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# Stability Analysis and Stabilization Synthesis for Periodically Switched Linear Systems with Uncertainties

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*Abstract*— **This paper addresses the stability analysis and stabilization problems for periodically switched linear systems(PSLSs) with uncertainties in the discrete-time domain. An LMI(Linear Matrices Inequality)-based necessary and sufficient condition is given for the autonomous uncertain PSLS to be asymptotically stable. Then, an LMI-based condition for state feedback stabilization is established. If the condition holds, the computation of feedback gains is also given.**

## I. INTRODUCTION

In recent years, the study of switched systems has received growing attention in control theory and its application. Switched linear systems are an important class of hybrid dynamic systems consisting of a family of linear time-invariant subsystems and a switching law specifying the switching between them. There have been many works for switched linear systems, primarily on stability analysis and synthesis[1]-[11].

Uncertainty is ubiquitous in control systems. It inevitably exists in system model due to the complexity of system itself, exogenous disturbance, measurement errors and so on. There are a few existing results concerning uncertain switched systems. [12] studied quadratic stabilizability via state feedback for both continuous-time and discrete-time switched linear systems that are composed of polytopic uncertain subsystems. [13] considered discrete-time uncertain switched linear systems which are affected both by parameter variations and exterior disturbances. The problem of synthesis of the switching control laws, which assure that the system state is ultimately bounded within a given compact set containing the origin with an assigned rate of convergence, was investigated by the method based on set-induced Lyapunov functions.

A switched linear system with a fixed and periodically switching signal is called a *periodically switched linear system, PSLS*. [14] first studied controllability and observability of PSLSs, some sufficient and necessary conditions for one-period controllability and one-period observability were established, respectively. Then [15] introduced the multiple-periods controllability and multiple-periods observability concepts naturally extended from the one-period ones, and derived necessary and sufficient criteria as well. It was also pointed out that controllability can be realized in $n$ periods at most, where $n$ is the state dimension. Subsequently, [16] extended these controllability results to periodically switched systems with time-delays in control inputs. [17] extended the above results to the discrete time case. Moreover, the bounded controllability for PSLSs with input saturation case were also investigated and necessary and sufficient conditions for null controllability and asymptotic null controllability were obtained, respectively. [18] studied state feedback stabilization problem for periodically switched linear systems in the discrete-time domain. The stabilization problem was solved by the computation of feedback matrices expressed in terms of LMIs. However, uncertainty was not considered in these works.

In this paper, we focus our attention on PSLSs with uncertainties in the discrete-time domain. Each constituent subsystem involves parameter uncertainties appearing in all the matrices of the state-space model. We consider the stability analysis and stabilization problems for this class of uncertain switched systems. A necessary and sufficient condition is given which guarantees that the autonomous uncertain PSLS is asymptotically stable. Similar to the stability analysis, a necessary and sufficient condition for the state feedback stabilization is derived as well. The conditions are both expressed in terms of LMIs which are suitable for application [19].

The paper is organized as follows. In Section II, the problem is formulated and some preliminaries are presented. Section III is the stability analysis result. Section IV contains the state feedback stabilization result. An illustrating example is presented in Section V. Finally Section VI concludes the paper.

**Notations:** $M^T$ is the transpose of the matrix $M$ and $\mathsf{Sym}\{M\}$ is used for the symmetric expression $M + M^T$. $M > 0$ (resp. $< 0$) means that $M$ is positive definite (resp. negative definite). $I$ and $0$ denote the identity and zero matrices of appropriate size respectively. Within symmetric matrices, the symbol $(\bullet)^T$ denotes the corresponding symmetric block.

## II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a switched linear system with uncertainty given by

$$x(t+1) = \widetilde{A}_{r(t)}x(t) + \widetilde{B}_{r(t)}u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^{q_x}$ is the state, $u(t) \in \mathbb{R}^{q_u}$ is the control input, $r(t)$ is the switching signal which takes its value in the finite set $\mathcal{I} = \{1, \ldots, N\}$. $N > 1$ is the number of subsystems. Moreover, $r(t) = i$ means that the subsystem $(\widetilde{A}_i, \widetilde{B}_i)$ is activated. The uncertain parameter matrices are

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in the forms of

$$\begin{bmatrix} \widetilde{A}_i & \widetilde{B}_i \end{bmatrix} = \begin{bmatrix} A_i & B_i \end{bmatrix} + \begin{bmatrix} \Delta A_i & \Delta B_i \end{bmatrix} = \begin{bmatrix} A_i & B_i \end{bmatrix} + H_i\Gamma_i \begin{bmatrix} E_i & T_i \end{bmatrix} \quad (2)$$

where $(A_i, B_i)$ are given constant matrices which describe the $i$-th nominal subsystem, $\Gamma_i$ is the uncertainty of the $i$-th subsystem which satisfies $\Gamma_i^T\Gamma_i \leq I$, and $H_i, E_i, T_i$ are given constant matrices which characterize the structure of the uncertainty.

If $r(t)$ satisfies that

$$r(t) = r_i, \text{ if } t = kN+i-1, \quad i = 1, \cdots, N; k = 0, 1, 2, \cdots \quad (3)$$

where $r_i \in \mathcal{I}$ and $r_l \neq r_m, \forall\, l \neq m$, system (1) is called a *periodically switched linear system, PSLS*. Without loss of generality, we assume that in (3)

$$r_i = i, \, i \in \mathcal{I} \quad (4)$$

In this work, we are interested in the stability analysis and stabilization problems for *PSLSs* with uncertainties. Stability analysis is to establish a condition under which the autonomous uncertain PSLS

$$x(t+1) = \widetilde{A}_{r(t)}x(t) \quad (5)$$

is asymptotically stable, and stabilization is to design a switched state feedback control law as

$$u(t) = K_{r(t)}x(t) \quad (6)$$

ensuring the corresponding closed-loop system

$$x(t+1) = (\widetilde{A}_{r(t)} + \widetilde{B}_{r(t)}K_{r(t)})x(t) \quad (7)$$

is asymptotically stable.

Next, we present two lemmas of which we will make an extensive use.

*Lemma 1:* [20][21] Let $\Psi, M, R$ be given matrices of compatible dimensions, $\Psi = \Psi^T$, then

$$\Psi + M\Gamma R + (M\Gamma R)^T < 0$$

holds for all $\Gamma$ satisfying $\Gamma^T\Gamma \leq I$ if and only if there exists a constant $\alpha > 0$ such that

$$\Psi + \alpha MM^T + \frac{1}{\alpha}R^TR < 0$$

*Lemma 2:* [22] Let $\Phi, U, W$ be given matrices of compatible dimensions, $\Phi = \Phi^T$, then the following statements are equivalent:

i) There exists a matrix $V$ satisfying

$$UVW + (UVW)^T + \Phi < 0$$

ii) The following two conditions hold

$$\mathcal{N}_U\Phi\mathcal{N}_U^T < 0 \text{ or } UU^T > 0$$

$$\mathcal{N}_W^T\Phi\mathcal{N}_W < 0 \text{ or } W^TW > 0$$

where $\mathcal{N}_U$ and $\mathcal{N}_W^T$ are respectively orthogonal complements of $U$ and $W^T$; that is

$$\mathcal{N}_UU = 0$$

$$\mathcal{N}_W^TW^T = 0.$$

## III. STABILITY ANALYSIS RESULT

In this section, we investigate the stability of the uncertain PSLS (5).

*Theorem 1:* System (5) is asymptotically stable if and only if there exist positive definite matrix $S$, nonsingular matrices $V_i$ and positive scalars $\alpha_i$ ($\forall\, i \in \mathcal{I}$) such that the following LMI

$$\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_{1,2} & \Lambda_{1,3} \\ (\bullet)^T & \Lambda_2 & 0 \\ (\bullet)^T & 0 & \Lambda_3 \end{bmatrix} > 0 \quad (8)$$

is feasible, where

$$\Lambda_1 = \begin{bmatrix} S & A_NV_N & \cdots & 0 & 0 & 0 \\ (\bullet)^T & V_N + V_N^T & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & V_2 + V_2^T & A_1V_1 & 0 \\ 0 & 0 & \cdots & (\bullet)^T & V_1 + V_1^T & S \\ 0 & 0 & \cdots & 0 & (\bullet)^T & S \end{bmatrix},$$

$$\Lambda_{1,2} = \begin{bmatrix} \alpha_NH_N & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \alpha_2H_2 & 0 \\ 0 & \cdots & 0 & \alpha_1H_1 \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$\Lambda_{1,3}^T = \begin{bmatrix} \widehat{E}_N \\ \vdots \\ \widehat{E}_2 \\ \widehat{E}_1 \end{bmatrix} = \begin{bmatrix} 0 & E_NV_N & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & E_2V_2 & 0 & 0 \\ 0 & 0 & \cdots & 0 & E_1V_1 & 0 \end{bmatrix},$$

$$\Lambda_2 = \Lambda_3 = \begin{bmatrix} \alpha_NI & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \alpha_2I & 0 \\ 0 & \cdots & 0 & \alpha_1I \end{bmatrix}. \quad (9)$$

*Proof:* See Appendix A. ■

*Remark 1:* The condition in Theorem 1 is in the form of LMI, which is suitable for application.

## IV. State Feedback Stabilization

In this section, based on the analysis result, Theorem 1, a necessary and sufficient condition will be given to build a switched state feedback controller as (6) ensuring the closed-loop system (7) is asymptotically stable.

*Theorem 2:* System (7) is asymptotically stabilizable by means of the switched state feedback control law (6) if and only if there exist positive definite matrix $S$, nonsingular matrices $V_i$, matrices $F_i$ and positive scalars $\alpha_i$ $(\forall i \in \mathcal{I})$ such that the following LMI

$$\bar{\Lambda} = \begin{bmatrix} \bar{\Lambda}_1 & \Lambda_{1,2} & \bar{\Lambda}_{1,3} \\ (\bullet)^T & \Lambda_2 & 0 \\ (\bullet)^T & 0 & \Lambda_3 \end{bmatrix} > 0 \qquad (10)$$

is feasible where $\Lambda_{1,2}, \Lambda_2, \Lambda_3$ are same defined as (9) and

$$\bar{\Lambda}_1 = \begin{bmatrix} S & A_N V_N + B_N F_N & \cdots & 0 & 0 & 0 \\ (\bullet)^T & V_N + V_N^T & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & V_2 + V_2^T & A_1 V_1 + B_1 F_1 & 0 \\ 0 & 0 & \cdots & (\bullet)^T & V_1 + V_1^T & S \\ 0 & 0 & \cdots & 0 & (\bullet)^T & S \end{bmatrix},$$

$$\bar{\Lambda}_{1,3}^T = \begin{bmatrix} 0 & E_N V_N + T_N F_N & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & E_2 V_2 + T_2 F_2 & 0 & 0 \\ 0 & 0 & \cdots & 0 & E_1 V_1 + T_1 F_1 & 0 \end{bmatrix}.$$

The resulting feedback matrices are given by

$$K_i = F_i V_i^{-1}, \; i \in \mathcal{I} \qquad (11)$$

*Proof:* Define

$$\begin{aligned} \bar{A}_i &= \widetilde{A}_i + \widetilde{B}_i K_i, \\ \bar{\Psi} &= -\bar{\Lambda}_1, \end{aligned}$$

$$\begin{bmatrix} \bar{E}_N \\ \vdots \\ \bar{E}_2 \\ \bar{E}_1 \end{bmatrix} = \begin{bmatrix} 0 & (E_N + T_N K_N) V_N & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & (E_2 + T_2 K_2) V_2 & 0 & 0 \\ 0 & 0 & \cdots & 0 & (E_1 + T_1 K_1) V_1 & 0 \end{bmatrix}.$$

Then replacing $\widetilde{A}_i, \Psi, \widehat{E}_i$ with $\bar{A}_i, \bar{\Psi}, \bar{E}_i (\forall i \in \mathcal{I})$ respectively in the proof process of Theorem 1 and performing the change of variable $F_i = K_i V_i$ complete the proof. ■

## V. An Example

In this section, we give a numerical example to illustrate the utilization of our results.

*Example 1:* Consider system (1) with $\mathcal{I} = \{1, 2\}$ and

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 2 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ H_1 &= \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, E_1 = [\, 0.02 \quad 0.03 \,], \; T_1 = 1; \\ A_2 &= \begin{bmatrix} -3 & 0 \\ 0 & -0.6 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \\ H_2 &= \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, E_2 = [\, 0.15 \quad 0.1 \,], \; T_2 = 1. \end{aligned}$$

On the one hand, it is easy to verify that the LMI (8) is not feasible. Thus, by Theorem 1, the autonomous system (5) is not asymptotically stable. On the other hand, by solving the LMI (10), we can take the switched state feedback control law (6) with

$$\begin{aligned} K_1 &= [\, -0.0949 \quad -0.0155 \,], \\ K_2 &= [\, -0.0122 \quad 0.2778 \,], \end{aligned}$$

to asymptotically stabilize the closed-loop system (7).

## VI. Conclusion

In this paper, the stability analysis and stabilization problems for periodically switched linear systems(PSLSs) with uncertainties in the discrete-time domain have been addressed. An LMI-based condition bas been given for the autonomous uncertain PSLS to be asymptotically stable. Furthermore, an LMI-based condition also has been given to build a state feedback controller ensuring the closed-loop system is asymptotically stable. The conditions are both necessary and sufficient.


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## Appendix A

*Proof:* [Proof of Theorem 1] Denote

$$\eta(k) = x(kN), \quad k = 0, 1, 2, \cdots. \qquad (12)$$

From system (5), we get

$$\eta(k+1) = \widetilde{A}\,\eta(k) \qquad (13)$$

where $\widetilde{A} = \widetilde{A}_N \cdots \widetilde{A}_2\widetilde{A}_1$. It is easy to prove that system (5) is asymptotically stable if and only if system (13) is asymptotically stable, furthermore, if and only if $\widetilde{A}$ is Schur-stable. In the Lyapunov framework, the Schur-stability of $\widetilde{A}$ is equivalent to the existence of a positive definite matrix $P$ satisfying

$$-P + \widetilde{A}^T P \widetilde{A} < 0,$$

or equivalently with $S = P^{-1} > 0$,

$$-S + \widetilde{A} S \widetilde{A}^T < 0. \tag{14}$$

Define

$$\begin{aligned} Q_1 &= \widetilde{A}_1, \\ Q_i &= \widetilde{A}_i Q_{i-1}, \quad i \in \mathcal{I} \setminus \{1\}. \end{aligned}$$

Condition (14) can be written as

$$[\, I \quad -\widetilde{A}_N \,] \begin{bmatrix} -S & 0 \\ 0 & Q_{N-1} S Q_{N-1}^T \end{bmatrix} \begin{bmatrix} I \\ -\widetilde{A}_N^T \end{bmatrix} < 0$$

Define

$$\mathcal{N}_{U_1} = [\, I \quad -\widetilde{A}_N \,],$$

$$W_1 = [\, 0 \quad I \,],$$

and

$$\Phi_1 = \begin{bmatrix} -S & 0 \\ 0 & Q_{N-1} S Q_{N-1}^T \end{bmatrix},$$

then we have

$$\mathcal{N}_{U_1} \Phi_1 \mathcal{N}_{U_1}^T < 0,$$

$$\mathcal{N}_{W_1}^T \Phi_1 \mathcal{N}_{W_1} = -S < 0.$$

Hence, by Lemma 2, (14) is equivalent to

$$\begin{bmatrix} -S & 0 \\ 0 & Q_{N-1} S Q_{N-1}^T \end{bmatrix} + \mathsf{Sym}\left\{ \begin{bmatrix} -\widetilde{A}_N \\ -I \end{bmatrix} V_N [\, 0 \quad I \,] \right\} < 0,$$

that is

$$\begin{bmatrix} -S & -\widetilde{A}_N V_N \\ (\bullet)^T & -V_N - V_N^T + Q_{N-1} S Q_{N-1}^T \end{bmatrix} < 0. \tag{15}$$

Denote

$$\Psi_2 = \begin{bmatrix} -S & -\widetilde{A}_N V_N \\ (\bullet)^T & -V_N - V_N^T \end{bmatrix}.$$

Since $S > 0$, we get from (15) that $\Psi_2$ is necessarily negative definite. Noticing that (15) can be written as

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & -\widetilde{A}_{N-1} \end{bmatrix} \times \begin{bmatrix} -S & -\widetilde{A}_N V_N & 0 \\ (\bullet)^T & -V_N - V_N^T & 0 \\ 0 & 0 & Q_{N-2} S Q_{N-2}^T \end{bmatrix} \times \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & -\widetilde{A}_{N-1}^T \end{bmatrix} < 0$$

Define

$$\mathcal{N}_{U_2} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & -\widetilde{A}_{N-1} \end{bmatrix},$$

$$W_2 = [\, 0 \quad 0 \quad I \,],$$

and

$$\Phi_2 = \begin{bmatrix} -S & -\widetilde{A}_N V_N & 0 \\ (\bullet)^T & -V_N - V_N^T & 0 \\ 0 & 0 & Q_{N-2} S Q_{N-2}^T \end{bmatrix},$$

then we have

$$\mathcal{N}_{U_2} \Phi_2 \mathcal{N}_{U_2}^T < 0,$$

$$\mathcal{N}_{W_2}^T \Phi_2 \mathcal{N}_{W_2} = \Psi_2 < 0.$$

Thus, we apply Lemma 2 and get the equivalent inequality

$$\begin{bmatrix} -S & -\widetilde{A}_N V_N & 0 \\ (\bullet)^T & -V_N - V_N^T & 0 \\ 0 & 0 & Q_{N-2} S Q_{N-2}^T \end{bmatrix} + \mathsf{Sym}\left\{ \begin{bmatrix} 0 \\ -\widetilde{A}_{N-1} \\ -I \end{bmatrix} V_{N-1} [\, 0 \;\; 0 \;\; I \,] \right\} < 0,$$

that is

$$\begin{bmatrix} -S & -\widetilde{A}_N V_N & 0 \\ (\bullet)^T & -V_N - V_N^T & -\widetilde{A}_{N-1} V_{N-1} \\ 0 & (\bullet)^T & -V_{N-1} - V_{N-1}^T + Q_{N-2} S Q_{N-2}^T \end{bmatrix} < 0. \tag{16}$$

Denote

$$\Phi_3 = \begin{bmatrix} -S & -\widetilde{A}_N V_N & 0 \\ (\bullet)^T & -V_N - V_N^T & -\widetilde{A}_{N-1} V_{N-1} \\ 0 & (\bullet)^T & -V_{N-1} - V_{N-1}^T \end{bmatrix}.$$

$\Psi_3$ is also necessarily negative definite since $S > 0$. Similar to the above process, rewriting (16) to get $\mathcal{N}_{U_3} \Phi_3 \mathcal{N}_{U_3}^T < 0$ and $\mathcal{N}_{W_3}^T \Phi_3 \mathcal{N}_{W_3} = \Psi_3 < 0$, then by Lemma 2, we can get the equivalent inequality of $4n$-dimension and $\Psi_4$ which is necessarily negative definite. After repeating $N$ times, we can get the equivalent inequality

$$\left[\begin{array}{cccccccc} -S & -\widetilde{A}_N V_N & 0 & \cdots & 0 & 0 & 0 \\ (\bullet)^T & -V_N - V_N^T & -\widetilde{A}_{N-1}V_{N-1} & \cdots & 0 & 0 & 0 \\ 0 & (\bullet)^T & -V_{N-1} - V_{N-1}^T & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -V_3 - V_3^T & -\widetilde{A}_2 V_2 & 0 \\ 0 & 0 & 0 & \cdots & (\bullet)^T & -V_2 - V_2^T & -\widetilde{A}_1 V_1 \\ 0 & 0 & 0 & \cdots & 0 & (\bullet)^T & -V_1 - V_1^T + S \end{array}\right] < 0, \quad (17)$$

which is equivalent, by the Schur complement, to

$$\left[\begin{array}{cccccccc} -S & -\widetilde{A}_N V_N & 0 & \cdots & 0 & 0 & 0 \\ (\bullet)^T & -V_N - V_N^T & -\widetilde{A}_{N-1}V_{N-1} & \cdots & 0 & 0 & 0 \\ 0 & (\bullet)^T & -V_{N-1} - V_{N-1}^T & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -V_2 - V_2^T & -\widetilde{A}_1 V_1 & 0 \\ 0 & 0 & 0 & \cdots & (\bullet)^T & -V_1 - V_1^T & -S \\ 0 & 0 & 0 & \cdots & 0 & (\bullet)^T & -S \end{array}\right] < 0. \quad (18)$$

Define

$$\begin{aligned} M &= [\, M_N \ \cdots \ M_2 \ \ M_1 \,] = \begin{bmatrix} H_N & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & H_2 & 0 \\ 0 & \cdots & 0 & H_1 \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix}, \\ R &= \begin{bmatrix} R_N \\ \vdots \\ R_2 \\ R_1 \end{bmatrix} = \begin{bmatrix} -\widehat{E}_N \\ \vdots \\ -\widehat{E}_2 \\ -\widehat{E}_1 \end{bmatrix}, \\ \Psi &= -\Lambda_1, \end{aligned}$$

where $\widehat{E}_i, \Lambda_1$ are same defined as (9). According to (2), condition (18) can be rewritten as

$$\Psi + \sum_{i=1}^{N}[M_i\Gamma_i R_i + \left(M_i\Gamma_i R_i\right)^T] < 0. \qquad (19)$$

By the iterative application of Lemma 1, (19) holds for all $\Gamma_i$ satisfying $\Gamma_i^T\Gamma_i \leq I,\ i \in \mathcal{I}$ if and only if there exist positive scalars $\alpha_i > 0,\ i \in \mathcal{I}$ such that

$$\Psi + \sum_{i=1}^{N}[\alpha_i M_i M_i^T + \frac{1}{\alpha_i}R_i^T R_i] < 0 \qquad (20)$$

which is equivalent, by the Schur complement, to

$$\begin{bmatrix} \Psi & -M\Lambda_2 & R^T \\ (\bullet)^T & -\Lambda_2 & 0 \\ (\bullet)^T & 0 & -\Lambda_2 \end{bmatrix} < 0$$

which is nothing but $-\Lambda < 0$. ■

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