Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Approximate Model Predictive Control for Gas Turbine Engines

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***Abstract* — A novel model predictive control strategy using instantaneous linearization of nonlinear models incorporating the Generalized Predictive Control (GPC) called Approximate Model Predictive Control (AMPC) is used to control a shaft speed of a gas turbine engine. This method gives advantages over the Nonlinear Model Predictive Control (NMPC), which is computationally demanding and has local minimums. The performance of the model based control schemes is dependent on the accuracy of the process model, so firstly the paper examines the estimation of global nonlinear gas turbine models using NARMAX and neural network representations. The performance of the proposed methods is examined using a range of small and large random step tests. The results illustrate the improvements in control performance that can be achieved to that of gain-scheduling PID controllers.**

I. Introduction

Gas turbines are now extensively used in aerospace, marine and industrial application. With this increasing use in a diverse range of application, designing of controllers for the optimal performance is an important consideration. This paper deals with the nonlinear modelling and control between the fuel flow and shaft speed of an aircraft gas turbine. The work presented here is based on a Rolls Royce Spey Mk202 aircraft gas turbine. Although it is no longer in service, for the control purposes, the Spey possesses the same characteristics as a modern engine.

Model Predictive Control (MPC) has been introduced mainly to deal with processes with complex dynamics [1] and now is one of the most widely used advanced control methods in the process control industry. MPC denotes a broad range of control strategies, which uses a model to predict future process behavior and calculate the control trajectory through the optimization of an objective function within a specified horizon.

Model based control schemes are highly related to the accuracy of the process model. Recent work by Evans *et al*. [2] concentrated on testing the engine using small-amplitude multisine signals and frequency domain techniques to identify linear models of high accuracy at a range of different operating points. The errors due to noise and nonlinearities were assessed and found to be small for these small-signal models. The fact that the dc gains and the dynamics of these models change with operating points showed that the gas turbine is nonlinear, so the need was apparent for a more accurate nonlinear modelling of the gas turbine. The work was developed further by Chiras *et al*. [3,4] who used Nonlinear AutoRegressive Moving Average with eXogenous inputs (NARMAX) and neural network models, to represent the global dynamics of the engine. It was demonstrated that both models were suitable for representing engine dynamics throughout its operating range.

Since the relationship between the fuel flow and shaft speed of the gas turbine is nonlinear, Nonlinear Model Predictive Control (NMPC) provided a possible solution to the control problem. This is an alternative to a gain-scheduling PID strategy. However, NMPC involves a complex nonlinear programming problem. The need to use a simple model predictive control method is apparent. The proposed Approximate Model Predictive Control (AMPC) is based on instantaneous linearization of a nonlinear model at each sampling instant, thus enabling the application of linear model predictive control techniques. In this paper, Generalized Predictive Control (GPC) originally proposed by Clarke *et al*. [5,6] is applied to control the gas turbine engine, enabling the control trajectory to be found by direct analysis. The advantages of using AMPC over conventional nonlinear design are less computational time and the avoidance of the problem of local minimums.

In this paper nonlinear gas turbine modelling using NARMAX and neural networks is presented. Since the authors did not have access to a gas turbine to implement the control strategy, the neural network model will be used for the model predictor, and the NARMAX model to represent the "true dynamics" of the system. The motivation for using two different models to design AMPC is to show the control robustness when model mismatch occurs. The control performance of a gas turbine engine using AMPC is examined using a wide range of small and large random signal tests and it is shown that the control performance of the AMPC is superior to that of the gain-scheduling PID controllers.

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II. Nonlinear Gas Turbine Modelling

A. NARMAX Model

In order to identify a global model, which is capable of representing the engine dynamics for both the small and large input amplitudes, Chiras *et al*. [3] used nonparametric data analysis in both time- and frequency-domains and an orthogonal estimation algorithm to estimate NARMAX models of the engine defined by the nonlinear function:

$$y(t) = F\begin{pmatrix} y(t-1),\ldots, y(t-n_y), u(t-1),\ldots, \\ u(t-n_u), e(t-1),\ldots, e(t-n_e) \end{pmatrix} + e(t) \quad (1)$$

where F is a nonlinear function; $y(t)$, $u(t)$, $e(t)$ represent the output, input and noise signals, respectively; and n_y, n_u, and n_e are their associated maximum lags. A well-established procedure for structure selection of a NARMAX polynomial model is based on the error reduction ratio (ERR) defined by Billings *et al*. [7] as

$$ERR_i = \frac{g_i \sum_{t=1}^{N} w_i^2(t)}{\sum_{t=1}^{N} y^2(t)} \quad (2)$$

where g_i are auxiliary model coefficients, $w_i(t)$ are terms of an auxiliary model that are orthogonal over the data records. A forward-regression orthogonal estimation algorithm is employed to select at each step the term with the highest ERR, in other words the term which contributes most to the reduction of the residual variance. The procedure is usually stopped using an information criterion such as Akaike's Information Criterion (AIC), defined as

$$AIC = N \log_e(\sigma_\varepsilon^2(\theta_p)) + kp \quad (3)$$

where $\sigma_\varepsilon^2(\theta_p)$ is the variance of the residuals associated with a p-term model and k is a penalizing factor. One of the estimated NARMAX models in (4) is validated in Fig. 1 and proved to be suitable for both small and large signal tests.

$$\begin{aligned} y(t) &= 0.7206y(t-1) + 0.2885y(t-2) \\ &+ 0.0066u(t-1) - 3.31e^{-5}u(t-1)y(t-2) \\ &- 0.7202 - 2.56e^{-4}y(t-1)y(t-2) \end{aligned} \quad (4)$$

B. Neural Network Model

Chiras *et al*. [4] used another representation, a two-layer feedforward neural network, to model the fuel flow to shaft speed of the gas turbine. This structure is based on a result by Cybenco [8] who proved that a neural network with one hidden layer of sigmoid or hyperbolic tangent units, equation (5), and an output layer of linear units is capable of approximating any continuous function.

$$f_{\tanh}(x) = \frac{2}{1+e^{-x}} - 1 \quad (5)$$

The network is described by the magnitude of the weights and biases, and should be determined by training the network on the estimation data. In this paper a Neural Network AutoRegressive with eXogenous inputs (NNARX) using past input and output terms as regressors is used to model the gas turbine. Parameter estimation involves the minimization of the sum of square errors given by

$$V_N(\theta, Z_e^N) = \frac{1}{2N}\sum_{t=1}^{N}\left[y(t) - \hat{y}(t/\theta)\right]^2 = \frac{1}{2N}\sum_{t=1}^{N} e^2(t/\theta) \quad (6)$$

where θ are the model parameters, $y(t)$ the system output, $\hat{y}(t)$ the model estimate, N the number of samples and Z_e^N a matrix known as training data given by (7), and the parameter estimate $\hat{\theta}$ is obtained by (8).

$$Z_e^N = \left[y(t)\ \ y(t-1)\ \cdots\ y(t-n_y)\ \ u(t-1)\ \cdots\ u(t-n_u)\right] \quad (7)$$

$$\hat{\theta} = \arg\min V_N(\theta, Z_e^N) \quad (8)$$

Chiras *et al*. [4] showed that a neural network with 6 hidden layers and one linear output layer provides the best performance on the existing validation data shown in Fig. 1, and is capable of modelling both the low and high amplitude dynamics of the engine. Fig. 1 shows that the neural network model performs better than the NARMAX model for the small signal tests, however for large signal tests the NARMAX model is better.

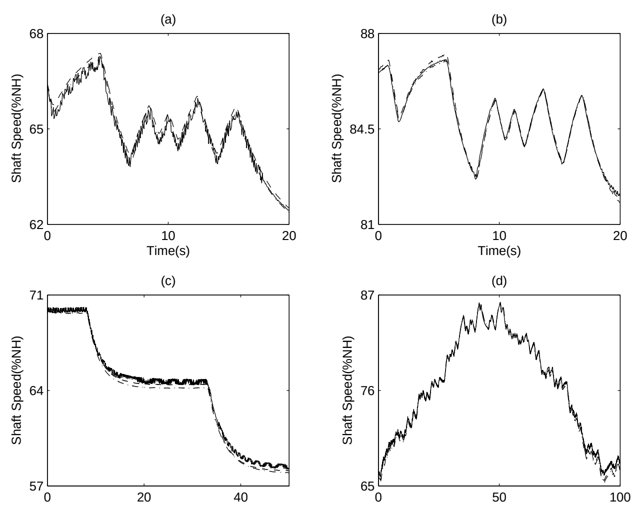

Fig. 1. Outputs of validation data test based on NARMAX and neural network models. (Measured engine output (solid), NARMAX model output (dashed), neural network model output (dashdot)). (a) IRMLBS (Inverse Repeat Maximum Length Binary Sequence) test at 65%N_H, (b) IRMLBS test at 85%N_H, (c) three-level periodic test at 58-70%N_H, (d) triangle wave + IRMLBS test at 65-85%N_H.

C. Linearization of the Neural Network Model

In order to implement the AMPC, linear models must be extracted from a nonlinear model at each sampling instant. The coefficients of the extracted linear models can be calculated by the derivative of the output against each input [9]. For a two-layer feedforward neural network model with one hidden layer of *tanh* units and a linear output,

$$\hat{y}(t) = \sum_{j=1}^{n_h} W_j \tanh\left[\sum_{k=1}^{n_\phi} w_{jk}\varphi_k(t) + w_{j0}\right] + W_0 \quad (9)$$

where w_{jk} and w_{j0} are the weights and biases of the hidden layer, W_k and W_0 are the weights and biases of the output layer, respectively, $\varphi_k(t)$ are the output and input lag terms. The derivative of the output with respect to input $\varphi_i(t)$ is calculated in accordance with

$$\frac{\partial\hat{y}(t)}{\partial\varphi_i(t)} = \sum_{j=1}^{n_h} W_j w_{jk}\left(1 - \tanh^2\left[\sum_{k=1}^{n_\phi} w_{jk}\varphi_k(t) + w_{j0}\right]\right) \quad (10)$$

III. Approximate Model Predictive Control

AMPC is a flexible criterion based design and requires the linearization of a nonlinear model incorporating a GPC. At each sampling time, a linear model is extracted from a neural network model and applied to predict the future process output within the prediction range. Predicted output depends on known past values of input and output signals and on the future trajectory, assuming that beyond a certain control horizon further increments in control are zero. The control trajectory can be found by minimizing the following criterion

$$J(t, U(t)) = \sum_{k=N_1}^{N_2} [r(t+k) - \hat{y}(t+k)]^2 + \rho\sum_{k=1}^{N_u}[\Delta u(t+k-1)]^2 \quad (11)$$

with respect to the N_u future control inputs and subject to the control constraint

$$U(t) = [u(t) \quad \cdots \quad u(t+N_u-1)]^T$$

$$\Delta u(t+k) = 0, \quad N_u \le k \le N_2 - d$$

where N_1 denotes the minimum prediction horizon, N_2 the maximum prediction horizon and N_u the control horizon, ρ is a weight factor penalizing changes in the control input to get smooth control input signals and d is the system time delay.

To remove the offset due to regular disturbances and to model mismatch, it is necessary to let the controller include integral action. An integrated ARX model (ARIX) is as following

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t) + \frac{e(t)}{\Delta} \quad (12)$$

where $A(z^{-1})$ and $B(z^{-1})$ are the denominator and nominator of the linearized models, $y(t)$ and $u(t)$ are the current output and input signals respectively. $e(t)$ is integrated white noise. $\frac{1}{\Delta} = \frac{1}{1-z^{-1}}$ acts as a discrete integral term. Considering the time instant $t+k$, the ARIX model is as follows

$$\Delta A(z^{-1})y(t+k) = z^{-d}B(z^{-1})\Delta u(t+k) + e(t+k) \quad (13)$$

To systematically derive a predictor, the model is now reorganized by introducing the following Diophantine equation

$$1 = \Delta A(z^{-1})E_k(z^{-1}) + z^{-k}F_k(z^{-1}) \quad (14)$$

where $E_k(z^{-1})$ is of degree k-1 and $F_k(z^{-1})$ is of degree n_a. Multiplying both sides of (13) by $E_k(z^{-1})$ and using (14) gives

$$\begin{aligned} y(t+k) &= z^{-d}E_k(z^{-1})B(z^{-1})\Delta u(t+k) \\ &\quad + F_k(z^{-1})y(t) + E_k(z^{-1})e(t+k) \end{aligned} \quad (15)$$

If the sequence of future control inputs is known, the minimum variance predictor for $y(t+k)$ is the expectation conditioned on the information gathered up to time t.

$$\hat{y}(t+k) = G_k(z^{-1})\Delta u(t+k-d) + F_k(z^{-1})y(t) \quad (16)$$

$G_k(z^{-1}) = E_k(z^{-1})B(z^{-1})$ is clearly a polynomial of order n_b+k-1. The only unknown quantities in (16) are now the future control inputs. In order to derive the control law it is necessary to separate these from the part of the expression containing past data

$$\begin{aligned} \hat{y}(t+k) &= \overline{G}(z^{-1})\Delta u(t+k-d) \\ &+ z^{k-d}[G_k(z^{-1}) - \overline{G}(z^{-1})]\Delta u(t) + F_k(z^{-1})y(t) \end{aligned} \quad (17)$$

$$\overline{G}(z^{-1}) = g_0 + g_1 z^{-1} + ... + g_{k-d}z^{d-k} \quad (18)$$

The remaining problem is now to solve all the Diophantine equations. Based on Clarke *et al.* [5,6], a method referred to as recursion of the Diophantine equation is used to solve the Diophantine equation. Initialize the recursion by setting

$$E_1(z^{-1}) = 1 \quad \text{then} \quad F_1(z^{-1}) = z[1 - \Delta A(z^{-1})] \quad (19)$$

$$E_{k+1}(z^{-1}) = E_k(z^{-1}) + e_k^{(k+1)}z^{-k} \quad (20)$$

where $e_k^{(k+1)}$ specifies the coefficient to z^{-k} in the polynomial E_{k+1}.

$$F_{k+1}(z^{-1}) = z[F_k(z^{-1}) - \Delta A(z^{-1})e_k^{(k+1)}] \quad (21)$$

due to ΔA being monic, so

$$e_k^{(k+1)} = f_0^k \tag{22}$$

$$\begin{aligned} G_{k+1}(z^{-1}) &= B(z^{-1})E_{k+1}(z^{-1}) \\ &= G_k(z^{-1}) + z^{-k}B(z^{-1})f_0^k \end{aligned} \tag{23}$$

To derive the control law, firstly the predictors derived from (17) are expressed using the following vector notation

$$\hat{Y} = \Gamma\tilde{U} + \Phi \tag{24}$$

where

$$\hat{Y} = [\hat{y}(t+N_1) \quad \hat{y}(t+N_1+1) \quad ... \quad \hat{y}(t+N_2)]^T$$
$$\tilde{U} = [\Delta u(t) \quad \Delta u(t+1) \quad ... \quad \Delta u(t+N_u-1)]^T$$
$$\Phi = [\varphi(t+N_1) \quad \varphi(t+N_1+1) \quad ... \quad \varphi(t+N_2)]^T$$
$$\varphi(t+k) = z^{k-d}[G_k(z^{-1}) - g_0 - g_1 z^{-1} ... - g_{k-d} z^{d-k}]\Delta u(t) + F_k(z^{-1})y(t)$$

Γ is a matrix of dimension $(N_2 - d + 1) \times N_u$:

$$\Gamma = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & & 0 \\ & g_1 & & \vdots \\ \vdots & \vdots & & g_0 \\ g_{N_2-d} & g_{N_2-d-1} & \cdots & g_{N_2-d-N_u+1} \end{bmatrix} \tag{25}$$

From the definitions of the vectors above and with

$$R = \left[r(t+N_1) \quad r(t+N_1+1) \quad \cdots \quad r(t+N_2)\right] \tag{26}$$

the cost function (11) can be written as

$$J(t, U(t)) = [R - \Gamma\tilde{U} - \Phi]^T[R - \Gamma\tilde{U} - \Phi] + \rho\tilde{U}^T\tilde{U} \tag{27}$$

The sequence of future control action is obtained by setting the derivative of the criterion equal to zero,

$$\tilde{U} = [\Gamma^T\Gamma + \rho I_{N_u}]^{-1}\Gamma^T(R - \Phi) \tag{28}$$

Since real processes are subject to constraints, which limit the range and gradient of the control signals. The following constraints need to be satisfied.

$$\begin{aligned} u_{\min} \le u(t) \le u_{\max} \qquad & \forall t, \\ -\Delta u_{\max} \le \Delta u(t) \le \Delta u_{\max} \qquad & \forall t. \end{aligned} \tag{29}$$

IV. Gas Turbine Control

The control arrangement shown in Fig. 2 is employed to enable the control of a shaft speed of the engine. The engine throttle gives the operating point required. Due to the limit on fuel feed, saturation occurs at 440cc/s. Also it is necessary to employ a rate limiter in order to prevent engine surge. The rising slew rate is 40cc/s^2 and the falling slew rate is –40cc/s^2. A future control trajectory is generated as a possible solution by the optimizer based on the linearized models using proposed AMPC methods. At each sampling instant, only the first predicted input signal from the obtained control trajectory is applied to control the gas turbine engine.

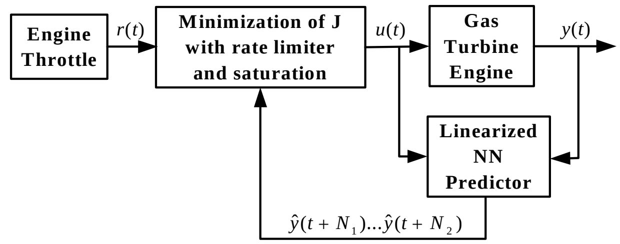

Fig. 2. Control arrangement for the gas turbine engine using AMPC.

A. Tuning of AMPC for Gas Turbine Engines

Tuning of AMPC is quite intuitive compared to other control methods. Soeterboek [10] gives an elaborate discussion on how to tune predictive controllers. This is widely supported by illustrative simulation studies. N_1, N_2, N_u and ρ should be set as suggested in the following:

Minimum prediction horizon N_1: It is always set to the model time-delay d. There is no reason for choosing it smaller because the d-1 first predictions depend on past control inputs only and cannot be affected by the first action $u(t)$. On the other hand it is not recommended to choose it bigger since this can lead to quite unpredictable results [10]. For the gas turbine, it is set to 1 (sampling period) and not tuned.

Maximum Prediction horizon N_2: A rule of thumb is that the prediction horizon should be selected approximately to the rise time of system [5,6]. However, often it is not possible to choose it this long because the calculation time required by AMPC is too demanding. Usually it is empirically tuned based on actual performance. From repeated experiments on the gas turbine system, the maximum prediction horizon is set around 30 (sampling periods) for the best control performance for both small and large random step changes.

Control horizon N_u: Soeterboek [10] suggest N_u is equal to the number of output lag terms. If N_u is made longer, the control performance is slightly improved and the calculating time is also increased. Based to the simulation results, it is set to 2 (sampling periods), which is the same as the number of output lag terms.

The control penalty factor ρ: The purpose of the control penalty factor is to penalize large changes in the process input and reduce actuator wear. It is usual to set ρ as a constant in the range [0,1]. For the gas turbine system, to achieve the best control performance, it is set 0.05.

With the AMPC variables set to N_1 =1, N_2 =30, N_u =2 and ρ =0.05, the global nonlinear controller results in the system responses shown in Fig. 3 and 4. This is for large random step changes (Fig. 3) and for small step changes

(Fig. 4). The results show that a fast rise time is achieved, with almost no overshoot, demonstrating proposed method provides a near optimal performance for both small and large random step changes.

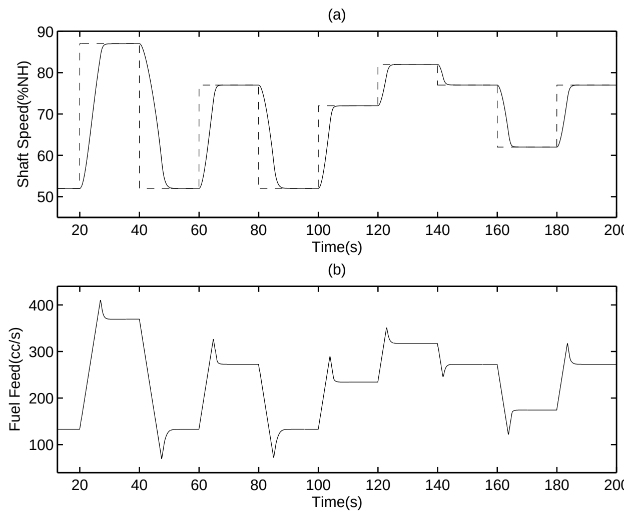

Fig. 3. Performance of AMPC on several large random set point changes. Set points (dashed). Gas turbine output (solid).

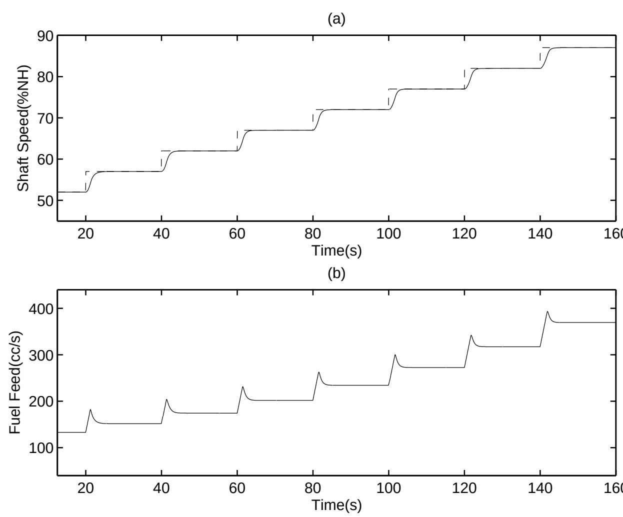

Fig. 4. Performance of AMPC on small set point changes. Set points (dashed). Gas turbine output (solid).

B. Disturbance Rejection using AMPC

The performance of the predictive control scheme is also explored in the face of disturbances. This is shown in Fig. 5, where an input disturbance of 100cc/s occurs at 30s and an output disturbance of 10%N_H takes place at 40s. The response shows that the scheme ensures zero steady state error in the face of the large disturbances. Also the proposed method is offset free in the face of model mismatch. This is demonstrated in the results since the neural network model is used for the model predictor with the NARMAX model representing the real plant. The two models exhibit slightly different dynamics, but the response is offset free.

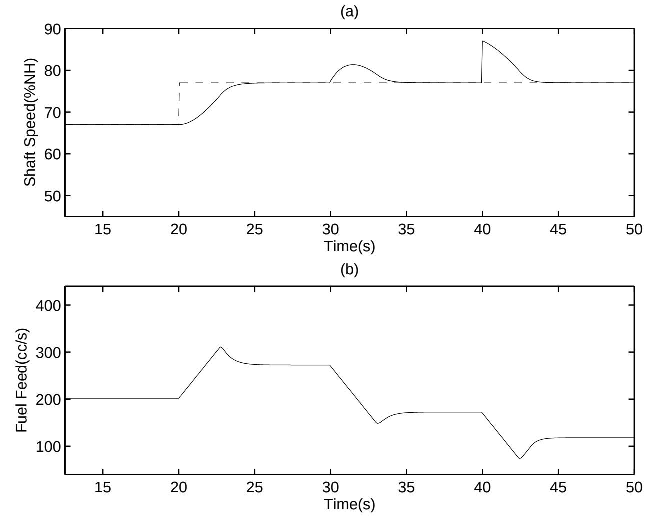

Fig. 5. Performance of AMPC with input-output disturbances. Set points (dashed). Gas turbine output (solid).

V. Comparison Between AMPC and Gain Scheduling PID Controllers

PID controllers were developed based on NARMAX and neural network models for gas turbine engines using Integral of Time Absolute Error (ITAE) performance index and is reported in [11]. It is shown that one set of PID parameters can not provide the optimal control because of the nonlinearities of the engine. So the gain-scheduling PID controllers are deemed essential in order to obtain optimal control performance. To enable a comparison to be made, the PID parameters obtained using a neural network model are used to control the gas turbine, which is modelled using a NARMAX model.

Initially, one set of PID parameters obtained based on a neural network model and a large step change from 52%N_H to 87%N_H and AMPC controller (N_1 =1, N_2 =30, N_u =2 and ρ =0.05) are applied to a NARMAX model. The control performances are shown in Fig. 6 using a set of large random step changes. It shows that one PID controller is not suitable for controlling the random set changes, especially for the smaller step changes, but AMPC gives a good performance across the ranges. The resulting ITAE using one PID controller is 2654.9, which is rather higher than that of using AMPC (745.3). The next comparison is between the gain-scheduling PID controllers shown in Table 1 and AMPC for the small step changes. The control performances are shown in Fig. 7. It shows that both of them can provide the optimal control for the gas turbine. On the basis of these results, the performance of AMPC is better and less affected by the model mismatch than that of gain-scheduling PID for small step changes. Also from

Table 1, ITAE using the gain-scheduling PID are slightly higher than using AMPC. So AMPC performance compares favorably with that of gain-scheduling PID controllers.

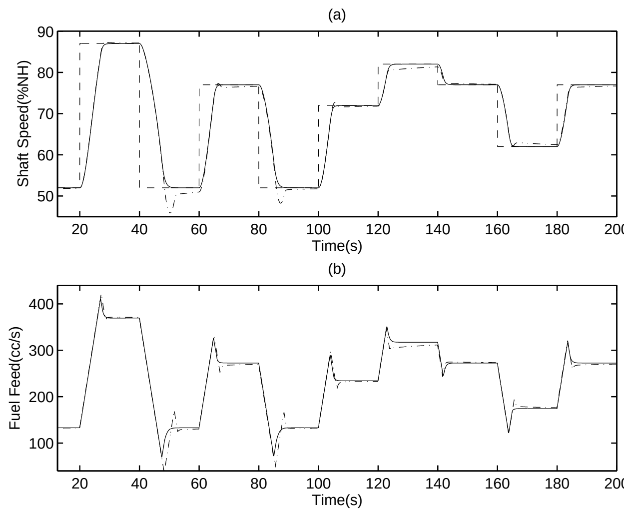

Fig. 6. Performance of AMPC vs. one PID *(*K_p = 25.62, K_i = 1.68, K_d = 0) on several large random set point changes. Set point (dashed), AMPC controlled output (solid), PID controlled output (dashdot).

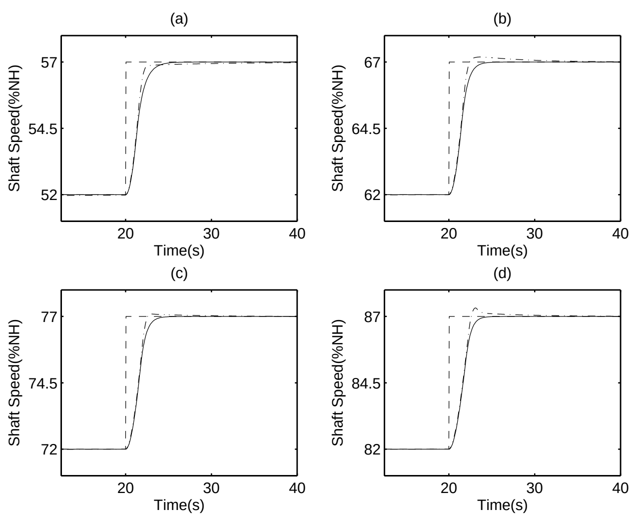

Fig. 7. Performance of AMPC vs. gain-scheduling PID on small set point changes. Set point (dashed), AMPC controlled output (solid), PID controlled output (dashdot).

TABLE 1. ITAE using gain-scheduling PID (K_d = 0) and AMPC across the operating range.

Set Points	Gain-scheduling PID			AMPC
	K_p	K_i	ITAE	ITAE
52-57	22.56	2.35	14.2	8.9
62-67	23.65	4.94	15.0	8.2
72-77	28.62	5.66	10.2	8.7
82-87	39.20	7.62	14.3	9.5

VI. Conclusions

NARMAX and neural network representations were identified to provide models capable of representing the engine dynamics throughout different operating ranges. These models provided the basis for the design of a model predictive control. AMPC based on neural network linearization incorporating GPC techniques was presented, which was applied to the gas turbine engine to control a shaft speed of a Spey engine. The proposed method provided the optimal performance not only for the small step changes but also for large random step changes. It has also been shown that the implementation can deal with disturbances and model mismatch. It was shown that the AMPC performance is superior to that of gain-scheduling PID controllers for the ranges examined.

AMPC as a global nonlinear controller has a smaller computational burden to that of NMPC and avoids the problem of local minimums. It can provide an efficient approach to the adaptive control of the gas turbine engine.

Acknowledgment

This work was conducted on the data gathered at the Defence Evaluation and Research Agency (DERA) at Pyestock with the support of Rolls Royce plc. The authors would like to thank all staff involved.

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