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Recursive control using structure analysis of control systems

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***Abstract*— In this paper, some operators which denote the systems stabilized by recursive control, are defined. This class of systems is called extended interlaced systems (EIS), and is formed by the compositions of systems with upper-triangular structure and systems with lower-triangular structure. According to the method in this paper, the stabilizing controller of the systems can be constructed easily since the design procedure is constructive and the stabilizing controller can be constructed without complex change of coordinates. What's more, the relation between EIS and fractal is got. From the result, one can understand the systems easily. We can make sure that a system belongs to EIS after analyzing its structure and testing some assumptions.**

I. INTRODUCTION

There have been many studies on structural analysis of control systems since the concept of structural controllability was provided by Lin [1] in 1974. For the linear time-invariant systems

$$\dot{x} = Ax + Bu$$

with $x \in R^n, U \in R^m$, the pair (A, B) denotes the system. The pair $(\bar{A}, \bar{B})$ has the same structure as the pair (A, B) of the same dimension if for every of the entry fixed(zero) of the matrix $(A\ B)$, the corresponding entry of the matrix $(\bar{A}\ \bar{B})$ is also fixed(zero), and for every of the entry fixed(zero) of the matrix $(\bar{A}\ \bar{B})$, the corresponding entry of the matrix $(A\ B)$ is also fixed(zero). The pair (A, B) is structural controllable if a pair $(\bar{A}, \bar{B})$ that has the same structure as the pair (A, B) can be controlled. As a general extension of controllability of linear systems, the concept of structural controllability doesn't help for constructing the controller, and only help to exclude some systems which can not be controlled by analysis of the systems' structure (illustrated by Figure 1).

In the 1990s, the constructive procedures such as backstepping and forwarding, appeared [2]. The stabilizing controller can be constructed by these procedures according to the structure of control systems, such as lower-triangular systems and upper-triangular systems. The study of the lower-triangular systems comes from the study of cascade systems, which lead to the construction procedure of the stabilizing controller, called backstepping. The procedure of backstepping has acquired many developments since it was initiated in [2]. In [3], the geometric conditions are given, under which a class of systems with unknown parameters can be translated into lower-triangular systems and can be stabilized by adaptive backstepping method. The existence of a lower-triangular form also play a crucial part in robust control [4][5][6][7], decouple control [8] and networks control [9]. The more complex lower-triangular structure, such as nested lower-triangular structure, is proposed and is used to construct the stabilizing controller [10][11][12]. The upper-triangular systems is seemed as a compliment of the lower-triangular systems [13], and can be construct the stabilizing controller under some conditions [14]. By the procedure of forwarding, the stabilizing controller can be constructed without the coordinate change. The combinations of backstepping and forwarding is not widely studied, and is proposed little in [14].

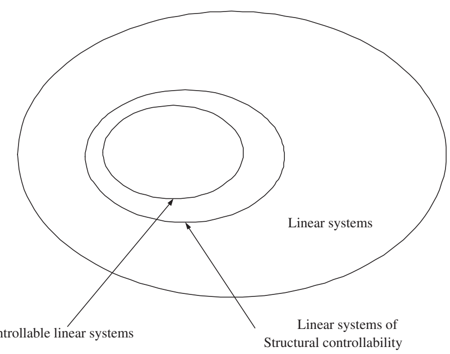

Fig. 1. relation between structural controllability and controllability

Fractal is a nature scene in the nature [15]. It can be known from fractal that the simple systems can form the complex systems through the composition. People can construct many kinds of fractals by iterated functions systems (IFS). On the one hand, the fractal image can be formed easily by simple functions. On the other hand, we can find these describe functions according to a certain fractal image. Fractal image compression is such an application of the method [16]. Similar to the fractal image compression, the control goal can be transformed into finding the combination forms of triangular structures of the systems, and constructing the stabilizing controller according to the structure.

In this paper, a class of control systems, called EIS is propose. EIS is the combination of lower-triangular systems and the upper-triangular systems and can be stabilized through procedures of backstepping and forwarding. The relations between the EIS and cantor set is provided by a compact function. Consider the control system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

with $x \in R^n, u \in R$. What we want to do is to find the structure formations of the systems. Then we construct the stabilizing controller according to the structure formations. The analyzing method can help people to understand the complex systems more easily, and are helpful to construct the controller to implement the control goal.

The paper is organized as follows: In Section II, lower-triangular systems and upper-triangular systems and their control methods are described. In Section III , the definition of some operators and EIS are provided. The main result of the paper and an identifying algorithm of single-input EIS are given. Illustrated examples are given in Section IV. The conclusion is provided in Section V.

II. Preliminaries

For the control system

$$\dot{x} = f(x,u) \quad x \in R^n, u \in R \tag{2}$$

configuration matrix ([14]pp.280) of the system is defined as follows:

$$P = \frac{\partial\left(f(x,u)\right)}{\partial\left(x,u\right)} = [p_{ij}] \tag{3}$$

Define the structure matrix $S = [s_{ij}]$

$$s_{ij} = \begin{cases} 0 & \text{if} \quad p_{ij} \equiv 0 \\ 1 & \text{if} \quad p_{ij} \neq 0 \\ 2 & \text{other cases} \end{cases} \tag{4}$$

We shall say that the matrix S has the same structure as the matrix $\bar{S}$ if for every of the entry fixed(zero) of the matrix S, the corresponding entry of the matrix $\bar{S}$ is also fixed(zero), and for every of the entry fixed(zero) of the matrix $\bar{S}$, the corresponding entry of the matrix S is also fixed(zero).

The lower-triangular system, called strict feedback system, is as follows:

$$\begin{aligned}
\dot{x} &= f_0(x) + g_0(x)z_1 \\
\dot{z}_1 &= f_1(x,z_1) + g_1(x,z_1)z_2 \\
\dot{z}_2 &= f_2(x,z_1,z_2) + g_2(x,z_1,z_2)z_3 \\
&\vdots \\
\dot{z}_{r-1} &= f_{r-1}(x,z_1,\cdots,z_{r-1}) + g_{r-1}(x,z_1,\cdots,z_{r-1})z_r \\
\dot{z}_r &= f_r(x,z_1,\cdots,z_r) + g_r(x,z_1,\cdots,z_r)u
\end{aligned} \tag{5}$$

The procedure, called backstepping, with which the stabilizing controller of the above systems can be constructed, is proposed in [14].

The upper-triangular system, called strict feedforward system, is as follows:

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1,\cdots,x_r,\xi) + \psi_1(x_1,\cdots,x_r,\xi) \\
&\quad + g_1(x_1,\cdots,x_r,\xi)u \\
\dot{x}_2 &= f_2(x_2,\cdots,x_r,\xi) + \psi_2(x_2,\cdots,x_r,\xi) \\
&\quad + g_2(x_2,\cdots,x_r,\xi)u \\
&\vdots \\
\dot{x}_r &= f_r(x_r,\xi) + \psi_r(x_r,\xi) + g_r(x_r,\xi)u \\
\dot{\xi} &= a(\xi) + b(\xi)u
\end{aligned} \tag{6}$$

The procedure, called forwarding, which can construct the stabilizing controller of the above systems, is proposed in [14].

As a combination of special lower-triangular systems and upper-triangular systems, interlaced systems is defined in [14] as follows: A system (2) is called interlaced system if its Jacobian linearization is stabilized and its configuration matrix $P(x,u)$ satisfies the following requirements:

1) If $j > i+1$ and $p_{ij} \neq 0$, then $p_{kl}(x) \equiv 0$ for all $k \geq l, k \leq j-1$, and $l \leq i$.
2) If $p_{ij} \neq 0$ for some $j \leq i$, then p_{ij} is independent of x_{ii+1} and $p_{ii+1}(x) \neq 0$ for all x

In [14], the stabilizing controller of interlaced systems can be constructive by alternative use of backstepping and forwarding. Consider (2), when $n = 3$, the structure matrix of lower-triangular systems is as follows:

$$S_L = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 2 & 2 & 2 & 1 \end{bmatrix} \tag{7}$$

Consider (2), $n = 3$, the structure matrix of upper-triangular systems is as follows;

$$S_U = \begin{bmatrix} 2 & 2 & 2 & 1 \\ 0 & 2 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \tag{8}$$

The structure matrix of interlaced systems have four kinds of matrix as follows:

$$S_{I1} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 2 & 2 & 2 & 1 \end{bmatrix} \tag{9}$$

$$S_{I2} = \begin{bmatrix} 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix} \tag{10}$$

$$S_{I3} = \begin{bmatrix} 0 & 2 & 2 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 2 & 1 \end{bmatrix} \tag{11}$$

$$S_{I4} = \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \tag{12}$$

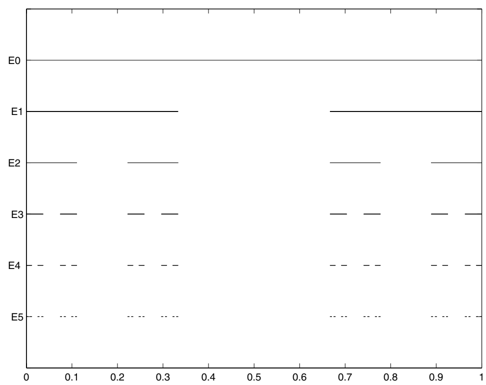

Fig. 2. third cantor set

From the above six structure matrixes, it is easy to find that the interlaced systems cannot include the lower-triangular systems and upper-triangular systems completely.

Consider the third cantor set formed by $S_0(x) = \frac{1}{3}x$, $S_1(x) = \frac{2}{3} + \frac{1}{3}x$, $E_k = S_0(E_{k-1}) \bigcup S_1(E_{k-1})$ as Figure 2. In the next section, the relation between third cantor sets and systems which can be controlled using recursive control, will be given.

III. PROBLEM AND MAIN RESULTS

The following will give the definitions of some operators which can denote the control systems that can be stabilized.

Definition 1: $\Sigma_{GA}(x,u)$ denotes the systems $\dot{x} = f(x)+g(x)u$, and the controller $u = \alpha(x)$ can be designed, and make the system stable at the origin $x = 0$.

Definition 2: $\Sigma_{GAS}(x,u)$ denotes the systems $\dot{x} = f(x) + g(x)u$, and the controller $u = \alpha(x)$, $\alpha(0) = 0$ can be designed, and make the system stable at the origin $x = 0$.

Definition 3: Σ_B denotes a form, with which systems increase, and $\Sigma_{GAS}\Sigma_B(x,\xi,u) = \Sigma_{GAS}(x,\xi)\Sigma_B(\xi,u)$ denotes the systems

$$\begin{aligned} \dot{x} &= f_1(x) + g_1(x)\xi \\ \dot{\xi} &= f_2(x,\xi) + g_2(x,\xi)u \end{aligned} \tag{13}$$

which can be stabilized at the origin $[x^{\mathbf{T}}, \xi^{\mathbf{T}}]^{\mathbf{T}} = 0$ by constructing the controller $u = \alpha(x,\xi)$.

Definition 4: Σ_{OB} denotes a form, with which systems increase, and $\Sigma_{GAS}\Sigma_{OB}(x,\xi,u) = \Sigma_{GAS}(x,\xi)\Sigma_{OB}(\xi,u)$ denotes the systems (13). Using backstepping, people can design the stabilizing controller $u = \alpha(x,\xi)$, $\alpha(0) = 0$, which make system stable at the origin $[x^{\mathbf{T}}, \xi^{\mathbf{T}}]^{\mathbf{T}} = 0$.

Definition 5: Σ_F denotes a form, with which systems increase, and $\Sigma_{GAS}\Sigma_F(x,u,\xi) = \Sigma_{os}(x,u)\Sigma_F(\xi,u)$ denotes the systems

$$\begin{aligned} \dot{x} &= f_1(x) + g_1(x)u \\ \dot{\xi} &= f_2(x,\xi) + g_2(x,\xi)u \end{aligned} \tag{14}$$

where system satisfies assumptions proposed in [14], and then it can design the controller $u = \alpha(x,\xi)$ using the method of forwarding, make it stable at the origin $[x^{\mathbf{T}}, \xi^{\mathbf{T}}]^{\mathbf{T}} = 0$.

Definition 6: Σ_{OF} denotes a class of increasing form, with which systems increase, and $\Sigma_{GAS}\Sigma_F(x,u,\xi) = \Sigma_{GAS}(x,u)\Sigma_{OF}(\xi,u)$ denotes the systems (14) where the stabilizing controller $u = \alpha(x,\xi)$, $\alpha(0) = 0$ can be constructed by using forwarding, and stabilizes the system (14) at the origin $[x^{\mathbf{T}}, \xi^{\mathbf{T}}]^{\mathbf{T}} = 0$.

Definition 7: If system $\dot{x} = f(x,u)$, $x \in R^n, u \in R$ have the following statement

$$\Sigma_{GAS} \prod_{i=1}^{r} \Delta_i(z_1, \cdots, z_{r+1}) \tag{15}$$

where

$$\begin{cases} \Delta_i = \Sigma_{os} & \text{when } i = 1 \\ \Delta_i \in \{\Sigma_{OF}, \Sigma_{OB}\} & \text{when } 1 < i < r \\ \Delta_i \in \{\Sigma_B, \Sigma_F\} & \text{when } i = r \end{cases}$$

σ is a transitive group, and $z = \sigma(x)$ is a transitive change of coordinates, meaning

$$\begin{pmatrix} x_1 & x_2 & \cdots & x_n & u \\ z_{\sigma(1)} & z_{\sigma(2)} & \cdots & z_{\sigma(n)} & z_{\sigma(n+1)} \end{pmatrix}$$

Then, the class of systems is called EIS,

$$\Sigma_{GAS} \prod_{i=1}^{r} \Delta_i(z_1, \cdots, z_{r+1})$$

is the system's Σ statement.

Form definition 1-6, we get the following properties.

1) $\Sigma_{GAS}(x,u) \subset \Sigma_{GA}(x,u)$.
2) $\Sigma_{GAS}(x_1,x_2)\Sigma_B(x_2,u) \subset \Sigma_{GA}(x,u)$, where $x = [x_1^T, x_2^T]^T$.
3) $\Sigma_{GAS}(x_1,x_2)\Sigma_{OB}(x_2,u) \subset \Sigma_{GAS}(x,u)$, where $x = [x_1^T, x_2^T]^T$.
4) $\Sigma_{GAS}(x_1,u)\Sigma_F(x_2,u) \subset \Sigma_{GA}(x,u)$, where $x = [x_1^T, x_2^T]^T$.
5) $\Sigma_{GAS}(x_1,u)\Sigma_{OF}(x_2,u) \subset \Sigma_{GAS}(x,u)$, where $x = [x_1^T, x_2^T]^T$.

Therefore, we can get the following theorem.

Theorem 1: If a system belongs to EIS, the system's stabilizing controller can be constructed according to its relative Σ statement by recursive control procedures.

Through analyzing EIS system, we can get the structure matrix S_n, corresponding to the EIS with Σ statement as (15). Let $S_0 = [a_{ij}(0)]$, we can get the $S_{k+1} = [a_{ij}(k+1)]$

TABLE I
RELATION TABLE OF SYSTEM, STRUCTURE MATRIX AND $F(\cdot)$

EIS statement of a system	structure matrix of EIS	value of $F(\cdot)$
$\Sigma_{GAS}(x,u)$	[1 1]	[0,1]
$\Sigma_{GAS}\Sigma_{B}(x_1,x_2,u)$	$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$	$[0,\frac{1}{3}]$
$\Sigma_{GAS}\Sigma_{F}(x_1,u,x_2)$	$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$[\frac{2}{3},1]$
$\Sigma_{GAS}\Sigma_{OB}\Sigma_{B}$ (x_1,x_2,x_3,u)	$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	$[0,\frac{1}{9}]$
$\Sigma_{GAS}\Sigma_{OB}\Sigma_{F}$ (x_1,x_2,u,x_3)	$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	$[\frac{2}{3},\frac{7}{9}]$

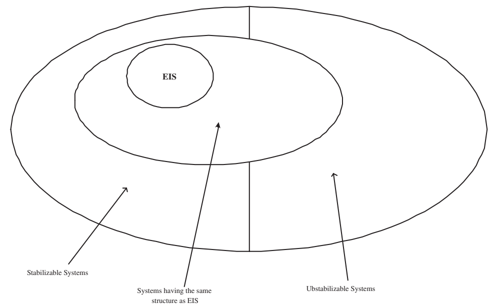

Fig. 3. Relation between EIS and the system having the same structure as EIS

by

$$a_{ij}(k+1)=\begin{cases} a_{ij}(k) & i,j\le k \\ a_{ij}(k) & i=k+1 \\ & \Delta_k\in\Sigma_{BO} \\ 1 & i=k+1 j\le k=2 \\ & \Delta_k\in\Sigma_{BO} \\ a_{i,j-1}(k) & j=k+2 \\ & 1\le i\le k, \Delta_k\in\Sigma_{FO} \\ 1 & j=k+2 \\ & i=k+1, \Delta_k\in\Sigma_{FO} \\ 1 & i=k+1, 1\le j\le k+1 \\ & \Delta_k\in\Sigma_{FO} \end{cases}$$

$a_l(1)=a_u(1)\in[0 \quad 1]$

$$\begin{cases} \Delta_k\in\Sigma_B, u(k)=z_{k+1}, a_{u(k)}=a_{k,k+1} \\ \Delta_k\in\Sigma_F, u(k)=u(k-1), a_{u(k)}=a_{u(k-1)}+1 \end{cases}$$

$a_u(k)=\frac{1}{3}a_u(k-1), a_l(k)=\frac{2}{3}+\frac{1}{3}a_l(k-1)$

$a(k)=a_u(k)\cdot(1-a_{k-1,k+1}(k))+a_l(k)\cdot a_{k-1,k+1}(k)$

$a_l(k)=a_u(k)=a(k)$

From above equations, we can get

$$\begin{aligned} a(k) &= F(a(k-1)) \\ &= \frac{1}{3}a(k-1)(1-a_{k-1,k+1}(k)) \\ &\quad + \left(\frac{2}{3}+\frac{1}{3}a(k-1)\right)a_{k-1,k+1}(k) \\ &= \frac{1}{3}a(k-1)+\frac{2}{3}a_{k-1,k+1}(k) \end{aligned} \tag{16}$$

It is easy to know that the function $F: R\times R\longrightarrow R$ is a compact function. The relation among EIS, their structure S_n and $F(S_n)$ is shown in Table I. It is known that the $F(S_n)$ is a cantor set, a kind of fractal, and the value of F, corresponding to every EIS, locates in a part of the cantor set.

It is known from theorem 1 that if a system belongs to EIS, its stabilizing controller can be constructed. First, it is necessary to confirm whether the system has the same structure as a kind of EIS. Second, the necessary assumptions proposed in [14] are need to be satisfied. Then, the system can be proved to belong to EIS and the stabilizing controller of the systems can be constructed. The algorithm about how to get Σ statement is as follows.

1) There are two arrays d_n, which stores the number of the control variables, and d_m which is used to store the method of control. In d_m, $'b'$ denotes the method of backstepping, and $'f'$ denotes the method of forwarding.
2) Take $i=n+1$.
3) Identify the size of structure matrix S of the system. If the number of columns is equal to 1, stop, otherwise continue.
4) Let $\Omega=\{j|S_{ji}=1,2\}$.
5) Compute number m of elements in the set Ω. If $m=0$, stop. If $m=1$, the input variable may be input of backstepping, store i into d_n, and store b into d_m, goto step 6. If $m>1$, the input may be input of forwarding, goto step 5.
6) Take $\Omega\times\{\Omega,i\}$, $m\times(m+1)$ Q, find c_m, store c_m into d_n, let $i=c_m(m)$, store f_m into d_m.
7) Take $j\in\Omega$, $S^{'}=S$, reduce jth column and ith row of S, and get new matrix S'. Compute jth column of S location m compared to new matrix S. Let $i=m$.
8) Goto step2.

Remark 1: The algorithm can confirm that the systems which under transitive change of coordinates, have the same structure as EIS. Then, those necessary assumptions, related with backstepping and forwarding, will need to be tested. The design procedure using structural analysis is a tentative procedure. The relation between EIS and the systems having the same structure as EIS is shown in Figure.3.
In the next section, the examples, including a practical example, will be given to test the effect of the recursive control.

IV. ILLUSTRATED EXAMPLES

There are two examples in this section. The controller of the first example can be constructed analytically, while the stabilizing controller of the second example need to constructed by numerical integration.
Example 1:Consider the following control system

$$\dot{x}_1 = -x_1+x_1x_2+x_1x_2x_3 \tag{17}$$
$$\dot{x}_2 = x_2^2+x_3 \tag{18}$$
$$\dot{x}_3 = (x_1+x_2+x_3)^2+u \tag{19}$$

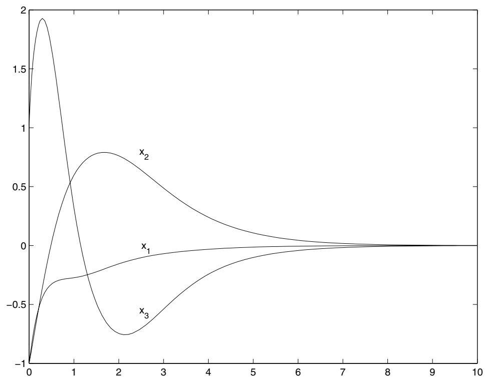

Fig. 4. Closed-loop response of example 1

its structure matrix is

$$S=\begin{bmatrix} 2 & 2 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 2 & 2 & 2 & 1 \end{bmatrix}$$

we can get $S_m=\{'b','f'\}$, $S_d=\{3,1,1\}$, S_d, and the sequence of control is $\{2,1,3\}$. Therefore, we can think that (17)-(18) subsystem may be stabilized using forwarding. To identify it, to $[x_1,x_2]$ subsystem we need to test relative assumptions proposed in [14]. If assumptions are satisfied, we can seem x_3 as virtual input and use the forwarding to design the controller. Similar, we can think that (17-18)-(19) system may be stabilized using backstepping. If inverse conditions is satisfied . We can get the finally controller by backstepping.

$$\begin{aligned} u &= -3x_2-3x_3-2x_1x_2-4x_2^2-x_1^2+2x_1^2x_2 \\ &\quad -9x_1^2x_2^2-12x_1^3x_2^2+6x_1^3x_2-2x_1x_3-4x_2x_3 \\ &\quad -x_3^2-2x_2^3-6x_1^2x_2^2x_3-9x_1 \end{aligned}$$

the simulation is as Figure 4. It is effective to stabilize the system at origin $[x_1,x_2,x_3]^{\mathbf{T}}=[0,0,0]^{\mathbf{T}}$.
Example 2: Consider the reduced order model of the reaction wheel pendulum in [17].

$$\begin{aligned} \dot{x}_1 &= x_2 && (20) \\ \dot{x}_2 &= -\frac{d_{22}}{detD}\phi(x_1)-\frac{d_{12}}{detD}\tau && (21) \\ \dot{x}_3 &= \frac{d_{21}}{detD}\phi(x_1)+\frac{d_{11}}{detD}\tau && (22) \end{aligned}$$

its structure matrix is

$$S=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 \end{bmatrix} \qquad (23)$$

we can get $S_m=\{'f','b'\}$, $S_d=\{3,1,1\}$, S_d, and the sequence of control is $\{2,1,3\}$. Therefore, we can think that (20)-(21) subsystem may be stabilized using backstepping. To identify it, to $[x_1,x_2]$ subsystem we need to test relative assumptions proposed in [14]. If assumptions are satisfied, we can seem x_3 as virtual input and use backstepping to construct the controller. Similar, we can think that (20-21)-(22) system may be stabilized using forwarding. If inverse conditions is satisfied . We can get the finally controller by forwarding. The process is as following. Let

$$z_1=x_1, z_2=x_2+z_1 \qquad (24)$$

we get from (20)

$$\begin{aligned} \dot{z}_1 &= -z_1+z_2 && (25) \\ \dot{z}_2 &= -\frac{d_{22}}{detD}\phi(z_1)-\frac{d_{12}}{detD}\tau-z_1+z_2 && (26) \end{aligned}$$

Let

$$\tau=\frac{1}{\frac{d_{12}}{detD}}\left(-\frac{d_{22}}{detD}\phi(x_1)+2z_2-v\right) \qquad (27)$$

we get

$$\begin{aligned} \dot{z}_1 &= -z_1+z_2 && (28) \\ \dot{z}_2 &= -z_1-z_2+v && (29) \\ \dot{x}_3 &= \psi(z_1,z_2)-bv && (30) \end{aligned}$$

where $\psi(x_1,x_2)=\frac{d_{21}-d_{11}}{detD}\phi(z_1)+\frac{d_{11}}{d_{22}}(2z_2), b=\frac{d_{11}}{d_{22}}$. When $v=0$, the model (28-30)becomes

$$\begin{aligned} \dot{z}_1 &= -z_1+z_2 && (31) \\ \dot{z}_2 &= -z_1-z_2 && (32) \\ \dot{x}_3 &= \psi(z_1,z_2) && (33) \end{aligned}$$

and the solution of (31) is

$$\begin{aligned} \tilde{z}_1(t) &= e^{-t}(\cos(t)z_1+\sin(t)z_2) && (34) \\ \tilde{z}_2(t) &= -e^{-t}(\sin(t)z_1-\cos(t)z_2) && (35) \\ \tilde{x}_3(t) &= \int_0^t \psi(\tilde{z}_1(s),\tilde{z}_2(s))ds+x_3 && (36) \end{aligned}$$

Then, the cross-term is

$$\begin{aligned} \Psi(z_1,z_2,x_3) &= \int_0^\infty \tilde{x}_3(s)\psi(\tilde{z}_1(s),\tilde{z}_2(s))ds \\ &= \frac{1}{2}\tilde{x}_3^2(s)|_0^\infty && (37) \end{aligned}$$

Take

$$\begin{aligned} V(z_1,z_2) &= \frac{1}{2}(z_1^2+z_2^2) \\ U(x_3) &= \frac{1}{2}x_3^2 && (38) \\ W &= U(z_1,z_2)+V(x_3)+\Psi(z_1,z_2,x_3) && (39) \end{aligned}$$

Then

$$\begin{aligned} v &= -L_GW \\ &= -G\begin{bmatrix} \frac{\partial W}{\partial z_1} \\ \frac{\partial W}{\partial z_2} \\ \frac{\partial W}{\partial x_3} \end{bmatrix} \\ &= -[0,1,-b]\begin{bmatrix} \frac{\partial W}{\partial z_1} \\ z_2+\tilde{x}_3(\infty)\frac{\partial \tilde{x}_3(\infty)}{\partial z_2} \\ \tilde{x}_3(\infty) \end{bmatrix} \\ &= bx_3(\infty)-z_2-x_3(\infty)\frac{\partial x_3(\infty)}{\partial z_2} && (40) \end{aligned}$$

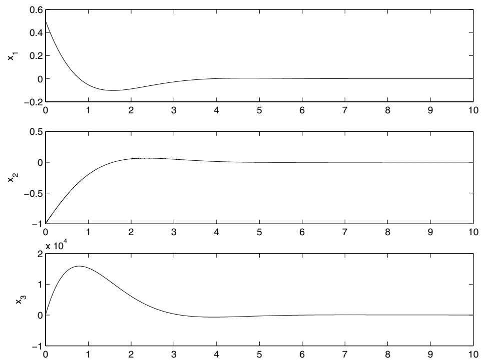

Fig. 5. Result of recursive control of reaction wheel pendulum

can stabilize the system (28-30). Therefore, Using the stabilizing controller (40)(27) of the system (20-22), we can get the simulation result plotted in Figure 5.

Remark 2: The stabilizing controller of example 2 is need to compute the numerical integration $\tilde{x}_3(\infty)$ and $\frac{\partial \tilde{x}_3(\infty)}{\partial z_2}$.

V. Conclusion

In this paper, the sufficient conditions, which can ensure the nonlinear system can be stabilized, are given. The system satisfying the conditions are called EIS. This class of the systems can be seemed as the composition of the strict feedback systems and the feedforward systems. The stabilizing controller can be designed according to the structure of EIS. EIS can include many systems with different structures. Since the geometric conditions under general changes of coordinates have not been got, we can only study the simplest changes of coordination, transitive changes of coordinations. An algorithm for Single-input EIS, which compare the structure of EIS and the systems, is proposed, and in the future algorithms for multi-input will be given. The effectiveness is proved by examples.

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