ψ matrix, "dead beat" convergence is not achievable with a diagonal gain matrix K_{I-ILC}.

A variant of terminal I-ILC control can be defined by relaxing the diagonal positive definite constraint on K_{I-ILC}. And then, a good choice for the matrix gain is $K_{I-ILC}=\Psi^{-1}$. With this choice of matrix gain we can achieve "dead beat" control and have the MIMO system converge in only one cycle.

Proposition 5: Assume a constant initial state vector $x_0(k)=\chi_0$ and the desired terminal output vector $y_d=\gamma$. Assume also we have defined $K_{I-ILC}=\Psi^{-1}$. The MIMO terminal I-ILC control will converge to the desired terminal value in only one cycle.

Proof: Using an approach similar to the proof of proposition 3, we can rewrite (31) as:

$$\hat{y}_T(z)=z^{-1}\hat{y}_d(z)+\left(1-z^{-1}\right)\Gamma\hat{x}_0(z) \quad (40)$$

since $K_{I-ILC}=\Psi^{-1}$.

Using the inverse z-transform, one can write:

$$y_T(k) = y_d(k-1) + \Gamma\left(x_0(k) - x_0(k-1)\right). \quad (41)$$

So at the second cycle (k=1), we have directly the desired terminal value at each output and we stay on it for all subsequent cycles like in the SISO case.

□

On the next section, simulation results will show the effectiveness of the I-ILC algorithm on MIMO system.

V. Simulation Results

To show the effectiveness of the control, we will take as example the following MIMO system obtained by discretizing a continuous time system with a sampling period h = 1 s:

$$\begin{aligned} x_k(t+h) &= \begin{bmatrix} 1 & 0.4323 & 0 & 0.3167 \\ 0 & 0.1353 & 0 & 0 \\ 0 & 0 & 1 & 0.3167 \\ 0 & 0 & 0 & 0.04979 \end{bmatrix} x_k(t) \\ &+ \begin{bmatrix} 0.2838 & 0.2278 \\ 0.4323 & 0 \\ 0 & 0.2278 \\ 0 & 0.3167 \end{bmatrix} u_k \quad (42) \\ y_k(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x_k(t) \end{aligned}$$

The initial state of the system is $x_k(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$ ($\forall k \in \mathbb{Z}_+$) and the initial input applied is $u_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. We want to reach the desired terminal value $y_d = \begin{bmatrix} 2 & 3 \end{bmatrix}^T$ at t = 10 s.

To design a terminal I-ILC control, we need to calculate:

$$\Psi = \begin{bmatrix} 4.75 & 3.2222 \\ 0 & 3.2222 \end{bmatrix}. \quad (43)$$

Case 1: Design with a diagonal gain matrix K_{I-ILC}.

In this case, the terminal I-ILC control will be (in the z-domain):

$$\begin{aligned} \hat{u}(z) &= G_{ILC}(z)\left(\hat{y}_d(z) - \hat{y}_T(z)\right) \\ &= \frac{z^{-1}}{1-z^{-1}} \begin{bmatrix} k_{I1} & 0 \\ 0 & k_{I2} \end{bmatrix} \left(\hat{y}_d(z) - \hat{y}_T(z)\right) \end{aligned} \quad (44)$$

The gains are adjusted to have the roots of the characteristic equation:

$$\Delta(z) = \left\{z + \left(4.75k_{I1} - 1\right)\right\}\left\{z + \left(3.2222k_{I2} - 1\right)\right\} \quad (45)$$

strictly inside the unit circle. Then the gains must be in the following ranges $0 < k_{I1} < 0.4211$ and $0 < k_{I2} < 0.6206$ for stability (convergence). Here, we select $k_{I1} = 0.2105$ and $k_{I2} = 0.3103$ to have a characteristic equation with two poles at 0.

Figure 1 show the convergence of the inputs based on cycle simulation results. Convergence is achieved in two cycles and is not “dead beat” since K_{I-ILC} is diagonal but Ψ is not.

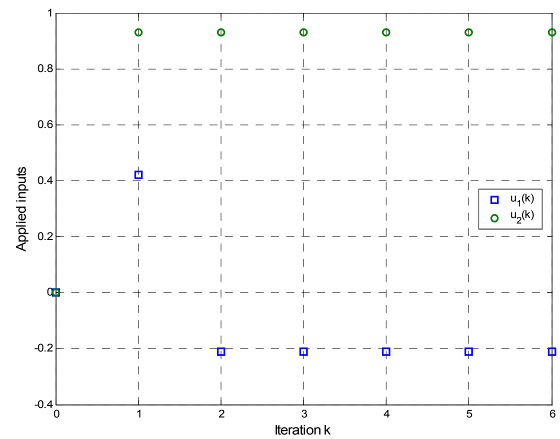

Figure 1: Applied input from cycle to cycle for Case 1.

Case 2: Design with a gain matrix $K_{I-ILC} = \Psi^{-1}$.

This is the MIMO “dead beat” convergence design. Now the terminal I-ILC control is:

$$\begin{aligned} \hat{u}(z) &= G_{ILC}(z)\left(\hat{y}_d(z) - \hat{y}_T(z)\right) \\ &= \frac{z^{-1}}{1-z^{-1}}\left(\Psi^{-1}\right)\left(\hat{y}_d(z) - \hat{y}_T(z)\right) \end{aligned} \quad (46)$$

Therefore, we have:

$$K_{I-ILC} = \Psi^{-1} = \begin{bmatrix} 0.2105 & -0.2105 \\ 0 & 0.3103 \end{bmatrix} \quad (47)$$

Figure 2 shows the “dead beat” convergence of the inputs based on cycle simulation results. As one can see, the inputs converged in only one step.

Note that, since we have perfect knowledge of the system, we can calculate directly the optimal input with:

$$u^* = \Psi^{-1}\left(y_d - \Gamma x_0\left(k\right)\right). \tag{48}$$

With the value defined earlier one can obtain this optimal input vector $u^* = \left[-0.2105 \quad 0.9310\right]^T$. As we can see from Figures 1 and 2, the applied inputs have converged to this optimal input vector.

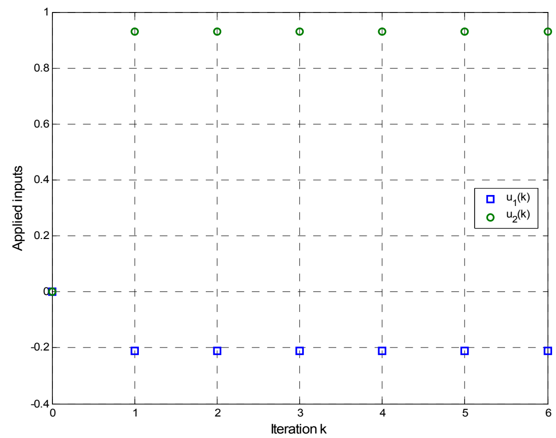

Figure 2: Applied input from cycle to cycle for Case 2.

Case 3: Effect of error in the evaluation of Ψ.

We will repeat Case 2, but we assume we have a wrong estimate of Ψ. Suppose we have evaluated:

$$\Psi = \begin{bmatrix} 6 & 2 \\ 0 & 5 \end{bmatrix}, \tag{49}$$

but the real system matrix Ψ stays as show in (43).

Then, the gain matrix K_{I-ILC} will be:

$$K_{I-ILC} = \begin{bmatrix} 0.1667 & -0.0333 \\ 0 & 0.2 \end{bmatrix} \tag{50}$$

and we use this matrix for the cycle-to-cycle control on the real system.

Figure 3 shows the effect of the error in estimation of Ψ. The convergence is slower than the two other cases because our value of Ψ is not exact. The robustness of our approach has to be evaluated in further work. But certainly if the error on Ψ is too large, it is possible to have a non converging cycle-to-cycle control.

VI. Conclusion

We used an approach different from the usual one to analyze the convergence of terminal I-ILC algorithm of SISO and MIMO systems. This novel approach reduces convergence analysis of terminal ILC to stability analysis on the z-domain transfer function built from an equivalent system in cycle domain. The simulation results show how effective the controller can be.

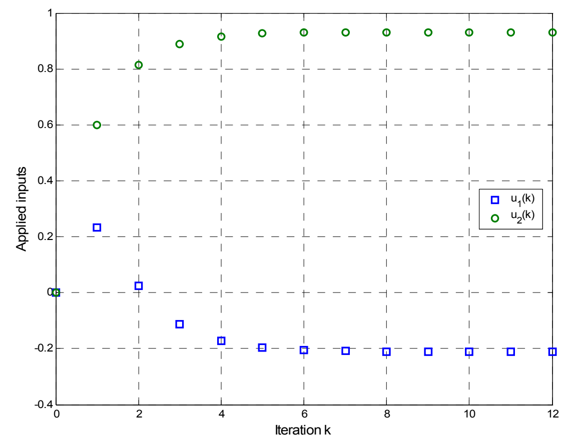

Figure 3: Applied input from cycle to cycle for Case 3.

Future work will address the robustness of the proposed approach to the uncertainty in Ψ and to changes in initial conditions. We will also conduct the same kind of analysis with robust control theory applied in the cycle domain to improve cycle-to-cycle control in the time domain.

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