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HIGH-GAIN TRACKING FOR NONLINEAR SYSTEMS

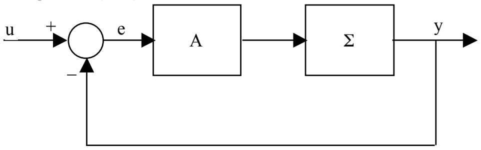
Jacob Hammer, *Fellow, IEEE*

***Abstract* — The classical control principle of using high forward gain to improve tracking accuracy of linear control systems is revisited in a nonlinear context. It leads to a simple methodology for the design of nonlinear tracking systems and to a solution of certain nonlinear approximate model matching problems.**

I. INTRODUCTION

A. *Ideas from classical control theory.*

An important design principle of classical control theory is the use of high forward gain in the following Black diagram ([2]).



(1)

Here, Σ is the system being controlled, and A is an ideal amplifier with gain A. The use of unity feedback in the diagram requires that the number of outputs of Σ be equal to the number of inputs (single input/single output is used in classical control). Letting Σ_A be the input/output relation of the closed loop (1), a simple calculation yields

$\Sigma_A = \Sigma[(1/A)I + \Sigma]^{-1}$. (2)

In fact, (2) is valid even when Σ is a nonlinear system. Ignoring for a moment mathematical rigor (see later sections for a rigorous discussion), one may presume that

$\lim_{A\infty\infty} [(1/A)I + \infty] = \infty$. (3)

If (3) is accepted as correct and substituted into (2), and if ∞ is continuous and invertible, one obtains

$\lim_{A\infty\infty} \infty[(1/A)I + \infty]^{-1} = \infty\infty^{-1} = I$. (4)

In other words, when A is sufficiently large, we have

$y \approx u$, (5)

i.e., for large A, Configuration 1 is an accurate tracking system, as long as ≈ is invertible and strictly causal.

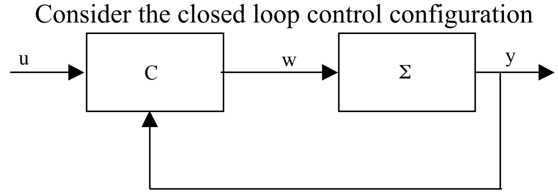
Great caution has to be exercised when drawing such far reaching conclusions. A brief examination of (4) reveals a major difficulty in case the system $\approx^{-1}$ is not Bounded-Input Bounded-Output (BIBO) stable. Indeed, the expression $\approx\approx^{-1}$ implies that, for large gain A, the input signal of ≈ in (1) is (almost) equal to the output signal of $\approx^{-1}$. Consequently, when $\approx^{-1}$ is not BIBO-stable, the input of ≈ in (1) will be unbounded for at least some input signals u. This makes Configuration 1 unusable, as it will not be internally stable. In other words, Configuration 1 cannot be used with large gain A when $\approx^{-1}$ is not BIBO-stable.

The present note shows that this difficulty can be alleviated by introducing a certain form of hysteresis into the control configuration (1) (see also [10]). With this modification, it is possible to control a rather large class of nonlinear systems and achieve accurate tracking.

Alternative approaches to the control of nonlinear systems can be found in [11], [12], [6], [7], [8], [9], [4], [19], [18], [3], [15], [20], [17], [16], [1], [5], [13], the references cited in these publications, and others.

II. BASIC CONSIDERATIONS

Consider the closed loop control configuration



(6)

Here, I is the system being controlled, C is a controller, and I_c is the system represented by the closed loop. For notational convenience, we assume that C is constructed so that I_c has the same input space as I.

To investigate the stability of Configuration 6, we limit our attention to bounded input signals u. We define the *bounded-input image* $Im_b I_c$ as the set of all output signals of I_c generated by bounded input signals. Similarly, the bounded-input image $Im_b I$ of I is formed by all responses of I to bounded input sequences. As the output of I_c is the output of I, we conclude that $Im_b I_c \subset Im_b \subset$.

(7) DEFINITION. THE APPROXIMATE MODEL MATCHING PROBLEM. Given a system Φ, a bounded domain S, and a real number ε > 0, determine whether there is a controller C such that

$|\varepsilon u - \varepsilon_c u| \le \varepsilon$ for all $u \in S$. (8)

If such a controller exists, then $\in_c$ is an *∈approximant* of ∈ *over* S, and ∈ is called the *model*. ♦

When the model is the identity system ♦ = I, then the approximate model matching problem reduces to the classical problem of designing a tracking system.

For a real number ♦> 0 and an element v of a normed space, denote by N♦(v) the neighborhood of v given by

N♦(v) = {w : |w – v| ≤ ♦}.

For a set A, the corresponding neighborhood is

$N_♦(A) = \bigcup_{v♦A} N_♦(v)$.

Assume now that $♦_c$ is a ♦approximant of the system ♦. By (8), we have ♦u ♦ N♦(♦$_c$u) for all u ♦ S, so that

$\Phi[S] \subset N_\varepsilon(\Sigma_c[S])$. Recalling that S is a bounded domain, we clearly have $N_\Sigma(\Sigma_c[S])\ \Sigma\ N_\Sigma(Im_b\ \Sigma_c)$, so that
$\Sigma[S]\ \Sigma\ N_\Sigma(Im_b\ \Sigma)$. When Σ is an invertible system, this leads to the relation
$S\ \Sigma\ \Sigma^{-1}[N_\Sigma(Im_b\ \Sigma)]$.
For tracking systems, i.e., for $\Sigma = I$, this yields

$$S\ \Sigma\ N_\Sigma(Im_b\ \Sigma). \quad (9)$$

Inclusion (9) implies that only signals that are within Σ of the bounded-input image of Σ can be tracked with an error not exceeding Σ. This simple fact imposes a fundamental restriction on the operation of tracking systems. It plays an important role in our ensuing discussion. Setting $\Sigma = 0$ in (9), we obtain the following requirement for accurate tracking

$$S\ \Sigma\ Im_b\ \Sigma. \quad (10)$$

The performance limitation (9) brings into focus the need to properly specify tracking signals. It highlights a distinction between the present approach and the traditional method of designing tracking systems. Traditionally, tracking signals are specified without regard to the restriction (9), and, consequently, cannot usually be tracked in their entirety. Instead, one lets the tracking system choose a path that converges asymptotically to the tracking signal. This leaves the designer with incomplete control over the tracking process. In the approach taken in this paper, the tracking signal is selected so that it satisfies the inclusion (9). In this way, the system does not deviate from the tracking signal by more than the permissible error throughout the entire tracking process.

As an example, consider the case of a ground-to-air missile tracking an airplane. In traditional tracking, the missile is given the airplane's flight data, and is left to create its own approach path to the airplane. This leaves the operator without control over the initial part of the tracking process. In the approach developed in this paper, the entire path of the missile, including takeoff, is specified, subject to the constraint (9). This allows the operator more complete control over tracking and missile performance.

In many applications, selecting a tracking signal near the image of the system is not an overly taxing process. Often, appropriate signals can be selected based on general characteristics, such as bandwidth, signal magnitude bounds, or maximal rates-of-change.

A. Preliminaries

The presentation here is for discrete-time systems, but similar principles apply to continuous-time systems. Let R be the set of real numbers, let R^m be the set of all m-dimensional real vectors, and let $S(R^m)$ be the set of all sequences $u = \{u_0, u_1, u_2, \dots\}$ of m-dimensional real vectors. A system Σ with specified initial conditions induces a map $\Sigma : S(R^m) \to S(R^p)$, transforming input sequences of m dimensional real vectors into output sequences of p dimensional real vectors. The output sequence y generated by $\to$ from the input sequence u is $y = \to u$. It will be convenient to assume that $\to 0 = 0$.

As usual, a system $\to$ is *causal* if its response does not depend on future input values. The system is *strictly causal* if there is a delay of at least one step before input changes are reflected in its response. Finally, the system $\to$ is *bicausal* if it is invertible, and if $\to$ and its inverse $\to^{-1}$ are both causal systems (e.g., [6]).

The systems we consider are given in terms of a *state representation*

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) \\ y_k &= h(x_k),\ k = 0, 1, 2, \dots \end{aligned} \quad (11)$$

Here, $x_k \in R^n$ is the *state* of the system at the step k, while u_k and y_k represent the input value and the output value, respectively, at that step. The function $f : R^n \times R^m \times R^n$ is the *recursion function* and $h : R^n \times R^p$ is the *output function*. For convenience, we use the initial condition $x_0 = 0$ for the system. A system described by (11) is strictly causal, since the output function h does not depend on the input value (e.g., [6]). The realization (11) is *uniformly continuous* if f and h are uniformly continuous functions. The *input/state part* $\times_s$ of $\times$ is given by $x_{k+1} = f(x_k, u_k)$, $k = 0, 1, \dots$

For a real number a, let $|a|$ be the absolute value of a. For a vector $r = (r^1, r^2, \dots, r^q) \times R^q$, denote
$|r| := \max\{|r^i|, i = 1, \dots, q\}$;
it will be convenient to refer to $|r|$ as the *ℓ^∞-norm* of r. The ℓ^∞-norm of an element $s \infty S(R^q)$ is
$|s| := \sup_{i \geq 0} |s_i|$.
A subset $S \infty S(R^q)$ is *ℓ^∞-bounded* (or, simply, *bounded*) if there is a real number $M \geq 0$ such that $|s| \leq M$ for all elements $s \infty S$; when the latter holds, we write $|S| \leq M$. Given a real number $\theta \leq 0$, denote by $S(\theta^q)$ the set of all sequences $s\ \theta\ S(R^q)$ satisfying $|s| \leq \theta$, i.e., the set of all sequences of q-dimensional real vectors bounded by θ.

The *ℓ^1-norm* is, for a vector $v = (v^1, v^2, \dots, v^p)\ \theta\ R^p$, given by
$|v|_1 := |v^1| + |v^2| + \dots + |v^p|$.
The *weighted ℓ^1-norm* $|\bullet|_{1w}$ is defined, for a sequence $y\ \theta\ S(R^p)$, by

$$|y|_{1w} := \theta_{i=0}^{\theta} 2^{-i} |y_i|_1. \quad (12)$$

It is easy to see that the weighted ℓ^1-norm exists for every bounded sequence $y\ \theta\ S(R^p)$.

A norm $\langle\bullet\rangle$ over $S(R^p)$ is *compatible* with the weighted ℓ^1-norm if there is a constant $a > 0$ such that $\rangle u \rangle \leq a|u|_{1w}$ for all $u \rangle S(R^p)$. Denote by $\|\bullet\|$ a norm that has the following properties: (i) it is compatible with the weighted ℓ^1-norm, and (ii) under it, every closed and $\ell^\rangle$-bounded subset of $S(R^p)$ is compact. The weighted ℓ^1-norm (12) is an example of such a norm $\|\bullet\|$.

A system $\rangle : S(R^m) \rangle S(R^p)$ is *BIBO-stable* (Bounded-Input Bounded-Output stable) if, for every real number $M \leq 0$, there is a real number $N \leq 0$ such that $|\rangle u| \leq N$ whenever $|u| \leq M$. The notion of BIBO-stability underlies every other stability concept. We say that Σ is *stable* if it is BIBO-stable, and if it is continuous with respect to the norm $\|\bullet\|$.

Let Σ be an invertible system. If Σ^{-1} is BIBO-stable, then Σ is a *BIBO-minimum phase system*. When Σ is both BIBO-stable and BIBO-minimum phase, then it is a *BIBO-unimodular system*. Similarly, if Σ^{-1} is stable, then Σ is a *minimum phase system*, and if Σ is both stable and minimum phase, then it is a *unimodular system*.

For composite systems, stronger notions of stability are needed. Consider a composite system Ψ that consists of q subsystems. Add an external signal to the output of each subsystem. This results in a system with $q+1$ external input signals - the original input signal and the q newly added signals. Then, the composite system Ψ is *internally BIBO-stable* if the following holds for each one of the $(q+1)$ external input signals: the map from the external signal to any signal within the configuration is a BIBO-stable system. Further, Ψ is *internally stable* if each such map is a stable system. Internal stability guaranties that a composite system is implementable.

To investigate stability properties as the gain approaches infinity requires the following stronger notions.

(13) DEFINITION. Let $\Psi(A)$ be a system depending on a real parameter A. Then, $\Psi(A)$ is *uniformly BIBO-stable* if there is a real number A_0 such that the following is true for all $A \geq A_0$: for every real $M \geq 0$, there is a real $N \geq 0$ such that $|\Psi(A)u| \leq N$ for all input sequences $|u| \leq M$.

The system $\Psi(A)$ is *uniformly BIBO-minimum phase* if there is a real number B_0 such that $\Psi(A)$ is invertible and $\Psi^{-1}(A)$ is uniformly BIBO-stable for all $A \leq B_0$.

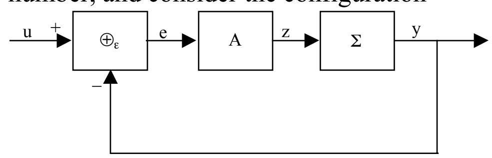
Finally, $\Psi(A)$ is *uniformly BIBO unimodular* if it is uniformly BIBO-stable and uniformly BIBO-minimum phase. ♦

(14) DEFINITION. Let $\Phi(A)$ be a composite system composed of subunits $\Phi^1(A), ..., \Phi^q(A)$ that depend on a parameter A. Insert an adder at the output of each subunit, and add an external signal u^i to the output of $\Phi^i(A)$, $i = 1, ..., q$. Denote by u^0 the input sequence of the composite system. For a given value of the parameter A, let $v^0(A)$ be the output sequence of the composite system, and let $v^i(A)$ be the output sequence of the subunit $\Phi^i(A)$, $i = 1, ..., q$. Then, $\Phi(A)$ is *uniformly internally BIBO-stable* if there is a real number A_0 such that the following is true for all $A \leq A_0$: for every real $M > 0$, there is a real $N > 0$ such that $|v^i(A)| \leq N$ for all $i = 0, ..., q$ whenever $|u^i| \leq M$ for all $i = 0, ..., q$. Φ

III. HIGH GAIN CONTROL

A. General considerations

In order to use high gain compensators with non-minimum-phase systems, we must depart from one of the basic tenets of traditional control theory: the requirement to have a unique response. This will lead to a broader class of compensators and to improved performance. Needless to say, there is no harm in allowing a non-unique response, as long as possible responses do not differ by more than a permissible error bound. Specifically, we shall use a control configuration with a hysteresis-type response. Let $\Phi : S(R^m) \to S(R^m)$ be the system that needs to be controlled, let A be a constant gain compensator, let $\varepsilon > 0$ be a real number, and consider the configuration



(15)

Here, the symbol $\oplus_\oplus$ indicates the following operation: given two real numbers a and b,

$$a \oplus_\oplus b := \begin{cases} 0 & \text{if } |a+b| \leq \lceil, \\ a+b-\lceil \operatorname{sign}(a+b) & \text{if } |a+b| > \lceil. \end{cases}$$

In words, the outcome of the operation is zero if the sum is $\lceil$ or less; otherwise, the operation reduces the magnitude of the regular sum by $\lceil$. This can be restated as follows.

(16) LEMMA. $a \lceil_\lceil b$ is the number v of minimal magnitude for which $|a+b-v| \leq \lceil$. $\lceil$

For two vectors $x = (x^1, x^2, ..., x^p)$, $z = (z^1, z^2, ..., z^p) \in R^p$, we define the operation componentwise:

$$x \in_\in z := (x^1 \in_\in z^1, x^2 \in_\in z^2, ..., x^p \in_\in z^p). \tag{17}$$

The next statement is a consequence of Lemma 16.

(18) LEMMA. For vectors $x, z \in R^p$, set $w := x \in_\in z$, and let $A(x,z)$ be the set of all vectors $v \in R^p$ for which $|x+z-v| \leq \in$ Then, w is the vector of minimal ℓ^1-norm in $A(x,z)$; it also has the minimal ℓ^∞-norm in $A(x,z)$. ∞

Finally, let $v, w \propto S(R^p)$ be two sequences of vectors. The sequence $y := v \propto_\infty w$ is defined elementwise by

$y_k := v_k \propto_\infty w_k$, $k = 0, 1, 2, ...$

Then, Lemma 18 leads to the following.

(19) LEMMA. For two bounded sequences $v, w \propto S(R^p)$, set $s := v \propto_\infty w$, and let $S(v,w)$ be the set of all sequences $t \propto S(R^p)$ satisfying $|v+w-t| \leq \infty$ Then, s is the sequence of minimal weighted ℓ^1-norm in $S(v,w)$; it also has the minimal ℓ^∞-norm in $S(v,w)$. ∞

In Configuration 15, the minus sign indicates that

$$e = u \propto_\infty (-y). \tag{20}$$

We denote the input/output map of (15) by $\propto_A^\infty$, to indicate its dependence on the gain A and on the parameter $\infty > 0$.

For a preliminary examination of the control loop (15), assume that all signals are scalar, and that the system ∞ represents a scalar constant gain amplifier. Then, the combination ∞A is again a constant gain amplifier, say $\infty A = a$. To examine the response, consider the case when the input signal $u > \infty > 0$. Then, two output values are possible: $y' > u$ and $y'' < u$, as follows. In the first case, the loop induces the equation $(u - y'' + '')a = y''$, which yields

$$y'' = a(u+'')/(1+a). \tag{21}$$

In the second case, we have $(u - y'' - '')a = y''$, or

$y'' = a(u-\varepsilon)/(1+a)$.

As we can see, the output value of Configuration 15 is not uniquely determined by its input value. In this example, the output value of (15) depends on the "initial value" of the output y. In other words, the system exhibits a hysteresis property. Since the discrepancy between the two output values converges to zero when $\varepsilon \to 0$, this non-uniqueness of the response causes no adverse effects, as long as $\to$ is sufficiently small. Still, mathematically, the non-uniqueness broadens the class of controllers, leading to potential performance improvements.

We turn now to a more general examination of Configuration 15. First, in view of Lemma 19, we can write

$|u - (y + e)| \le \dashrightarrow$

By the same Lemma, e is the signal $\mu \in S(R^m)$ of minimal weighted ℓ^1-norm for which $(y + \mu)$ is within an $\in$-neighborhood of the input signal u. Also, since $e = z/A$ with A being a scalar constant gain, we can say that z is the signal $\varpi\ \varpi\ S(R^m)$ of minimal weighted ℓ^1-norm for which $(y + \varpi/A)$ is within an ϖ-neighborhood of the input signal u. Further, since $y = \Sigma Ae$ and A represents a scalar constant gain, we have $e + y = [I + \Sigma A]e = [(1/A)I + \Sigma]Ae = [(1/A)I + \Sigma]z$. This implies

(22) LEMMA. For a system $\Sigma : S(R^m)\ \Sigma\ S(R^m)$ and a sequence $u\ \Sigma\ S(R^m)$, let $S(u)$ be the set of all sequences $\mu\ \Sigma\ S(R^m)$ for which $|[(1/A)I + \Sigma]\mu - u| \le \Sigma$. Assume that $S(u)$ is not empty. Then, the signal z of Configuration 15 is the sequence of minimal weighted ℓ^1-norm in $S(u)$. Also, z has minimal ℓ^∞-norm in $S(u)$. ♦

In the next subsection we show that, under appropriate conditions, the signal z remains bounded as A ♦ ♦. This implies that $e = z/A$ ♦ 0 as A ♦ ♦. Accordingly, the discrepancy between the tracking signal u and the output signal y approaches ♦ as A ♦ ♦.

B. Tracking

Recall that a system ♦ is stable if it is BIBO-stable and continuous with respect to the norm $\|\bullet\|$ (subsection II.A). The following shows that tracking can be achieved with Configuration 15 simply by using a high gain A.

(23) THEOREM. Let ♦ : $S(R^m)$ ♦ $S(R^m)$ be a strictly causal and stable system, let ♦ > 0 be a real number, and let u ♦ $N_{♦/2}(\mathrm{Im}_b$ ♦ $)$ be a tracking signal. Then, Configuration 15 is uniformly BIBO-internally stable, and its response satisfies $\lim_{A♦♦} |u - ♦_A u| \le$ ♦.

Proof. First, by (20),

$N_♦(u) \supset e + y = e + \supset Ae = [I + \supset A]e = [(1/A)I + \supset](Ae) = [(1/A)I + \supset]z$. (24)

Since $\supset$ is strictly causal, the inverse $[(1/A)I + \supset]^{-1}$ exists (e.g., [6]). Consider the set

$\Omega := [(1/A)I + \Omega]^{-1}[N_\Omega(u)]$.

Note that $[(1/A)I + \Omega]$ is continuous since Ω is continuous. Consequently, the fact that $N_\Omega(u)$ is a closed set, implies that Ω is a closed subset of $S(R^m)$.

Next, as $u\ \Omega\ N_{\Omega 2}(\mathrm{Im}_b\ \Omega)$ by assumption, there is a bounded sequence $v\ \Omega\ S(R^m)$ for which $|u - \Omega v| \le \Omega 2$; set $\chi := |v| < \chi$. Denoting $w := \chi v$, we have $w\ \chi\ N_{\chi/2}(u)$. Now, consider a gain $A \ge 2\chi/\chi$, so that $|v|/A \ge \chi/2$. Then, $|[(1/A)I + \chi]v - u| = |(1/A)v + (\chi v - u)| \ge |v|/A + |\chi v - u| \ge \chi$, so that $[(1/A)I + \chi]v\ \chi\ N_\chi(u)$. This shows that $v\ \chi\ [(1/A)I + \chi]^{-1}[N_\chi(u)] = \chi$, i.e., that χ includes the bounded sequence v. Consequently, the bounded intersection $\chi \cap S(\chi^m)$ is not empty; since χ is closed, so is this intersection. Recalling that, in our topology, every closed and bounded set is compact, we conclude that $\chi \cap S(\chi^m)$ is a compact set. Together with the fact that the norm $\|\bullet\|$ is compatible with the weighted ℓ^1-norm, this implies that every sequence of elements of $\chi \cap S(\chi^m)$ with decreasing weighted ℓ^1-norms must have a convergent subsequence with a limit in $\chi \cap S(\chi^m)$. This further implies that $\chi \cap S(\chi^m)$ contains an element of minimal weighted ℓ^1-norm, which we denote by ω^+; since $\omega^+\ \omega\ \omega \cap S(\omega^m)$, it follows that $|\omega^+| \ge \omega$.

We return now to Configuration 15. By Lemma 22, the sequence z is of minimal ℓ^1-norm in ω, so we have, say, $z = \omega^+\ \omega\ \omega \cap S(\omega^m)$. This directly implies that

$|z| \ge \omega$. (25)

As ω is independent of A, this shows that z is bounded with a bound independent of A, for all $A \ge 2\omega/\omega$ Thus, z is uniformly bounded as a function of A. An examination of (15) leads to the following conclusions:

(i) The stability of ω gives rise to a real number $M > 0$ such that $\omega[S(\omega^m)] \subset S(M^m)$; by (25), this implies that $|y| \ge M$ for all $A \ge 2\subset/\subsetneq$ i.e., y is uniformly bounded.

(ii) From $e = z/A$, we obtain $|e| = |z|/A \ge \subset/A$, (26)

so that $|e| \ge \subset$ for all $A \ge 1$. Consequently, the signal e is uniformly bounded for all $A \ge \max\{2\subset/\subsetneq, 1\}$.

Thus, Configuration 15 is uniformly internally BIBO-stable. Furthermore, using (26),

$\lim_{A\subset\subset} |e| = \lim_{A\subset\subset} |z|/A \ge \lim_{A\subset\subset} \subset/A = 0$ (27)

From (24) we have $|u - (e + y)| \ge \subsetneq$ so that $|u - y)| - |e| \ge |u - (e + y)| \ge \subsetneq$ or $|u - y| \ge \subset + |e|$. Invoking (27),

$\lim_{A\subset\subset} |u - y| \ge \subset + \lim_{A\subset\subset} |e| \ge \subsetneq$ (27)

and the proof concludes. $\subset$

Theorem 23 indicates that Configuration 15 can track a prescribed signal u with an error of about $\subset$ or less, as long as the forward gain A is sufficiently large.

An examination of the proof of Theorem 23 reveals the following point: the appropriate gain A may vary from one tracking signal u to another. In [10], we show that the gain A can be selected independently of the tracking signal u, when $\subset$ possesses a certain common reachability property. In the same report, it is also shown that a similar methodology leads to a solution a class of approximate nonlinear model matching problems.

IV. CONCLUSION

Starting from the classical control principle that advocates the use high forward gain in feedback control loops, we have developed a general methodology for the design of nonlinear tracking systems. An important advantage of the resulting design technique is its simplicity: it requires only two design parameters - the gain A and the hysteresis parameter ε. This makes the resulting approach particularly convenient for design through simulation, as one can easily experiment with the parameter values until a desirable outcome is obtained.

REFERENCES

[1] L. BARAMOV and H. KIMURA
[1995] "Nonlinear coprime factorizations and parametrization of a class of stabilizing controllers", Proceedings of the 1995 IEEE conference on Decision and Control, pp. 981-986.

[2] H. S. BLACK
[1934] "Stabilized Feedback Amplifiers", Bell Syst. Tech. J., vol. 14, pp.1-18.

[3] G. CHEN and R.J.P. de FIGUEIREDO
[1990] "Construction of the left coprime fractional representations for a class of nonlinear control systems", Systems and Control Letters, Vol. 14, pp. 353-361.

[4] C.A. DESOER and M.G. KABULI
[1988] "Right factorization of a class of nonlinear systems", Trans. IEEE, Vol. AC-33, pp. 755-756.

[5] T.T. GEORGIOU and M.C. SMITH
[1997] "Robustness analysis of nonlinear feedback systems: an input-output approach", Trans. IEEE, Vol. AC-42, pp. 1200-1221.

[6] J. HAMMER
[1984] "Nonlinear systems: stability and rationality", Int. J. Control, Vol. 40, pp. 1-35.

[7] [1985] "Nonlinear systems, stabilization, and coprimeness", Int. J. Control, Vol. 42, pp. 1-20.

[8] [1989] "Robust stabilization of nonlinear systems", Int. J. Control, Vol. 49 No. 2, pp. 629-653.

[9] [1994] "Internally stable nonlinear systems with disturbances: a parametrization", IEEE Transactions on Automatic Control, Vol. AC-39, pp. 300 - 314.

[10] [2004] "Tracking and approximate model matching for nonlinear systems", accepted for publication in the International Journal of Control.

[11] J.F. LASALLE and S. LEFSCHETZ
[1961] "Stability by Liapunov's direct method with applications", Academic Press, N.Y. .

[12] S. LEFSCHETZ
[1965] "Stability of nonlinear control systems", Academic Press, N.Y.

[13] H. LOGEMANN, E.P. RYAN, and S. TOWNLEY
[1999] "Integral control of linear systems with actuator nonlinearities: lower bounds for the maximal regulating gain", Trans. IEEE, Vol. AC-44, pp. 1315-1319.

[14] G.C. NEWTON, L.A. GOULD, AND J.F. KAISER
[1957] "Analytical design of linear feedback controls", Wiley, N.Y.

[15] A.D.B. PAICE and J.B. MOORE
[1990] "Robust stabilization of nonlinear plants via left coprime factorizations", Systems and Control Letters, Vol. 15, pp. 121-129.

[16] A.D.B. PAICE and A.J. van der SCHAFT
[1994] "Stable kernel representation and the Youla parametrization for nonlinear systems", Proc. of the 33rd IEEE Conference on Decision and Control, Lake Buena Vista, FL, pp. 781-786.

[17] I.W. SANDBERG
[1993] "Uniform approximation and the circle criterion", Trans. IEEE, Vol. AC-38, pp. 1450-1458.

[18] E.D. SONTAG
[1989] "Smooth stabilization implies coprime factorization", Trans. IEEE, Vol. AC-34, pp. 435-444.

[19] M.S. VERMA
[1988] "Coprime fractional representations and stability of nonlinear feedback systems", Int. J. Control, Vol. 48, pp. 897-918.

[20] M.S. VERMA and L.R. HUNT
[1993] "Right coprime factorization and stabilization for nonlinear systems", IEEE Trans. on Automatic Control, AC-38, pp. 222-231.