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Adaptive Synchronized Attitude Angular Velocity Tracking Control of Multi-UAVs

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***Abstract*— An adaptive synchronization strategy is proposed and applied to the attitude angular velocity tracking control of multiple Unmanned Aerial Vehicles (UAVs). It achieves global asymptotic convergence of both the attitude angular velocity tracking and the angular velocity synchronization, even in the presence of system parameter uncertainties. Simulation results of multiple UAVs in coordination verify the effectiveness of the proposed approach.**

I. Introduction

In order to achieve high efficiency and productivity, there is tremendous demand for the use of multicomposed systems in manufacturing, aerospace operations and other applications. The multicomposed systems work either under cooperative or under coordinated schemes. According to [1], synchronization may be defined as the mutual time conformity of two or more processes. Cooperation, coordination and synchronization are intimately linked subjects and usually they are used as synonyms to describe the same kind of behavior [2].

For mechanical systems, motion synchronization is of great importance as soon as two or more systems have to cooperate. The cooperative behavior gives flexibility and maneuverability that cannot be achieved by an individual system [3]. In recent years, motion synchronization has received a lot of attention. A cross-coupling concept is first proposed in [4] to address the motion synchronization problem. It is combined with adaptive feedforward controller to achieve speed synchronization of two motion axes [5], as well as position and speed synchronization of two gyro motions [6]. The position synchronization of multiple robot systems with only position measurements is studied in [2], where coupling errors are introduced to create interconnections that render mutual synchronization of the robots. This method is also applied to synchronization control of multiple mobile robots [3].

In this paper, we address the angular velocity synchronization of multiple Unmanned Aerial Vehicles (UAVs) during their coordinated flight or formation flight. Motivated by the aforementioned cross-coupling concept, we propose an adaptive synchronized attitude angular velocity tracking strategy for multiple UAVs. It achieves global asymptotic convergences of both the attitude angular velocity tracking and the angular velocity synchronization, even in the presence of system parameter uncertainties.

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The remainder of this paper is organized as follows. Section II introduces the equations of attitude motion of UAV in a body-fixed coordinate frame. Section III presents the generalized synchronization error concept and the development of the adaptive synchronization controller. Moreover, the stability analysis is given to demonstrate the global asymptotic convergence. Numerical simulation results of attitude angular velocity tracking control of 4 UAVs are given in Section IV. Finally, Section V offers some conclusions and recommendation on future work.

II. Attitude dynamics of UAV

From the traditional nonlinear aircraft model, we have the following attitude dynamics for UAVs

$$J_{xx}\dot{P} - J_{xz}\dot{R} + (J_{zz} - J_{yy})QR - J_{xz}PQ = L \tag{1}$$

$$J_{yy}\dot{Q} + (J_{xx} - J_{zz})PR + J_{xz}(P^2 - R^2) = M \tag{2}$$

$$J_{zz}\dot{R} - J_{xz}\dot{P} + (J_{yy} - J_{xx})PQ + J_{xz}QR = N \tag{3}$$

where $\mathbf{J} \in \mathbb{R}^{3\times3}$ is a positive-definite moment of inertia matrix, J_{xx}, J_{yy}, J_{zz} and J_{xz} are the corresponding elements, the applied moment $\mathbf{M} \in \mathbb{R}^3 \triangleq [L \quad M \quad N]^T$, $\Omega \in \mathbb{R}^3 \triangleq [P \quad Q \quad R]^T$ is the attitude rate vector and P, Q, R are the roll, pitch, yaw rate, respectively..

Our aim is to design a controller for attitude angular velocity tracking and angular velocity tracking synchronization between all UAVs. First, we introduce a pseudo attitude angle vector $\Theta \in \mathbb{R}^3 \triangleq [\theta_P \quad \theta_Q \quad \theta_R]^T$ and $\theta_P = \int P$, $\theta_Q = \int Q$, $\theta_R = \int R$. The pseudo attitude angle vector Θ is different from the classical Euler angle $[\phi \quad \theta \quad \psi]^T$ and is introduced only for controller design purpose.

Thus, we obtain the following augmented equations of attitude dynamics

$$J_{xx}\ddot{\theta}_P - J_{xz}\ddot{\theta}_R + (J_{zz} - J_{yy})QR - J_{xz}PQ = L \tag{4}$$

$$J_{yy}\ddot{\theta}_Q + (J_{xx} - J_{zz})PR + J_{xz}(P^2 - R^2) = M \tag{5}$$

$$J_{zz}\ddot{\theta}_R - J_{xz}\ddot{\theta}_P + (J_{yy} - J_{xx})PQ + J_{xz}QR = N \tag{6}$$

or in matrix format

$$\mathbf{J}\ddot{\Theta} + \mathbf{N}(\dot{\Theta}, \mathbf{J}) = \mathbf{M} \tag{7}$$

where the nonlinear term $\mathbf{N}(\cdot) \in \mathbb{R}^3$ is

$$\mathbf{N}(\dot{\Theta}, \mathbf{J}) = \begin{bmatrix} (J_{zz} - J_{yy})QR - J_{xz}PQ \\ (J_{xx} - J_{zz})PR + J_{xz}(P^2 - R^2) \\ (J_{yy} - J_{xx})PQ + J_{xz}QR \end{bmatrix} \tag{8}$$

If n similar UAVs are considered, we have the following n attitude dynamics equations

$$\begin{aligned}
\mathbf{J}_1\ddot{\Theta}_1 + \mathbf{N}_1(\Omega_1, \mathbf{J}_1) &= \mathbf{M}_1 \\
\mathbf{J}_2\ddot{\Theta}_2 + \mathbf{N}_2(\Omega_2, \mathbf{J}_2) &= \mathbf{M}_2 \\
&\vdots \\
\mathbf{J}_i\ddot{\Theta}_i + \mathbf{N}_i(\Omega_i, \mathbf{J}_i) &= \mathbf{M}_i \\
&\vdots \\
\mathbf{J}_n\ddot{\Theta}_n + \mathbf{N}_n(\Omega_n, \mathbf{J}_n) &= \mathbf{M}_n
\end{aligned}$$

where subscript i denotes the i-th UAV.

Writing these n attitude dynamic equations in a matrix format, we have

$$\mathbf{I}\ddot{\mathbf{\Theta}} + \overline{\mathbf{N}}(\mathbf{\Omega}, \mathbf{I}) = \overline{\mathbf{M}} \tag{9}$$

where $\mathbf{\Theta} \in \mathbb{R}^{3n}$, $\overline{\mathbf{N}} \in \mathbb{R}^{3n}$, $\overline{\mathbf{M}} \in \mathbb{R}^{3n}$ are vectors, $\mathbf{I} \in \mathbb{R}^{3n\times 3n}$ is a diagonal matrix, and they are of the following expressions

$$\begin{aligned}
\mathbf{I} &= \text{diag}[\mathbf{J}_1\ \ \mathbf{J}_2\ \ \cdots\ \mathbf{J}_i\ \cdots\ \mathbf{J}_n]^T \\
\mathbf{\Theta} &= [\Theta_1\ \ \Theta_2\ \ \cdots\ \Theta_i\ \cdots\ \Theta_n]^T \\
\overline{\mathbf{N}} &= [\mathbf{N}_1\ \ \mathbf{N}_2\ \ \cdots\ \mathbf{N}_i\ \cdots\ \mathbf{N}_n]^T \\
\overline{\mathbf{M}} &= [\mathbf{M}_1\ \ \mathbf{M}_2\ \ \cdots\ \mathbf{M}_i\ \cdots\ \mathbf{M}_n]^T \\
&= [L_1\ \ M_1\ \ N_1\ \ L_2\ \ M_2\ \ N_2\ \ \cdots\ L_i\ \ M_i\ \ N_i\ \cdots \\
&\quad\ L_n\ \ M_n\ \ N_n]^T
\end{aligned}$$

III. Controller design

A. Control objective

First, we define $\mathbf{E}(t) \in \mathbb{R}^{3n}$ as the attitude angular velocity tracking error vector of n UAVs, $\Xi(t) \in \mathbb{R}^{3n}$ as the angular velocity synchronization error vector. They have the following expressions

$$\mathbf{E}(t) \triangleq [\mathbf{e}_1(t)\ \ \mathbf{e}_2(t)\ \cdots\ \mathbf{e}_i(t)\ \cdots\ \mathbf{e}_n(t)]^T \tag{10}$$

$$\Xi(t) \triangleq [\varepsilon_1(t)\ \ \varepsilon_2(t)\ \cdots\ \varepsilon_i(t)\ \cdots\ \varepsilon_n(t)]^T \tag{11}$$

$$\mathbf{e}_i(t) \triangleq \Omega_{\text{d}}(t) - \Omega_i(t) \tag{12}$$

$$\int \mathbf{e}_i(t) \triangleq \Theta_{\text{d}}(t) - \Theta_i(t) \tag{13}$$

where $\Omega_{\text{d}} \in \mathbb{R}^3$ is the desired attitude angular velocity trajectory. The details on synchronization error will be discussed in the subsequent section.

For synchronized attitude angular velocity tracking of multiple UAVs, first of all, the designed controller should guarantee the stability of the attitude angular velocity tracking errors of all involved systems. Secondly, the controller should also guarantee the stability of the angular velocity synchronization errors. Thirdly, the controller should regulate the attitude angular velocity motions to track the desired trajectory at the same rate so that the corresponding synchronization errors go to zero simultaneously.

Thus, the control objective becomes: $\mathbf{E}(t) \to 0$ and $\Xi(t) \to 0$ as $t \to \infty$.

Without loss of generality, we reformat the attitude dynamic equations in Eq. (9) as follows

$$\mathbf{I\Phi} + \overline{\mathbf{N}}(\mathbf{\Omega}, \mathbf{I}) = \overline{\mathbf{Y}}(\mathbf{\Phi}, \mathbf{\Omega})\mathbf{\Psi} = \overline{\mathbf{M}} \tag{14}$$

where $\mathbf{Y}_i \in \mathbb{R}^{3\times 4}$ is the regression matrix and is composed of known variables, $\mathbf{\Psi}_i \in \mathbb{R}^4$ is a constant parameter vector, $\mathbf{\Phi}_i \in \mathbb{R}^3$ is a dummy variable vector and will be defined later, $\mathbf{\Psi} \in \mathbb{R}^{4n}$, $\overline{\mathbf{Y}} \in \mathbb{R}^{3n\times 4n}$. In practice, there exist uncertainties in the system parameters, such as the moments of inertia. We define $\widehat{\mathbf{\Psi}}(t)$ as the estimation of $\mathbf{\Psi}$. Moreover, the following expressions can be obtained

$$\begin{aligned}
\mathbf{\Psi} &= [\mathbf{\Psi}_1\ \ \mathbf{\Psi}_2\ \ \cdots\ \mathbf{\Psi}_i\ \cdots\ \mathbf{\Psi}_n]^T \\
\mathbf{\Psi}_i &= [J_{xxi}\quad J_{yyi}\quad J_{zzi}\quad J_{xzi}]^T \\
\mathbf{\Phi} &= [\mathbf{\Phi}_1\ \ \mathbf{\Phi}_2\ \ \cdots\ \mathbf{\Phi}_i\ \cdots\ \mathbf{\Phi}_n]^T \\
\mathbf{\Phi}_i &= [\Phi_{Pi}\quad \Phi_{Qi}\quad \Phi_{Ri}]^T \\
\overline{\mathbf{Y}} &= \text{diag}[\mathbf{Y}_1\ \ \mathbf{Y}_2\ \ \cdots\ \mathbf{Y}_i\ \cdots\ \mathbf{Y}_n] \\
\mathbf{Y}_i(\mathbf{\Phi}_i, \mathbf{\Omega}_i) &=
\end{aligned} \tag{15}$$

$$\begin{bmatrix} \Phi_{Pi} & -Q_iR_i & Q_iR_i & -(\Phi_{Ri} + P_iQ_i) \\ P_iR_i & \Phi_{Qi} & -P_iR_i & P_i{}^2 - R_i{}^2) \\ -P_iQ_i & P_iQ_i & \Phi_{Ri} & Q_iR_i - \Phi_{Pi} \end{bmatrix}$$

B. Synchronization error

Synchronization error is used to identify the performance of synchronization controller, i.e. how the trajectory of each UAV converges with respect to each other. There are various ways to choose the synchronization error. In general, the attitude angular velocity synchronization error $\Xi(t)$ can be formulated as the following generalized expression.

$$\Xi(t) = \mathbf{TE}(t) \tag{16}$$

where $\Xi(t)$ is a linear combination of attitude angular velocity tracking error $\mathbf{E}(t)$, $\mathbf{T} \in \mathbb{R}^{3n\times 3n}$ is the generalized synchronization transformation matrix.

If we choose the following synchronization transformation matrix

$$\mathbf{T} = \begin{bmatrix} 2\text{I} & -\text{I} & & & -\text{I} \\ -\text{I} & 2\text{I} & -\text{I} & & \\ & \ddots & \ddots & \ddots & \\ & & -\text{I} & 2\text{I} & -\text{I} \\ -\text{I} & & & -\text{I} & 2\text{I} \end{bmatrix} \tag{17}$$

we can get the corresponding attitude angular velocity synchronization error

$$\begin{aligned}
\varepsilon_1(t) &= 2\mathbf{e}_1(t) - \mathbf{e}_2(t) - \mathbf{e}_n(t) \\
\varepsilon_2(t) &= 2\mathbf{e}_2(t) - \mathbf{e}_3(t) - \mathbf{e}_1(t) \\
\varepsilon_3(t) &= 2\mathbf{e}_3(t) - \mathbf{e}_4(t) - \mathbf{e}_2(t) \\
&\vdots \\
\varepsilon_n(t) &= 2\mathbf{e}_n(t) - \mathbf{e}_{n-1}(t) - \mathbf{e}_1(t)
\end{aligned} \tag{18}$$

In a similar way, other kinds of synchronization errors can be formed. It can be seen from Eq. (18) that each individual UAV's synchronization error is a linear combination of its tracking error and two adjoining UAVs' tracking errors.

C. Adaptive synchronization controller

According to the control objective discussed previously, the designed control moment $\overline{\mathbf{M}}(t)$ should guarantee convergences of both the attitude angular velocity tracking error $\mathbf{E}(t)$ and the angular velocity synchronization error $\Xi(t)$ simultaneously, and realize expected transient characteristic of attitude angular velocity tracking motion.

To realize motion synchronization between all UAVs, we may include all synchronization errors related to a certain axis into this axis's controller for each UAV. However, this method can only be feasible when the UAVs number n is small. If n is extreme large, including all synchronization errors into each controller will lead to intensive on-line computational work. Thus, in this section, we try to design controller for each axis to stabilize its attitude angular velocity tracking motion and synchronize its motion with several adjacent attitude axes.

First, the coupled attitude angular velocity error $\mathbf{E}^*(t)$ that contains both the attitude angular velocity tracking error $\mathbf{E}(t)$ and a linear combination of the angular velocity synchronization error $\Xi(t)$ is introduced

$$\mathbf{E}^*(t) = \mathbf{E}(t) + \mathbf{B}\mathbf{T}^T \int \Xi(t) \tag{19}$$

where $\mathbf{B} \triangleq \text{diag}[\mathbf{B}_1 \quad \mathbf{B}_2 \quad \cdots \quad \mathbf{B}_n]$ is a positive coupling gain matrix, $\mathbf{E}^* \triangleq [\mathbf{e}_1^* \quad \mathbf{e}_2^* \quad \cdots \quad \mathbf{e}_n^*]^T$ and its expanded expression is.

$$\begin{aligned}
\mathbf{e}_1^*(t) &= \mathbf{e}_1(t) + \mathbf{B}_1 \int \big[2\varepsilon_1(\tau) - \varepsilon_2(\tau) - \varepsilon_n(\tau)\big] \\
\mathbf{e}_2^*(t) &= \mathbf{e}_2(t) + \mathbf{B}_2 \int \big[2\varepsilon_2(\tau) - \varepsilon_3(\tau) - \varepsilon_1(\tau)\big] \\
\mathbf{e}_3^*(t) &= \mathbf{e}_3(t) + \mathbf{B}_3 \int \big[2\varepsilon_3(\tau) - \varepsilon_4(\tau) - \varepsilon_2(\tau)\big] \quad (20) \\
&\vdots \\
\mathbf{e}_n^*(t) &= \mathbf{e}_n(t) + \mathbf{B}_n \int \big[2\varepsilon_n(\tau) - \varepsilon_{n-1}(\tau) - \varepsilon_1(\tau)\big]
\end{aligned}$$

From Eq. (20) we can see that, the synchronization error $\varepsilon_i(t)$ appears in $\mathbf{e}_{i-1}^*(t)$ and $\mathbf{e}_{i+1}^*(t)$ as negative value $-\varepsilon_i(t)$, while as positive value $2\varepsilon_i(t)$ in $\mathbf{e}_i^*(t)$. In this way, the coupled attitude errors are driven in opposite directions by $\varepsilon_i(t)$, which contributes to the elimination of the synchronization error $\varepsilon_i(t)$.

With notation of the coupled attitude angular velocity error $\mathbf{E}^*(t)$, the coupled filtered tracking error for the ith UAV, $\mathbf{r}_i(t) \in \mathbb{R}^3$, is further defined as

$$\mathbf{r}_i(t) = \mathbf{e}_i^*(t) + \mathbf{\Lambda}_i \int \mathbf{e}_i^*(t) \tag{21}$$

where $\mathbf{\Lambda}_i \in \mathbb{R}^{3\times 3}$ is a constant, diagonal, positive-definite, control gain matrix. For all n UAVs, the whole coupled filtered tracking error becomes

$$\mathbf{r}(t) = \mathbf{E}^*(t) + \mathbf{\Lambda} \int \mathbf{E}^*(t) \tag{22}$$

where $\mathbf{r}(t) \triangleq [\mathbf{r}_1(t) \quad \mathbf{r}_2(t) \quad \cdots \quad \mathbf{r}_n(t)]^T$, $\mathbf{\Lambda} \in \mathbb{R}^{3n\times 3n} \triangleq \text{diag}[\mathbf{\Lambda}_1 \quad \mathbf{\Lambda}_2 \quad \cdots \quad \mathbf{\Lambda}_n]$.

To account for the uncertainties in system parameters, the controller is designed to contain an adaptation law for on-line estimation of the unknown parameters. The error between the actual and estimated parameters is defined by

$$\widetilde{\mathbf{\Psi}}(t) \triangleq \mathbf{\Psi}(t) - \widehat{\mathbf{\Psi}}(t) \tag{23}$$

where the parameter estimation error $\widetilde{\mathbf{\Psi}}(t) \in \mathbb{R}^{4n}$.

The dummy variable $\mathbf{\Phi}$ introduced in Eq. (14) has the following expression

$$\mathbf{\Phi} = \ddot{\mathbf{\Theta}}_d + \mathbf{\Lambda}\mathbf{E}^* + \mathbf{B}\mathbf{T}^T\Xi \tag{24}$$

Based on the dynamics equation (14), the control moment vector $\overline{\mathbf{M}}(t)$ is designed to be

$$\overline{\mathbf{M}}(t) = \overline{\mathbf{Y}}(\mathbf{\Phi}, \mathbf{\Omega})\widehat{\mathbf{\Psi}}(t) + \mathbf{K}\mathbf{r}(t) + \mathbf{K}_s\mathbf{T}^T \int \Xi(t) \tag{25}$$

where $\mathbf{K} \in \mathbb{R}^{3n\times 3n}$ and $\mathbf{K}_s \in \mathbb{R}^{3n\times 3n}$ are two constant, diagonal, positive-definite, control gain matrices. The estimated parameter $\widehat{\mathbf{\Psi}}(t)$ is subject to the adaptation law

$$\dot{\widehat{\mathbf{\Psi}}} = \mathbf{\Gamma}\overline{\mathbf{Y}}^T(\cdot)\mathbf{r} \tag{26}$$

where $\mathbf{\Gamma} \in \mathbb{R}^{4n\times 4n}$ is a constant, diagonal, positive-definite, adaptation gain matrix. By differentiating Eq. (23) with respect to time, the following closed-loop dynamics for the parameter estimation error can be obtained

$$\dot{\widetilde{\mathbf{\Psi}}} = -\mathbf{\Gamma}\overline{\mathbf{Y}}^T(\cdot)\mathbf{r} \tag{27}$$

Theorem 1. The proposed adaptive synchronization coupling controller Eqs. (25) and (26) guarantees the global asymptotic convergences of both the attitude angualr velocity tracking error $\mathbf{E}(t)$ and the angular velocity synchronization error $\Xi(t)$, i.e.,

$$\lim_{t\to\infty} \mathbf{E}(t), \Xi(t) = 0 \tag{28}$$

Proof. Considering the following positive definite Lyapunov function

$$\begin{aligned}
V(\mathbf{r}, \widetilde{\mathbf{\Psi}}, \Xi) &\triangleq \frac{1}{2}\Big[\mathbf{r}^T\mathbf{I}\mathbf{r} + \widetilde{\mathbf{\Psi}}^T\mathbf{\Gamma}^{-1}\widetilde{\mathbf{\Psi}} + \Big(\int \Xi\Big)^T\mathbf{K}_s \int \Xi \\
&+ \Big(\int\int \mathbf{T}^T\Xi\Big)^T\mathbf{B}\mathbf{\Lambda}\mathbf{K}_s\Big(\int\int \mathbf{T}^T\Xi\Big)\Big] \quad (29)
\end{aligned}$$

Differentiating Eq. (29) with respect to time t yields

$$\begin{aligned}
\dot{V}(\mathbf{r}, \widetilde{\mathbf{\Psi}}, \Xi) &= \mathbf{r}^T\mathbf{I}\dot{\mathbf{r}} + \widetilde{\mathbf{\Psi}}^T\mathbf{\Gamma}^{-1}\dot{\widetilde{\mathbf{\Psi}}} + \int \Xi^T\mathbf{K}_s\Xi \\
&+ \Big(\int\int \mathbf{T}^T\Xi\Big)^T\mathbf{B}\mathbf{\Lambda}\mathbf{K}_s\mathbf{T}^T \int \Xi \quad (30)
\end{aligned}$$

Differentiating Eq. (22) with respect to time t and consider Eqs. (19, 24), we have

$$\begin{aligned}
\dot{\mathbf{r}} &= \dot{\mathbf{E}}^* + \mathbf{\Lambda}\mathbf{E}^* = \dot{\mathbf{E}} + \mathbf{B}\mathbf{T}^T \int \Xi + \mathbf{\Lambda}\mathbf{E}^* \\
&= \ddot{\mathbf{\Theta}}_d - \ddot{\mathbf{\Theta}} + \mathbf{B}\mathbf{T}^T \int \Xi + \mathbf{\Lambda}\mathbf{E}^* \\
&= \mathbf{\Phi} - \ddot{\mathbf{\Theta}} \quad (31)
\end{aligned}$$

Multiplying $\mathbf{I}$ at the both sides of Eq. (31), substituting Eqs. (9, 14, 25) into it, we obtain

$$\begin{aligned} \mathbf{I}\dot{\mathbf{r}} &= \mathbf{I}\mathbf{\Phi} - \mathbf{I}\ddot{\mathbf{\Theta}} \\ &= \overline{\mathbf{Y}}(\mathbf{\Phi}, \mathbf{\Omega})\mathbf{\Psi} - \overline{\mathbf{M}} \\ &= \overline{\mathbf{Y}}(\mathbf{\Phi}, \mathbf{\Omega})\widetilde{\mathbf{\Psi}} - \mathbf{K}\mathbf{r} - \mathbf{K}_s\mathbf{T}^T\int\mathbf{\Xi} \end{aligned} \tag{32}$$

Substituting Eqs. (27, 32) into Eq. (30) yields

$$\begin{aligned} \dot{V}(\mathbf{r}, \widetilde{\mathbf{\Psi}}, \mathbf{\Xi}) &= \mathbf{r}^T\mathbf{I}\dot{\mathbf{r}} + \widetilde{\mathbf{\Psi}}^T\mathbf{\Gamma}^{-1}\dot{\widetilde{\mathbf{\Psi}}} + \int\mathbf{\Xi}^T\mathbf{K}_s\mathbf{\Xi} \\ &\quad +(\int\int\mathbf{T}^T\mathbf{\Xi})^T\mathbf{B}\mathbf{\Lambda}\mathbf{K}_s\mathbf{T}^T\int\mathbf{\Xi} \\ &= \mathbf{r}^T[\overline{\mathbf{Y}}(\mathbf{\Phi}, \mathbf{\Omega})\widetilde{\mathbf{\Psi}} - \mathbf{K}\mathbf{r} - \mathbf{K}_s\mathbf{T}^T\int\mathbf{\Xi}] \\ &\quad +\widetilde{\mathbf{\Psi}}^T\mathbf{\Gamma}^{-1}[-\mathbf{\Gamma}\overline{\mathbf{Y}}^T(\cdot)\mathbf{r}] + \int\mathbf{\Xi}^T\mathbf{K}_s\mathbf{\Xi} \\ &\quad +(\int\int\mathbf{T}^T\mathbf{\Xi})^T\mathbf{B}\mathbf{\Lambda}\mathbf{K}_s\mathbf{T}^T\int\mathbf{\Xi} \\ &= -\mathbf{r}^T\mathbf{K}\mathbf{r} - \mathbf{r}^T\mathbf{K}_s\mathbf{T}^T\int\mathbf{\Xi} + \int\mathbf{\Xi}^T\mathbf{K}_s\mathbf{\Xi} \\ &\quad +(\int\int\mathbf{T}^T\mathbf{\Xi})^T\mathbf{B}\mathbf{\Lambda}\mathbf{K}_s\mathbf{T}^T\int\mathbf{\Xi} \end{aligned} \tag{33}$$

Replacing $\mathbf{r}$ in the second term of Eq. (33) with Eqs. (19, 22), we have

$$\begin{aligned} \dot{V}(\cdot) &= -\left[\mathbf{E} + \mathbf{B}\mathbf{T}^T\int\mathbf{\Xi} + \mathbf{\Lambda}\int\mathbf{E} + \mathbf{B}\mathbf{\Lambda}\mathbf{T}^T\int\int\mathbf{\Xi}\right]^T \\ &\quad \cdot\mathbf{K}_s\mathbf{T}^T\int\mathbf{\Xi} - \mathbf{r}^T\mathbf{K}\mathbf{r} + \int\mathbf{\Xi}^T\mathbf{K}_s\mathbf{\Xi} \\ &\quad +(\int\int\mathbf{T}^T\mathbf{\Xi})^T\mathbf{B}\mathbf{\Lambda}\mathbf{K}_s\mathbf{T}^T\int\mathbf{\Xi} \\ &= -\mathbf{r}^T\mathbf{K}\mathbf{r} - \int(\mathbf{T}^T\mathbf{\Xi})^T\mathbf{B}\mathbf{K}_s\mathbf{T}^T\int\mathbf{\Xi} \\ &\quad -\int\mathbf{\Xi}^T\mathbf{\Lambda}\mathbf{K}_s\int\mathbf{\Xi} \end{aligned}$$

Because $\mathbf{K}$, $\mathbf{B}\mathbf{K}_s$, and $\mathbf{\Lambda}\mathbf{K}_s$ are all positive-definite matrices, so we can conclude that

$$\begin{aligned} \dot{V}(\cdot) &= -\mathbf{r}^T\mathbf{K}\mathbf{r} - \int(\mathbf{T}^T\mathbf{\Xi})^T\mathbf{B}\mathbf{K}_s\int(\mathbf{T}^T\mathbf{\Xi}) \\ &\quad -\int\mathbf{\Xi}^T\mathbf{\Lambda}\mathbf{K}_s\int\mathbf{\Xi} \leq 0 \end{aligned} \tag{34}$$

Since $\dot{V}(\mathbf{r}, \widetilde{\mathbf{\Psi}}, \mathbf{\Xi}) \leq 0$ in Eq. (34), the $V(\mathbf{r}, \widetilde{\mathbf{\Psi}}, \mathbf{\Xi})$ given in Eq. (29) is either decreasing or constant. Due to the fact that $V(\mathbf{r}, \widetilde{\mathbf{\Psi}}, \mathbf{\Xi})$ is non-negative, we conclude that $V(\mathbf{r}, \widetilde{\mathbf{\Psi}}, \mathbf{\Xi}) \in \mathcal{L}_\infty$; hence $\mathbf{r} \in \mathcal{L}_\infty$ and $\widetilde{\mathbf{\Psi}} \in \mathcal{L}_\infty$. With $\mathbf{r} \in \mathcal{L}_\infty$, we conclude from Eq. (22) that $\mathbf{E}^*(t) \in \mathcal{L}_\infty$ and $\int\mathbf{E}^*(t) \in \mathcal{L}_\infty$, and $\int\mathbf{E}(t)$, $\mathbf{E}(t)$, $\int\mathbf{\Xi}(t)$, $\mathbf{\Xi}(t) \in \mathcal{L}_\infty$ based on the definition in Eq. (19). Because of the boundedness of $\mathbf{\Theta}_d(t)$ and $\mathbf{\Omega}_d(t)$, we conclude that $\mathbf{\Theta}(t) \in \mathcal{L}_\infty$ and $\mathbf{\Omega}(t) \in \mathcal{L}_\infty$ from Eq. (13). Taking $\widetilde{\mathbf{\Psi}} \in \mathcal{L}_\infty$ and $\mathbf{\Psi}$ a constant vector, $\widehat{\mathbf{\Psi}} \in \mathcal{L}_\infty$ is obtained from Eq. (23). With the previous boundedness statements and the fact that $\ddot{\mathbf{\Theta}}_d(t)$ is also bounded, the dummy variable $\mathbf{\Phi} \in \mathcal{L}_\infty$ and $\overline{\mathbf{Y}}(\cdot) \in \mathcal{L}_\infty$ can be concluded from their definitions in Eq. (24) and Eq. (15). Hence, the control input $\overline{\mathbf{M}}(t) \in \mathcal{L}_\infty$ is also determined from Eq. (25). The preceding information can also be used to Eq. (9) and Eq. (31) to get $\ddot{\mathbf{\Theta}}(t)$, $\dot{\mathbf{r}}(t) \in \mathcal{L}_\infty$. Until now, we have explicitly illustrated that all signals in the adaptive synchronization controller and system remain bounded during the closed-loop operation.

From Eq. (34), we show that $\mathbf{r}(t) \in \mathcal{L}_2$, $\mathbf{T}^T\int\mathbf{\Xi}(t) \in \mathcal{L}_2$ and $\int\mathbf{\Xi}(t) \in \mathcal{L}_2$. Hence, $\int\mathbf{E}^*(t) \in \mathcal{L}_2$ and $\mathbf{E}^*(t) \in \mathcal{L}_2$ can be concluded from Eq. (22). From Eq. (19), we conclude $\mathbf{E}(t) \in \mathcal{L}_2$ since $\mathbf{T}^T\int\mathbf{\Xi}(t) \in \mathcal{L}_2$ and $\mathbf{E}^*(t) \in \mathcal{L}_2$. Moreover, we have $\int\mathbf{E}(t) \in \mathcal{L}_2$ because of $\int\mathbf{\Xi}(t) \in \mathcal{L}_2$. When $\ddot{\mathbf{\Theta}}(t) \in \mathcal{L}_\infty$ and $\ddot{\mathbf{\Theta}}_d(t)$ is bounded, $\dot{\mathbf{E}}(t) \in \mathcal{L}_\infty$ can be concluded. Considering $\mathbf{E}(t) \in \mathcal{L}_2$ and $\dot{\mathbf{E}}(t) \in \mathcal{L}_\infty$, we can conclude that $\mathbf{\Xi}(t) \in \mathcal{L}_2$ and $\dot{\mathbf{\Xi}}(t) \in \mathcal{L}_\infty$. Thus, Corollary 1.1 in [7] can be applied to conclude that

$$\lim_{t\to\infty}\mathbf{E}(t) = 0 \quad \lim_{t\to\infty}\mathbf{\Xi}(t) = 0$$

IV. Simulation

In this section, we take one example of cooperative motion of four UAVs. The UAVs are chosen as Hanger 9 J-3 Piper Cub like UAV [8]. The parameters, moments of inertia, given in Table I are modified according to the values used in [9]. These four UAVs are required to track the following three-axis attitude angular rate profiles.

$$\begin{aligned} P_d(t) &= 0.5\pi\cos(0.1\pi t) \\ Q_d(t) &= 0.4\pi\cos(0.1\pi t) \\ R_d(t) &= 0.3\pi\cos(0.1\pi t) \end{aligned} \tag{35}$$

The control and adaptation gains are also given in Table I.

TABLE I: System parameters and control gains of multiple UAVs

Symbol	Value
$\mathbf{\Psi}_i$	$\mathbf{\Psi}_1$=[1.6, 2.37, 2.5, 0.55], $\mathbf{\Psi}_2$=[3.0, 6.7, 9.0, 0.7]
	$\mathbf{\Psi}_3$=[1.2, 1.6, 2.3, 0.3], $\mathbf{\Psi}_4$=[4.74, 6.03, 9.8, 0.8]
$\widehat{\mathbf{\Psi}}_i(0)$	$0.8\mathbf{\Psi}_1$, $0.9\mathbf{\Psi}_2$, $1.1\mathbf{\Psi}_3$, $1.2\mathbf{\Psi}_4$
$\mathbf{K}_i$	$\mathbf{K}_1$=diag[25, 25, 25], $\mathbf{K}_2$=diag[30, 30, 30]
	$\mathbf{K}_3$=diag[15, 15, 15], $\mathbf{K}_4$=diag[45, 65, 75]
$\mathbf{K}_{si}$	$\mathbf{K}_{s1}$=diag[4, 4, 4], $\mathbf{K}_{s2}$=diag[6, 6, 6]
	$\mathbf{K}_{s3}$=diag[3, 3, 3], $\mathbf{K}_{s4}$=diag[7, 7.5, 8]
$\mathbf{\Lambda}_i$	$\mathbf{\Lambda}_1$=diag[8, 8, 8], $\mathbf{\Lambda}_2$=diag[10, 10, 10]
	$\mathbf{\Lambda}_3$=diag[3, 3, 3], $\mathbf{\Lambda}_4$=diag[8, 8, 8]
$\mathbf{\Gamma}_i$	$\mathbf{\Gamma}_1$=diag[11, 11, 11, 11]
	$\mathbf{\Gamma}_2$=diag[25, 25, 25, 25]
	$\mathbf{\Gamma}_3$=diag[10, 10, 10, 10]
	$\mathbf{\Gamma}_4$=diag[30, 30, 30, 30]
$\mathbf{B}_i$	diag[10, 10, 10]

Figure 1 shows the simulation results of adaptive attitude angular velocity tracking control of four UAVs without synchronization strategy. Fig. 2 shows the corresponding simulation results with the proposed synchronization strategy. For evaluation of control performance, we calculated the 2-norm of the attitude angular velocity tracking error $\mathbf{E}(t)$ and the angular velocity synchronization error $\mathbf{\Xi}(t)$.

The values in Table II show that although the attitude angular velocity tracking error $\mathbf{E}(t) \rightarrow 0$ can be realized by using adaptive controller without synchronization straregy, there are significant differences between the attitude angular velocity tracking errors of all involved UAVs. It means that the attitude angular velocity synchronization errors are large and the attitude angular velocity tracking motions are not synchronized well. However, with the proposed synchronization strategy, the synchronization errors can be remarkably reduced. Take the attitude angular velocity tracking motion of z-axis as an example. Without synchronization strategy, the 2-norm of the attitude angular velocity tracking errors and the attitude angular velocity synchronization errors for four UAVs are 21.193, 35.304, 59.490, 14.830 and 35.228, 92.403, 110.478, 51.192 (deg/sec), respectively. With the synchronization strategy, the corresponding 2-norm values are 23.861, 24.191, 27.281, 24.631 and 3.846, 17.047, 21.701, 10.849 (deg/sec), respectively. The 2-norm of the attitude angular velocity tracking errors have been regulated to be very close values. In this way, the synchronization errors are reduced observably.

TABLE II: Performance evaluation of various control strategies

Errors	UAV I	UAV II	UAV III	UAV IV
No synchronization				
$\|\|\dot{e}_x\|\|_2$	49.285	81.439	53.184	42.411
$\|\|\dot{e}_y\|\|_2$	10.235	30.254	53.054	21.316
$\|\|\dot{e}_z\|\|_2$	21.193	35.304	59.490	14.830
$\|\|\dot{\varepsilon}_x\|\|_2$	59.009	136.095	97.633	36.513
$\|\|\dot{\varepsilon}_y\|\|_2$	40.887	67.880	98.714	47.949
$\|\|\dot{\varepsilon}_z\|\|_2$	35.228	92.403	110.478	51.192
With synchronization strategy				
$\|\|\dot{e}_x\|\|_2$	38.029	34.924	26.767	28.719
$\|\|\dot{e}_y\|\|_2$	18.197	19.005	17.311	18.523
$\|\|\dot{e}_z\|\|_2$	23.861	24.191	27.281	24.631
$\|\|\dot{\varepsilon}_x\|\|_2$	42.531	33.950	49.733	22.674
$\|\|\dot{\varepsilon}_y\|\|_2$	12.393	14.557	16.666	13.360
$\|\|\dot{\varepsilon}_z\|\|_2$	3.846	17.047	21.701	10.849

V. Conclusions

In this paper, an adaptive synchronization control approach is applied to the attitude angular velocity tracking of multiple Unmanned Aerial Vehicles (UAVs). Different from the previous work on adaptive tracking control, the global asymptotic convergences of both the attitude angular velocity tracking error and the angular velocity synchronization error can be achieved using the proposed adaptive synchronization controller, even in the presence of unknown parameters. The introduction of the generalized synchronization error in controller provides significant enhancement in achieving synchronization behavior among all UAVs. Future work currently under our consideration includes: 1) adaptive synchronization control of multiple UAVs based on more complicated modeling (such as 6 DOF aircraft model with both translational motion and attitude motion); 2) adaptive synchronization control for multiple UAVs in close formation flight, with consideration of the coupled aerodynamics.

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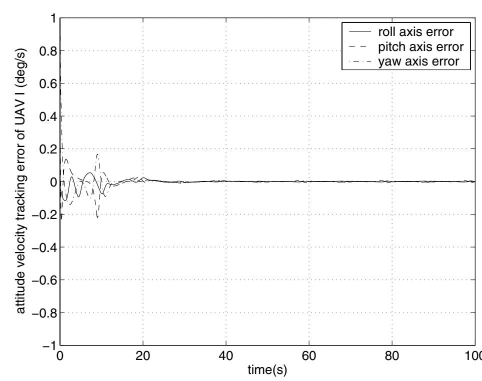

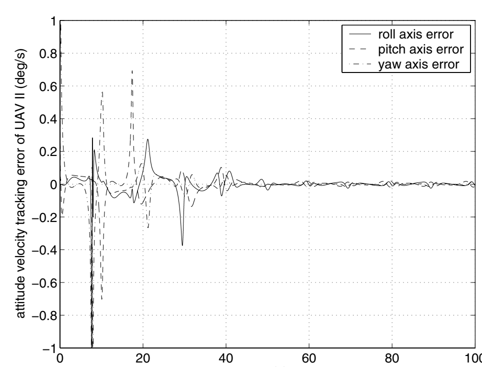

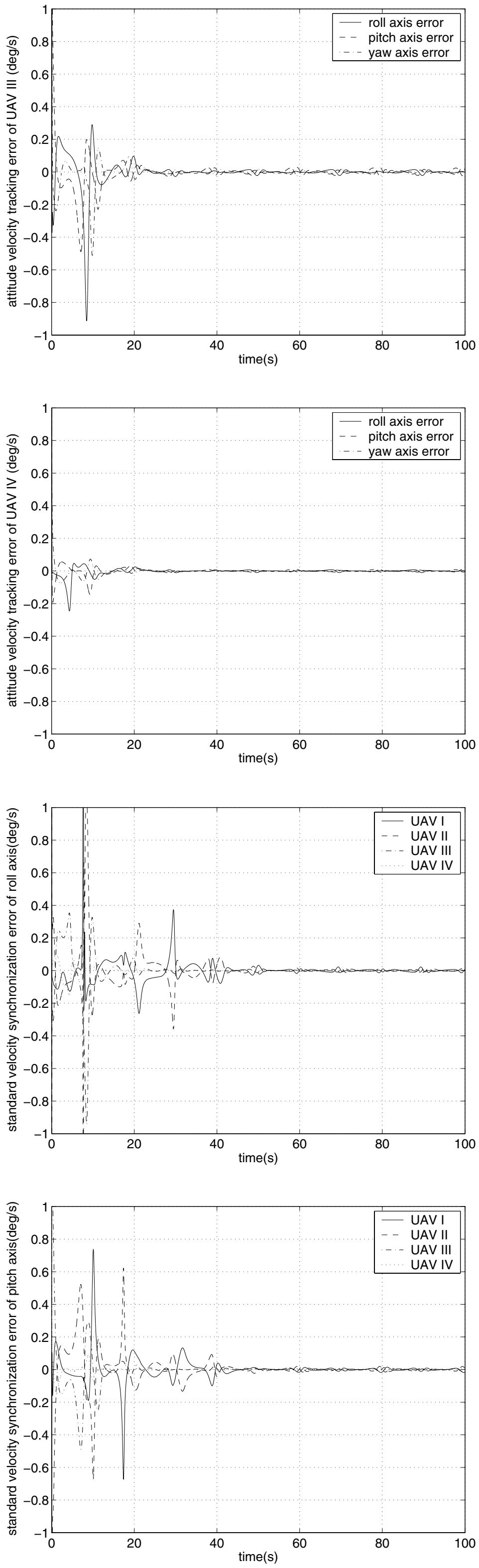

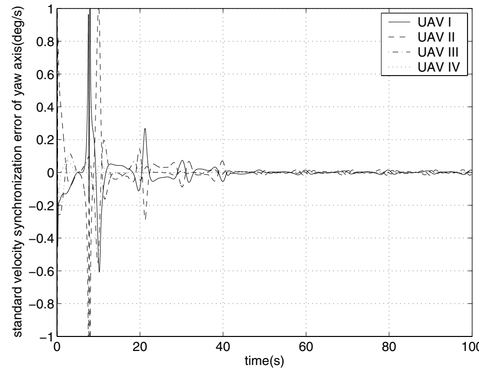

Fig. 1: Adaptive attitude angular velocity tracking control of multiple UAVs

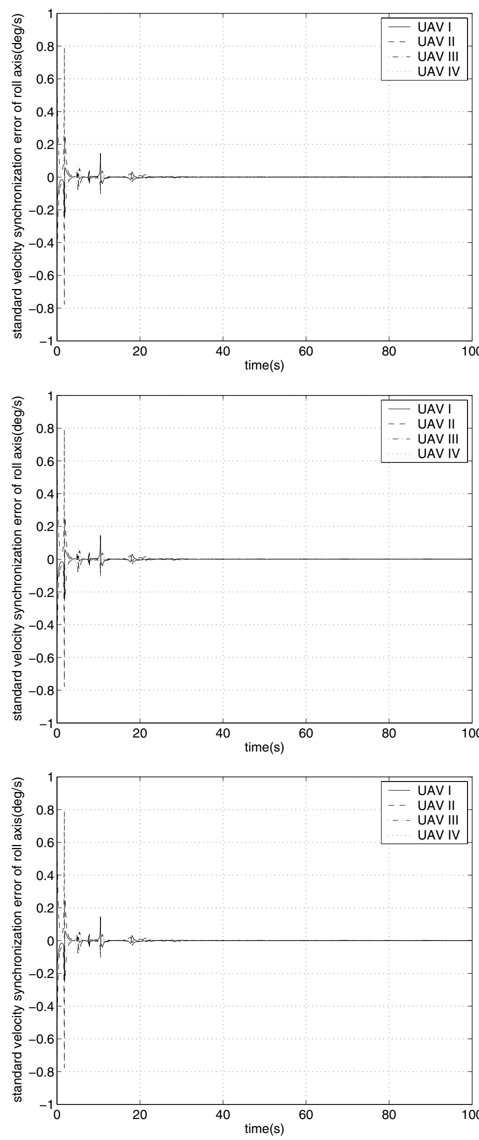

Fig. 2: Adaptive attitude angular velocity tracking control of multiple UAVs with synchronization strategy