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Sensor management Based on Cross-entropy in Interacting Multiple Model Kalman Filter

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Abstract: Multisensor systems have been widely used in a variety of civilian and military applications. While sensor management is one of the most important parts of multisensor system. Many techniques of sensor management have been proposed and applied. This paper describes a method using a cross-entropy-based sensor effectiveness metric for sensor assignment in interacting multiple model Kalman filter (IMMKF). The expected cross-entropy is computed for the sensor target pairing on each scan. Then the constrained globally optimum assignment of sensors to targets is calculated and applied.

Keywords: cross-entropy, IMMKF, sensor management, information

I. INTRODUCTION

The goal of sensor management is to perform the right task at the right time on the right object based on external performance measures or criteria. Recent emphasis is being placed on the effective use of sensor management and the value that adds to the data fusion process. Sensor management is trying to optimize the utilization of a finite set of sensors with a finite computational capability in a dynamic, non-stationary environment, which maximizes the flow of information about the environment so that a goal can be successfully completed. Many techniques have been proposed or applied to the area of sensor management. Nash[1] treated the sensor management as an optimization technique, in which he used linear programming to determine sensor-to-target assignment for targets being tracked. The trace of the Kalman filter error covariance matrices is chosen as the cost coefficients in the objective functions. Kastella[2,3] also uses the method of Nash, but he uses discrimination gain, which is based on the Kullback-Leibler discrimination information function as the cost coefficients. Liu Xianxing[4] uses entropy, which is based on information theory, to obtain information gain to optimize sensor-target-pair. Molina López[5] presents a sensor management scheme that accomplishes sensor tasking using knowledge-based reasoning and fuzzy decision theory. Other methods include back propagation neural network[6], covariance control method[7], et al.

Because of the maneuvers of targets, the track effect is poor only using standard Kalman filter. Recently, a new approach called the Interacting Multiple Model Kalman Filter (IMMKF) has been developed that seems to offer superior performance to target tracking problem. The IMMKF estimator is a suboptimal hybrid filter that has been shown to be one of the most cost-effective hybrid state estimation. The IMMKF estimates the state of a dynamic system with several models that can “switch” from one to another. IMMKF always maintains all of the models and blends their outputs with weights that are computed probabilistically. In addition to the state estimates for each motion model, the IMMKF maintains an estimate of the probability that the target is moving in accordance with each model. During target maneuvers, the prediction covariance increases in IMMKF.

In this paper, the uncertainty of probability distribution is used to obtain the effect information of targets being tracked. This method not only uses the information given by different models but also uses the information from fusion center.

II. INTERACTING MULTIPLE MODEL KALMAN FILTER (IMMKF)

Interacting multiple model Kalman filter is presented by Blom and Bar-Shalom[8]. The multisensor multiple model system can be described as

$$x(i,k+1) = A(i,k)x(i,k) + W(i,k) \qquad i = 1,2,\cdots,N_1 \tag{1}$$

$$z^j(k) = C^j(k)x^j(k) + V^j(k) \qquad j = 1,2,\cdots,N_2 \tag{2}$$

where i is the ith model, $x(i,k) \in R^{n\times l}$ is the base state, $A(i,k) \in R^{m\times m}$ is the transition matrix of model i. The model noise matrix is $W(i,k) \in R^{n\times l}$, and

$$\begin{cases} E\{W(i,k)\} = 0 \\ E\{W(i,k)W^T(i,r)\} = Q(i,k)\delta_{kr} \end{cases} \tag{3}$$

Using N_2 sensors to track the same target (sample time is the same), and j presents the jth sensor. Measurement vector is $z^j(k) \in R^{m_j}$. $C^j(k)$ is the measurement matrix. $V^j(k) \in R^{m_j}$ is white noise process with zero mean

$$\begin{cases} E\{V^j(k)\} = 0 \\ E\{V^j(k)[V^j(r)]^T\} = R^j(k)\delta_{kr} \end{cases} \tag{4}$$

The initial value $x(i,0)$ is

$$\begin{cases} E\{x(i,0)\} = X_{i0} \\ E\{(x(i,0) - X_{i0})(x(i,0) - X_{i0})^T\} = P_{i0} \\ \mathrm{i} = N_1, N_1 - 1, \cdots, 2, 1 \end{cases} \tag{5}$$

Probability of predicted model is

$$\mu(i,k+1\,|\,k) = \sum_{l=1}^{N_1} \pi_{li}\,\mu(l,k\,|\,k) \tag{6}$$

Model transition probability for time interval is

$$\pi_{li} = P\{M(k+1) = l \,|\, M(k) = i\} \qquad \mathrm{i,l} = 1,2,\cdots,N_1 \tag{7}$$

Mixed state estimate and state variance estimate for model i is

$$\hat{x}_0(i,k\,|\,k) = \sum_{l=1}^{N_1} \pi_{li} \frac{\mu(l,k\,|\,k)}{\mu(i,k+1\,|\,k)} \hat{x}(l,k\,|\,k) \tag{8}$$

$$P_0(i,k\,|\,k) = \sum_{l=1}^{N_1} \pi_{li} \frac{\mu(l,k\,|\,k)}{\mu(i,k+1\,|\,k)} \{P(l,k\,|\,k) + [\hat{x}(l,k\,|\,k) - \hat{x}_0(l,k\,|\,k)][\hat{x}(l,k\,|\,k) - \hat{x}_0(l,k\,|\,k)]^T\} \tag{9}$$

where $\hat{x}(l,k\,|\,k), P(l,k\,|\,k)$ is the optimal state estimate and state variance estimate of model l at time k. So the state prediction of model i based on model j is

$$\hat{x}^j(i,k+1\,|\,k) = A(i,k)\hat{x}_0^j(i,k\,|\,k) \tag{10}$$

Covariance prediction of model j is

$$P^j(i,k+1\,|\,k) = A(i,k)P_0^j(i,k\,|\,k)A^T(i,k) + Q_j(i,k) \tag{11}$$

Measurement prediction of model i is

$$\hat{z}^j(i,k+1\,|\,k) = C^j(k)\hat{x}^j(i,k+1\,|\,k) \tag{12}$$

Measurement residual is

$$v^j(i,k+1) = z^j(i,k+1) - \hat{z}^j(k+1,k) \tag{13}$$

Measurement update is

$$x^j(i,k+1\,|\,k+1) = \hat{x}^j(i,k+1\,|\,k) + K^j(i,k+1\,|\,k)v^j(i,k+1) \tag{14}$$

Covariance update is

$$P^j(i,k+1\,|\,k+1) = [I - K^j(i,k+1)C^j(k)]P^j(i,k+1\,|\,k) \tag{15}$$

Residual covariance estimate is

$$S^j(i,k+1) = C^j(k)P^j(i,k+1\,|\,k)[C^j(k)]^T \tag{16}$$

Filter gain is

$$K^j(i,k+1) = P^j(i,k+1\,|\,k)[C^j(k)]^T[S^j(i,k+1)]^{-1} \tag{17}$$

Model likelihood function is

$$\Lambda(i,k+1) = (2\pi)^{-1}\,|S^j(i,k+1)|^{-\frac{1}{2}} \bullet \exp\{-\frac{1}{2}[v^j(i,k+1)]^T[S^j(i,k+1)]^{-1}v^j(i,k+1)\} \tag{18}$$

Model probability is

$$\mu(i,k+1|k+1)=\frac{\mu(i,k+1|k)\Lambda(i,k+1)}{\sum_{l=1}^{N_1}\mu(l,k+1|k)\Lambda(l,k+1)} \quad (19)$$

Estimate combination of each model is

$$\tilde{x}^j(k+1|k+1)=\sum_{i=1}^{N_1}x^j(i,k+1|k+1)\mu(i,k+1|k+1) \quad (20)$$

$$\begin{aligned}&\tilde{P}^j(k+1|k+1)\\&=\sum_{i=1}^{N_1}\mu(i,k+1|k+1)\{P^j(i,k+1|k+1)\\&+[\tilde{x}^j(k+1|k+1)-x^j(i,k+1|k+1)]\\&\bullet[\tilde{x}^j(k+1|k+1)-x^j(i,k+1|k+1)]^T\}\end{aligned} \quad (21)$$

Estimation of fusion center whit N_2 sensors is

$$x(k+1|k+1)=\sum_{j=1}^{N_2}a^j\tilde{x}^j(k+1|k+1) \quad (22)$$

where choose of a^j can refer paper [9].

III. THE OPTIMAL SENSOR ASSIGNMENT BASED ON CROSS-ENTROPY

The probability density function of random variable X is $p_1(x)$ of hypothesis H_1 and $p_2(x)$ of hypothesis H_2, so the cross-entropy of $p_1(x)$ and $p_2(x)$ is

$$I(p_2,p_1)=\int p_2(x)\log(p_2(x)/p_1(x))dx \quad (23)$$

The cross-entropy $I(p_2,p_1)$ is the tendency information to H_2 during identifying hypothesis H_1 and hypothesis H_2 when random variable X is x. And we can also treat the cross-entropy as the information which observer obtains when random variable X changes from probability distribution $p_1(x)$ to $p_2(x)$.

Assume that the prior probability density function of random variable X is $p_1(x)$. After one test, the probability density function of random variable X is $p_2(x)$. The cross-entropy of $p_1(x)$ and $p_2(x)$ is $I(p_2,p_1)$. $I(p_m,p_1)$ is the maximum cross-entropy, which can be obtained through one test, and then we define

$$U(p)=I(p_m,p_1)-I(p_2,p_1) \quad (24)$$

where $U(p)$ indicates the unknown information about X, which is the uncertainty of probability distribution P(X). In IMMKF, the prior probability of sensor j which tracks target i at time k+1 is $p_0{}^j(i,x^j|z^j(k))$, the posterior probability is $p_1{}^j(i,x^j|z^j(k+1))$, $p_m(x|z(k+1))$ is the maximum probability, then the information of the target i is

$$\begin{aligned}U(p^j)&=\sum_{i=1}^{N_1}\{\int dxp_m(x|z(k+1))\log\frac{p_m(x|z(k+1))}{p_0{}^j(i,x^j|z^j(k))}\\&-\int dxP_{ji}(i,x^j|z^j(k+1))\log\frac{P_1^j(i,x^j(k+1))}{P_0^j(i,x^j|z^j(k))}\}\\&=\sum_{i=1}^{N_1}\{\int dxp_m(x|z(k+1))\log p_m(x|z(k+1))\\&-\int dxp_m(x|z(k+1))\log p_0{}^j(i,x^j|z^j(k))\\&-\int dxp_1^j(i,x^j|z^j(k+1))\log p_1{}^j(i,x^j|z^j(k+1))\\&+\int dxp_1^j(i,x^j|z^j(k+1))\log p_0{}^j(i,x^j|z^j(k))\}\end{aligned} \quad (25)$$

Since the probability of model and state vector x are statistic independence about measurement vector z

$$p_0{}^j(i,x^j|z^j(k))=\mu(i,k+1|k)p_0{}^j(x^j|z^j(k)) \quad (26)$$

$$\begin{aligned}&p_0^j(x^j|z^j(k))=(2\pi)^{-\frac{n}{2}}|P^j(k+1|k)|^{-\frac{1}{2}}\\&\bullet\exp\{-\frac{1}{2}[x^j-x^j(k+1|k)]^TP^j(k+1|k)[x^j-x^j(k+1|k)]\end{aligned} \quad (27)$$

$$P^j(k+1|k)=\sum_{i=1}^{N_1}\mu(i,k+1|k)P^j(i,k+1|k) \quad (28)$$

$$x^j(k+1|k)=\sum_{i=1}^{N_1}\mu(i,k+1|k)\hat{x}(i,k+1|k) \quad (29)$$

$$p_1{}^j(i,x^j|z^j(k+1))=\mu(i,k+1|k+1)p_1{}^j(x^j|z^j(k+1)) \quad (30)$$

$$p_1^j(x^j \mid z^j(k+1)) = (2\pi)^{-\frac{n}{2}} \mid \widetilde{P}^j(k+1 \mid k+1) \mid^{-\frac{1}{2}} \bullet$$
$$\exp\{-\frac{1}{2}[x^j - \widetilde{x}^j(k+1 \mid k+1)]^T \widetilde{P}^j(k+1 \mid k+1)$$
$$\bullet [x^j - \widetilde{x}^j(k+1 \mid k+1)]\}$$

(31)

equality (25) can be rewritten as

$$\sum_{i=1}^{N_1}\{\mu(i,k+1|k+1)\log\frac{\mu(i,k+1|k)}{\mu(i,k+1|k+1)} - \log\mu(i,k+1|k)\}$$
$$-\frac{1}{2}tr\{[P^j(k+1|k)]^{-1}[P_M(k+1) - \widetilde{P}^j(k+1|k+1)$$
$$+[x_m(k+1) - x^j(k+1|k)][x_m(k+1) - x^j(k+1|k)]^T$$
$$-[\widetilde{x}^j(k+1|k+1) - x^j(k+1|k)]$$
$$\bullet[\widetilde{x}^j(k+1|k+1) - x^j(k+1|k)]^T\}$$
$$+\frac{1}{2}\log|\widetilde{P}^j(k+1|k+1)| + \frac{1}{2}\log(2\pi e)$$

(32)

The more U(P), the little information about the targets known, and more attention should be paid to that target. So U(P) can be used as cost function to assign sensor to target.

The sensors are indexed s=1…S and the targets are indexed t=1…T. The uncertainty of probability distribution when sensor j is assigned to target i is denoted as $U(p)_{ji}$. The purpose is to maximize the total uncertainty of probability distribution across all of the targets.

$$\max \sum_{j=1}^{2^s-1}\sum_{i=1}^{t} U(p)_{ji} x_{ji} \tag{33}$$

subject to the constraints

$$\sum_{j=1}^{2^s-1} x_{ji} \le 2 \qquad i=1……t \tag{34}$$

$$\sum_{j\in J(k)}\sum_{i=1}^{t} x_{ji} \le \tau_k \qquad k=1……s \tag{35}$$

$$x_{ji} \ge 0 \qquad \text{for all pairs ji} \tag{36}$$

where τ_k is the maximum tracking capacity of the basic sensor k. This is the maximum number of targets that can be sensed on each sensor scan. $J(k)$ is the set of integers consisting of k and the integer numbers of the pseudo sensors which contain sensor k in their combination. There will be 2^{s-1} integers in each set $J(k)$. In the Linear Programming solution, each x_{ji} will be either 0 or 1. When $x_{ji} = 1$, senor j is assigned to target i.

IV. SIMULITION RESULT

There are 2 sensors track 5 targets, s=2,2^s-1=3. Sensor 1 can track two targets and sensor 2 can track three targets at the same time. Let S1, S2 be the designation of the sensors with S3={S1,S2}. The integer sets J(k) then contain 2^{2-1}=2 integers and are J(1)={1,3},J(2)={2,3}. Models of five targets are showed in figure 1. Targets 1,2,4,5 are in straight-line flight described as (37). Target 3 is in the turn rate flight showed as (38) during 20s and 40s. $q_1=1.4$, $q_2=1.4$, $q_3=1.6$, $q_4=1.3$, $q_5=1.5$, T=1s, $R_1=\begin{bmatrix}1.2^2 & 0\\ 0 & 1.2^2\end{bmatrix}$, $R_2=\begin{bmatrix}1.4^2 & 0\\ 0 & 1.4^2\end{bmatrix}$

$$A_1=\begin{bmatrix}1 & T & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & T\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad Q_1=\begin{bmatrix}\frac{T^4}{4} & \frac{T^3}{24} & 0 & 0\\ \frac{T^3}{2} & T^2 & 0 & 0\\ 0 & 0 & \frac{T^4}{4} & \frac{T^3}{24}\\ 0 & 0 & \frac{T^3}{2} & T^2\end{bmatrix}\times q_n^2$$

$$C_1=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix} \tag{37}$$

$$A_2=\begin{bmatrix}1 & T & \frac{T^2}{2} & 0 & 0 & 0\\ 0 & 1 & T & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & T & \frac{T^2}{2}\\ 0 & 0 & 0 & 0 & 1 & T\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix} \qquad Q_2=\begin{bmatrix}\frac{T^4}{4} & \frac{T^3}{2} & \frac{T^2}{2} & 0 & 0 & 0\\ \frac{T^3}{2} & T^2 & T & 0 & 0 & 0\\ \frac{T^2}{2} & T & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{T^4}{4} & \frac{T^3}{2} & \frac{T^2}{2}\\ 0 & 0 & 0 & \frac{T^3}{2} & T^2 & T\\ 0 & 0 & 0 & \frac{T^2}{2} & T & 1\end{bmatrix}\times q_n^2$$

$$C_2=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\end{bmatrix} \tag{38}$$

Single model cross-entropy sensing schedule is used as contrast. Fig.2~Fig.6 give the position track of target

1,2,3,4,5. From these figures, the IMMKF cross-entropy sensing schedule yields better performance than single model cross- entropy sensing schedules. By improving sensor allocation across all targets, average performance is improved for both the maneuvering targets and non-maneuvering targets. Similar results can be obtained in velocity. And Fig7 gives sensor-target pairs during tracking time.

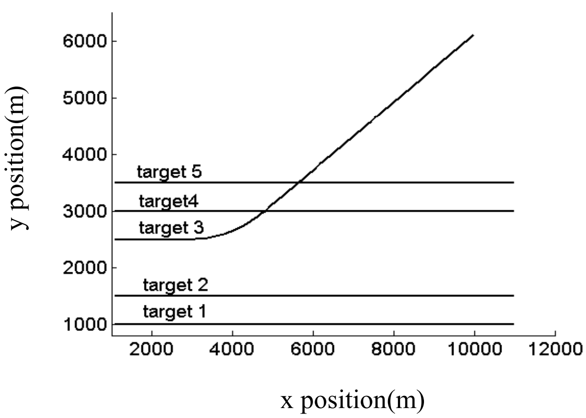

Fig 1: Targets Models

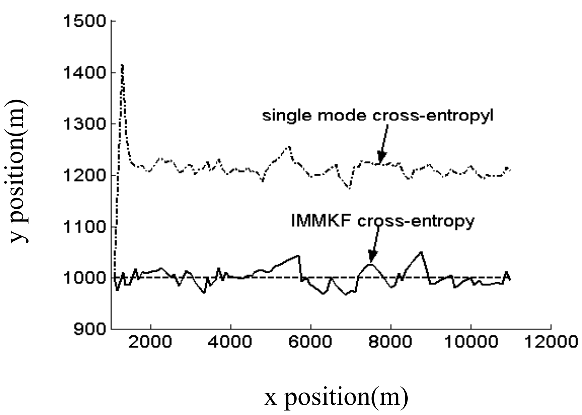

Fig 2: Position of target 1

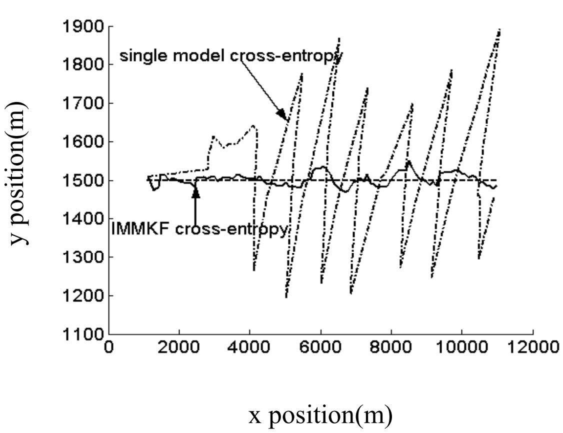

Fig 3: Position of target 2

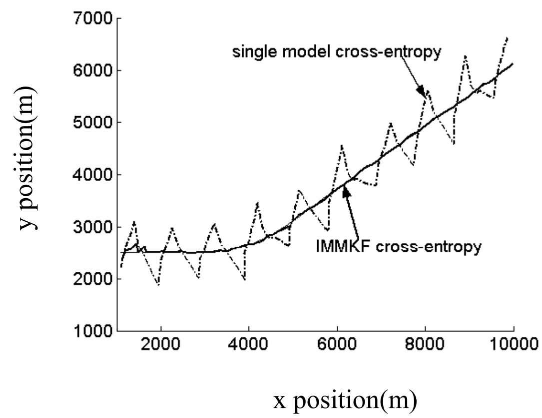

Fig 4: Position of target 3

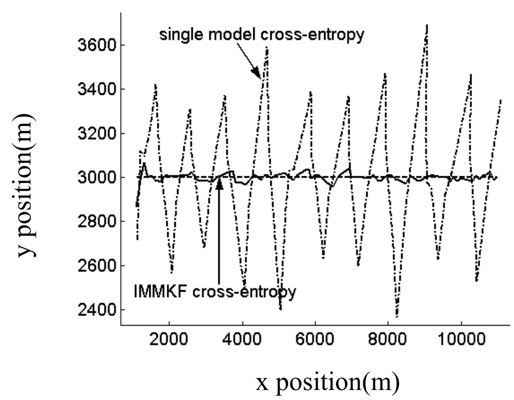

Fig 5: Position of target 4

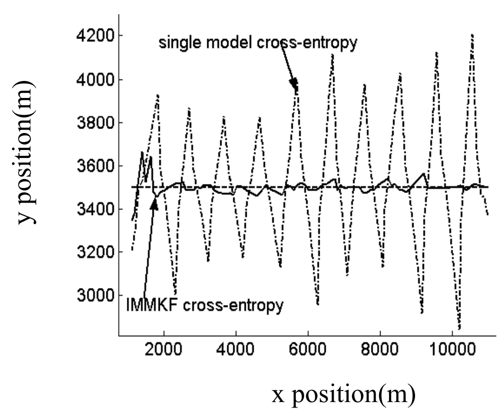

Fig 6: Position of target 5

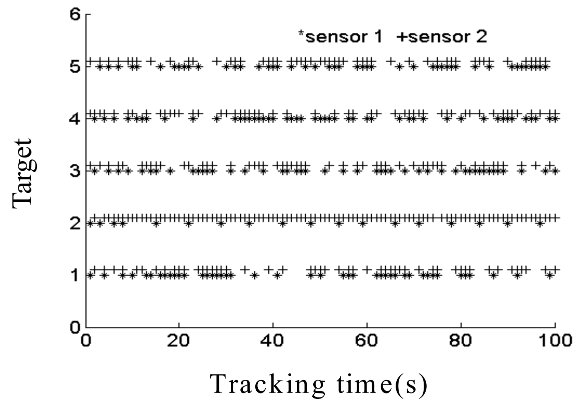

Fig 7: Sensor-target pairs

REFEREBCES

[1] J.M.Nash, 'Optimal Allocation of Tracking Resource',Proceedings of the 1977 IEEE Conference on Decision and Control,vol.1,New Orleans,LA,December 7-9 1977,pp.1177-1180

[2] K.Kastella, 'Discrimination Gain to Optimize Detctin and Classification',IEEE Trans on SMC,1997, 27(1), pp.112-116

[3] W.Schmaedeke and K.Kastella, 'Information Based Sensor Management and IMMKF', Signal and Data Processing of Small Targets 1998: Proceedings of the SPIE-The International Society for Optical Engineering, vol.3373, Orlando, FL, April 1998,pp.390-401

[4] Liu Xianxing, Zhao Wei, PanQuan, ZhangHongcai, 'An Algorithm of Multisensor Management Used in Detection and Classification', Journal of Northwestern, 19 (1), 2001,pp.27-30

[5] J.M. Molina López, F.J.Jiménez Rodríguez, and IJ.R.Casar Corredera, 'Fuzzy Reasoning For Multisensor Management,'1995 IEEE International Conference on Systems, Man, and Cybernetics, vol.2, Vancouver, British Columbia, Canada, October 22-25 1995, pp.1398-1403

[6] J.Zhongliang, X.Hong, and Z.Xueqin, 'Information Fusion and Tracking of Maneuvering Targets with Artificial Neural Networks,' Proceedings of the 1994 IEEE International Conference on Neural Networks, vol. 5, Orlando, FL, June 27-July 2 1994, pp.3403-3408

[7] L.PY.Pao and N.T.Baltz, 'Control of Sensor Information In Distributed Multisensor Systems,' Proc. American Control Conference, San Diego, CA, June 1999, pp.2397-2401

[8] Blom, H.A.P., and Bar-Shalom, 'The interacting multiple model algorithm for systems with Markovian switching coefficients', IEEE Trans. Aut. Con., vol AC-33, pp. 780-783

[9] Song Jin-pin, WenCheng-lin, Zhang Nan, 'An Algorithm of Correlated Data Fusion Estimation Based on Multi-Sensor Multi-Model Dynamic System', Journal of Henan University (Natural Science), 31(1), 2001, pp. 27-29