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BEARINGS-ONLY MEASUREMENTS FOR INS AIDING: THE THREE-DIMENSIONAL CASE *

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Abstract

The theory behind Inertial Navigation System (INS) aiding using passive, bearings-only measurements of an unknown ground object is developed. Passive, bearings-only measurements provide an effective means to aid an INS, but only the two-dimensional case was previously considered. This paper addresses the complexities of moving from two-dimensional to three-dimensional space. Stand-alone, bearings-only measurements of an unknown ground object are shown to yield estimates of the aircraft's aerodynamic angles, viz., the angle of attack and sideslip angle, which in turn are used to aid the INS. Using the synergy of INS state and passive, bearings-only measurements, a powerful navigation algorithm is developed.

1 INTRODUCTION

An Inertial Navigation System (INS) is a self-contained and nonjammable navigation system that provides redundancy for radio navigation systems that can experience interference or be jammed; however, an INS does suffer from the unbounded growth of errors over time [4]. Thus, an INS requires position and/or velocity aiding from external sources such as radar and more recently GPS. Unfortunately, radar signals are easy to detect and GPS is susceptible to jamming and spoofing.

A system using passive, bearings-only measurements of an unknown ground object is completely self-contained and impervious to jamming, spoofing, or interference. Moreover, passive measurements do not emit radiation and thus cannot be detected. The considered herein concept of INS aiding is thus fully compatible with inertial navigation by not detracting from the autonomy of an INS, and can aid the INS without the knowledge of the position of known landmarks. Thus, the autonomy of the INS primary navigation system is maintained.

*The views expressed in this article are those of the authors and do not reflect the official policy or position of the United States Air Force, Department of Defence, or U.S. Government.

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2 ANALYSIS

Consider the kinematic measurement scenario where bearings-only measurements on an unknown landmark are taken over time. It is shown that an optical flow sensor measures the angles α' and β' included between the aircraft's inertial velocity vector $\vec{V}$ and the body of the aircraft, as illustrated in Fig. 1. The angles α' and β' are

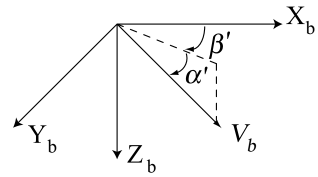

Figure 1: "Aerodynamic" Angles

related to the five angular navigation variables of the aircraft, viz., its Euler angles ψ, θ, ϕ and its course H and flight path angle γ. The navigation variables ψ, θ, and ϕ are the aircraft's Euler angles that represent yaw, pitch, and roll respectively - see, e.g., Fig. 2. The navigation variables

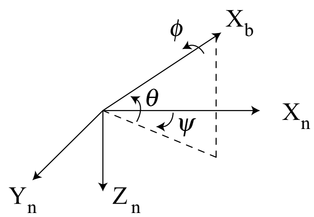

Figure 2: Euler Angles

γ and H are the angles used to specify the velocity vector in the navigation frame shown in Fig. 3.

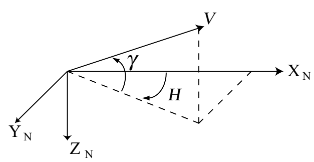

Figure 3: Velocity Vector

The air speed of the aircraft is equal to its ground speed in the absence of wind. Thus, by definition, in the absence of wind the aerodynamic angles α and β then satisfy

$$\alpha = \alpha' \text{ and } \beta = \beta' \tag{1}$$

2.1 Navigation Variables' Relationships

The orientation of the body frame with respect to the navigation frame is specified by the Euler angles ψ, θ, and ϕ. The Direction Cosine Matrix (DCM) C_b^n transforms vectors resolved in the body frame of the aircraft into vectors resolved in the navigation frame [4]. Conversely, the transpose of C_b^n, C_n^b, takes information from the navigation frame and transforms it into the body frame.

The unit vector $\vec{V}_1$ is in the direction of the aircraft's inertial velocity vector $\vec{V}$ and is specified with respect to the navigation frame by the angles γ and H as shown in Fig. 3, and therefore is

$$\vec{V}_{1n} = \begin{bmatrix} \cos\gamma\cos H \\ \cos\gamma\sin H \\ -\sin\gamma \end{bmatrix} \tag{2}$$

Hence, in the body frame

$$\vec{V}_{1b} = C_n^b \begin{bmatrix} \cos\gamma\cos H \\ \cos\gamma\sin H \\ -\sin\gamma \end{bmatrix} \tag{3}$$

The aircraft's unit inertial velocity vector is also represented in the body frame by the "aerodynamic" angles α' and β' - see, e.g., Fig. 1 - yielding the equation

$$\vec{V}_{1b} = \begin{bmatrix} \cos\alpha'\cos\beta' \\ \cos\alpha'\sin\beta' \\ \sin\alpha' \end{bmatrix} \tag{4}$$

Eqs. (3) and (4) are combined into Eq. (5) to show the relationship between the angles α' and β' and the five navigation variables ψ, θ, ϕ, γ, and H:

$$\begin{bmatrix} \cos\alpha'\cos\beta' \\ \cos\alpha'\sin\beta' \\ \sin\alpha' \end{bmatrix} = C_n^b \begin{bmatrix} \cos\gamma\cos H \\ \cos\gamma\sin H \\ -\sin\gamma \end{bmatrix} \tag{5}$$

This yields two independent equations relating the optical flow sensor's measurements α' and β' to the five navigation variables ψ, θ, ϕ, γ, and H.

2.2 The Main Equation

The Line of Sight (LOS) vector $\overrightarrow{LOS}$ is specified with respect to the body frame by the angles ψ_{LOS} and θ_{LOS} as seen in Fig. 4. The unit LOS

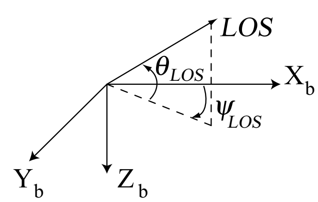

Figure 4: LOS in the Body Frame

vector resolved in the body frame is

$$\overrightarrow{LOS}_{1b} = \begin{bmatrix} \cos\theta_{LOS}\cdot\cos\psi_{LOS} \\ \cos\theta_{LOS}\cdot\sin\psi_{LOS} \\ \sin\theta_{LOS} \end{bmatrix} \tag{6}$$

Consider the plane P formed by the aircraft's velocity vector $\vec{V}$ and the initial LOS vector $\overrightarrow{LOS}_1$ to the unknown ground object P - see, e.g., Fig. 5. The angular rate $\vec{\omega}$ of the LOS

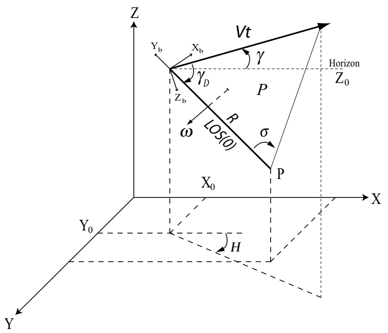

Figure 5: 3-D Measurement Scenario

from the aircraft to the unknown landmark is in the plane perpendicular to the LOS; thus, $\vec{\omega}$ will always contain a component of zero along the LOS. If, during the measurement interval t, the aircraft is flying with constant $\vec{V}_1$, then $\vec{\omega}$ is also perpendicular to the plane P. The LOS angle in the plane P is represented by σ with the magnitude of the angular rate represented by $\dot{\sigma}$. The LOS angular rate $\dot{\sigma}$, and the spatial orientation of the plane P relative to the navigation frame are obtained from the two measured nonzero components of $\vec{\omega}$. Measurements of $\vec{\omega}$ resolved in the telescope's frame are provided by an optical tracker consisting of a precision telescope mounted on a gimbal system. $\vec{\omega}$ is transformed into the aircraft's body frame using the gimbals' angle readings.

The vectors $\vec{\omega}$, $\vec{V}$, and $\overrightarrow{LOS}$ from Figure 5 are related through the cross product

$$\vec{V} \times \overrightarrow{LOS} = |V||LOS| \sin \gamma_D \frac{\vec{\omega}}{\|\omega\|} \tag{7}$$

where the unit vector

$$\vec{\omega}_1 = \frac{\vec{\omega}}{\|\omega\|}$$

Note: $\|\omega\| = |\dot{\sigma}|$ and γ_D is the angle included between the velocity vector and the initial LOS. Dividing both sides of Eq. (7) by $|V||LOS|$ produces the equation

$$\vec{V}_1 \times \overrightarrow{LOS}_1 = \vec{\omega}_1 \sin \gamma_D \tag{8}$$

where $\vec{V}_1$, $\overrightarrow{LOS}_1$, and $\vec{\omega}_1$ are all unit vectors. Eq. (8) is referred to as the "Main Equation" because it relates the five angular navigation variables with the measurements of the optical tracker: $\vec{\omega}_1$, $\overrightarrow{LOS}_1$, and the calculated angle γ_D, the latter being derived from the LOS angle measurements σ.

Representing the "Main Equation" in the body frame yields

$$\vec{V}_{1b} \times \overrightarrow{LOS}_{1b} = \sin \gamma_D \vec{\omega}_{1b} \tag{9}$$

Substituting Eqs. (4) and (6) into Eq. (9) yields Eq. (10).

$$\sin \gamma_D \cdot \vec{\omega}_{1b} = \begin{bmatrix} \cos\alpha' \cos\beta' \\ \cos\alpha' \sin\beta' \\ \sin\alpha' \end{bmatrix} \times \begin{bmatrix} \cos\theta_{LOS} \cos\psi_{LOS} \\ \cos\theta_{LOS} \sin\psi_{LOS} \\ \sin\theta_{LOS} \end{bmatrix} \tag{10}$$

The cross product is expanded and factored to produce

$$MC_n^b \begin{bmatrix} \cos\gamma \cos H \\ \cos\gamma \sin H \\ -\sin\gamma \end{bmatrix} = \sin \gamma_D \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{11}$$

where the matrix M is defined in Equation (12). Note that M is singular and $M^T = -M$. ω_x, ω_y, and ω_z are components of the unit vector $\vec{\omega}_1$ resolved in the body frame. A relationship is now established between the five angular navigation variables ψ, θ, ϕ, H, and γ, and the measurements ω_x, ω_y, ω_z, γ_D, ψ_{LOS}, and θ_{LOS}.

3 INS POSITION UPDATE

3.1 Transformation

INS aiding using passive, bearings-only measurements of an unknown ground object requires transforming position vectors resolved in the plane P as shown in Fig. 6, into position vectors resolved in the navigation frame. The DCM C_P^n is formed as shown below

$$C_P^n = \begin{bmatrix} \vec{V}_{1n} & \vdots & \vec{\omega}_{1n} \times \vec{V}_{1n} & \vdots & \vec{\omega}_{1n} \end{bmatrix}$$

It transforms vectors resolved in the plane P into vectors resolved in the navigation frame. The range vector $\vec{R}$ resolved in the navigation frame is

$$\vec{R}_n = \begin{bmatrix} X_P - X_0 \\ Y_P - Y_0 \\ Z_P - Z_0 \end{bmatrix} \tag{13}$$

$$M = \begin{bmatrix} 0 & \sin\theta_{LOS} & -\cos\theta_{LOS}\sin\psi_{LOS} \\ -\sin\theta_{LOS} & 0 & \cos\theta_{LOS}\cos\psi_{LOS} \\ \cos\theta_{LOS}\sin\psi_{LOS} & -\cos\theta_{LOS}\cos\psi_{LOS} & 0 \end{bmatrix} \tag{12}$$

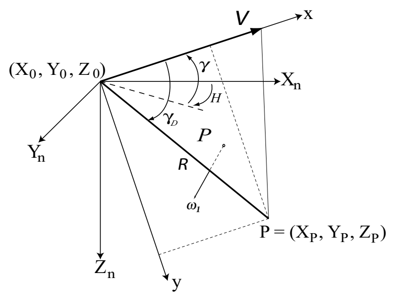

Figure 6: 3-D Measurement Scenario

and

$$\vec{R}_n = C_P^n \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \tag{14}$$

3.2 Linear Regression

It is apparent from Fig. 6 that

$$x = R\cos\gamma_D \quad \text{and} \quad y = R\sin\gamma_D \tag{15}$$

Realizing that only an estimate of γ_D is available, $\gamma_D = \hat{\gamma}_D + \upsilon_{\gamma_D}$ is substituted into Eq. (15) to yield

$$\begin{aligned} x &\approx R\cos\hat{\gamma}_D - R\sin\hat{\gamma}_D \cdot \upsilon_{\gamma_D} \\ y &\approx R\sin\hat{\gamma}_D + R\cos\hat{\gamma}_D \cdot \upsilon_{\gamma_D} \end{aligned}$$

υ_{γ_D} is modeled as white Gaussian noise with zero mean and represented as $\upsilon_{\gamma_D} = \mathcal{N}(0, \sigma^2_{\gamma_D})$. Using the relationships

$$x = K_x V \quad \text{and} \quad y = K_y V \tag{16}$$

optained in [2], [3], yields the linear regression in the primary parameters R and V.

$$\begin{aligned} \begin{bmatrix} V_m \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \cos\hat{\gamma}_D & -K_x \\ \sin\hat{\gamma}_D & -K_y \end{bmatrix} \begin{bmatrix} R \\ V \end{bmatrix} + \\ \begin{bmatrix} 0 & 1 \\ -R\sin\hat{\gamma}_D & 0 \\ R\cos\hat{\gamma}_D & 0 \end{bmatrix} \begin{bmatrix} \upsilon_{\gamma_D} \\ \upsilon_V \end{bmatrix} \end{aligned} \tag{17}$$

Eqs. (13), (14), (15), and (16) are used to produce Eq. (18).

$$\begin{bmatrix} X_P - X_0 \\ Y_P - Y_0 \\ Z_P - Z_0 \end{bmatrix} = RC_P^n \begin{bmatrix} \cos\gamma_D \\ \sin\gamma_D \\ 0 \end{bmatrix} \tag{18}$$

Substituting $H = \hat{H} + \upsilon_H$, $\gamma = \hat{\gamma} + \upsilon_\gamma$, and $\gamma_D = \hat{\gamma}_D + \upsilon_{\gamma_D}$ produces the linear regression in the parameter $(R, X_0, Y_0, Z_0, X_P, Y_P, Z_P)$ given as Eq. (19).

3.3 Nonlinear Equality Constraint

The range R satisfies the nonlinear Eq. (20). Linearization of Eq. (20) is performed by defining the vector

$$X \triangleq \begin{bmatrix} X_0 & Y_0 & Z_0 & X_P & Y_P & Z_P \end{bmatrix}^T$$

Let

$$f(X) \triangleq \sqrt{(X_P - X_0)^2 + (Y_P - Y_0)^2 + (Z_P - Z_0)^2}$$

The approximation for $f(X)$ is

$$\begin{aligned} f(X) = f(\hat{X}^- + X - \hat{X}^-) \approx \\ f(\hat{X}^-) + \nabla f|_{\hat{X}^-} (X - \hat{X}^-) \end{aligned} \tag{21}$$

where $\hat{X}^-$ is the prior estimate of the parameter X. $\hat{R}^-$ is the prior estimate of the range R. Inserting Eq. (21) into Eq. (20) produces

$$R - \nabla f|_{\hat{X}^-} X = - \nabla f|_{\hat{X}^-} \hat{X}^- + f(\hat{X}^-)$$

The gradient of f

$$\begin{aligned} \nabla f|_{\hat{X}^-} = \Big[\tfrac{\hat{X}_0^- - \hat{X}_P^-}{\hat{R}^-}, \tfrac{\hat{Y}_0^- - \hat{Y}_P^-}{\hat{R}^-}, \tfrac{\hat{Z}_0^- - \hat{Z}_P^-}{\hat{R}^-}, \\ \tfrac{\hat{X}_P^- - \hat{X}_0^-}{\hat{R}^-}, \tfrac{\hat{Y}_P^- - \hat{Y}_0^-}{\hat{R}^-}, \tfrac{\hat{Z}_P^- - \hat{Z}_0^-}{\hat{R}^-} \Big] \end{aligned} \tag{22}$$

Inserting Eq. (22) into Eq. (21) yields

$$\begin{aligned} & R + \tfrac{\hat{X}_P^- - \hat{X}_0^-}{\hat{R}^-} X_0 + \tfrac{\hat{Y}_P^- - \hat{Y}_0^-}{\hat{R}^-} Y_0 + \tfrac{\hat{Z}_P^- - \hat{Z}_0^-}{\hat{R}^-} Z_0 \\ & - \tfrac{\hat{X}_P^- - \hat{X}_0^-}{\hat{R}^-} X_P - \tfrac{\hat{Y}_P^- - \hat{Y}_0^-}{\hat{R}^-} Y_P - \tfrac{\hat{Z}_P^- - \hat{Z}_0^-}{\hat{R}^-} Z_P \\ & = \tfrac{\hat{X}_P^- - \hat{X}_0^-}{\hat{R}^-} \hat{X}_0^- + \tfrac{\hat{Y}_P^- - \hat{Y}_0^-}{\hat{R}^-} \hat{Y}_0^- + \tfrac{\hat{Z}_P^- - \hat{Z}_0^-}{\hat{R}^-} \hat{Z}_0^- \\ & - \tfrac{\hat{X}_P^- - \hat{X}_0^-}{\hat{R}^-} \hat{X}_P^- - \tfrac{\hat{Y}_P^- - \hat{Y}_0^-}{\hat{R}^-} \hat{Y}_P^- - \tfrac{\hat{Z}_P^- - \hat{Z}_0^-}{\hat{R}^-} \hat{Z}_P^- + \hat{R}^- \end{aligned}$$

$$
\begin{bmatrix}
\cos\hat{\gamma}_D\cos\hat{\gamma}\cos\hat{H}-\omega_y\sin\hat{\gamma}\sin\hat{\gamma}_D-\omega_z\cos\hat{\gamma}\sin\hat{H}\sin\hat{\gamma}_D & 1 & 0 & 0 & -1 & 0 & 0 \\
\cos\hat{\gamma}\sin\hat{H}\cos\hat{\gamma}_D+\omega_z\cos\hat{\gamma}\cos\hat{H}\sin\hat{\gamma}_D+\omega_x\sin\hat{\gamma}\sin\hat{\gamma}_D & 0 & 1 & 0 & 0 & -1 & 0 \\
-\sin\hat{\gamma}\cos\hat{\gamma}_D+\omega_x\cos\hat{\gamma}\sin\hat{H}\sin\hat{\gamma}_D-\omega_y\cos\hat{\gamma}\cos\hat{H}\sin\hat{\gamma}_D & 0 & 0 & 1 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix} R \\ X_0 \\ Y_0 \\ Z_0 \\ X_P \\ Y_P \\ Z_P \end{bmatrix}
$$

$$
+\hat{R}^-\left[\begin{array}{l}
-\sin\hat{\gamma}_D\cos\hat{\gamma}\cos\hat{H}-\omega_y\sin\hat{\gamma}\cos\hat{\gamma}_D-\omega_z\cos\hat{\gamma}\sin\hat{H}\cos\hat{\gamma}_D, \\
-\cos\hat{\gamma}\sin\hat{H}\sin\hat{\gamma}_D+\omega_z\cos\hat{\gamma}\cos\hat{H}\cos\hat{\gamma}_D+\omega_x\sin\hat{\gamma}\cos\hat{\gamma}_D, \\
\sin\hat{\gamma}\sin\hat{\gamma}_D+\omega_x\cos\hat{\gamma}\sin\hat{H}\cos\hat{\gamma}_D-\omega_y\cos\hat{\gamma}\cos\hat{H}\cos\hat{\gamma}_D, \\
-\cos\hat{\gamma}_D\sin\hat{\gamma}\cos\hat{H}-\omega_y\cos\hat{\gamma}\sin\hat{\gamma}_D+\omega_z\sin\hat{\gamma}\sin\hat{H}\sin\hat{\gamma}_D, \\
-\sin\hat{\gamma}\sin\hat{H}\cos\hat{\gamma}_D-\omega_z\sin\hat{\gamma}\cos\hat{H}\sin\hat{\gamma}_D+\omega_x\cos\hat{\gamma}\sin\hat{\gamma}_D, \\
-\cos\hat{\gamma}\cos\hat{\gamma}_D-\omega_x\sin\hat{\gamma}\sin\hat{H}\sin\hat{\gamma}_D+\omega_y\sin\hat{\gamma}\cos\hat{H}\sin\hat{\gamma}_D, \\
-\cos\hat{\gamma}_D\cos\hat{\gamma}\sin\hat{H}-\omega_z\cos\hat{\gamma}\cos\hat{H}\sin\hat{\gamma}_D \\
\cos\hat{\gamma}\cos\hat{H}\cos\hat{\gamma}_D-\omega_z\cos\hat{\gamma}\sin\hat{H}\sin\hat{\gamma}_D \\
\omega_x\cos\hat{\gamma}\cos\hat{H}\sin\hat{\gamma}_D+\omega_y\cos\hat{\gamma}\sin\hat{H}\sin\hat{\gamma}_D
\end{array}\right]
\begin{bmatrix} \upsilon_{\gamma_D} \\ \upsilon_{\gamma} \\ \upsilon_H \end{bmatrix}
=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\tag{19}
$$

$$
R-\sqrt{(X_P-X_0)^2+(Y_P-Y_0)^2+(Z_P-Z_0)^2}=0 \tag{20}
$$

The right hand side of the equation is simplified to

$$
-\frac{1}{\hat{R}^-}\Big[(\hat{X}_P^- - \hat{X}_0^-)^2+(\hat{Y}_P^- - \hat{Y}_0^-)^2+ (\hat{Z}_P^- - \hat{Z}_0^-)^2\Big]+\hat{R}^-=0
$$

Thus, the additional linear regression equation is obtained

$$
0=\Big[1, \frac{\hat{X}_P^- - \hat{X}_0^-}{\hat{R}^-}, \frac{\hat{Y}_P^- - \hat{Y}_0^-}{\hat{R}^-}, \frac{\hat{Z}_P^- - \hat{Z}_0^-}{\hat{R}^-}, -\frac{\hat{X}_P^- - \hat{X}_0^-}{\hat{R}^-}, -\frac{\hat{Y}_P^- - \hat{Y}_0^-}{\hat{R}^-}, -\frac{\hat{Z}_P^- - \hat{Z}_0^-}{\hat{R}^-}\Big]\begin{bmatrix} R \\ \cdots \\ X \end{bmatrix} \tag{23}
$$

The linear regression matrices are used to build up the INS position estimate updating algorithm.

3.4 The INS Aiding Equation

The INS provides measurements of the aircraft's positional variables, viz., the velocity V_m, and the aircraft's current position X_{0_m}, Y_{0_m}, and Z_{0_m}. The initial range error to the ground object is assumed small. The initial range measurement is obtained from an application of the Law of Sines to the triangle shown in Fig. 5. In other words, the stadiametric measurement is used

$$
R_m=\frac{\sin(\gamma_D+\sigma)}{\sin\sigma}V_mT
$$

where T is the measurement interval. Small errors in the angles σ and γ_D produce small errors in the estimated range R_m.

Eqs. (19), (17), and (23) are combined into a linear regression with the parameter $(R, V, X_0, Y_0, Z_0, X_P, Y_P, Z_P)$, as shown in Equation (24). A, B, C and D come from Eq. (19) in the form

$$
\begin{aligned}
&\begin{bmatrix} A_{3X3} & I_{3x3} & -I_{3X3} \end{bmatrix}\ldots+ \\
&\hat{R}^-\begin{bmatrix} B_{3X3} & C_{3X3} & D_{3X3} \end{bmatrix}\ldots=0_{3X3}
\end{aligned}
$$

Eq. (24) is in the form $Z=HX+\Gamma V$ where Z represents the 14-by-1 measurement vector, H represents the 14-by-8 regressor matrix, and V represents the 11-by-1 measurement noise vector. Assuming the noise components are not correlated, the 14-by-14 equation error covariance matrix is

$$
R=\Gamma\begin{bmatrix}
\sigma^2_{\gamma_D} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma^2_V & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma^2_\gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma^2_H & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma^2_{X_0} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma^2_{Y_0} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{Z_0} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{X_P} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{Y_P} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{Z_P} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_R
\end{bmatrix}\Gamma^T
$$

The measurement noise values are all modeled as white Gaussian noise with zero mean.

The linear regression Eq. (24) is solved using the Weighted Least Squares / Minimum Variance formulae [1]

$$
\begin{aligned}
\hat{X}^+ &= \left[H^TR^{-1}H\right]^{-1}H^TR^{-1}Z \\
P^+ &= \left[H^TR^{-1}H\right]^{-1}
\end{aligned} \tag{25}
$$

where $\hat{X}^+$ is the minimum variance parameter estimate and P^+ is the predicted parameter

$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\cos\hat{\gamma}_D & -K_X & 0 & 0 & 0 & 0 & 0 & 0 \\
\sin\hat{\gamma}_D & -K_Y & 0 & 0 & 0 & 0 & 0 & 0 \\
A_1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
A_2 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
A_3 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
1 & 0 & \frac{\hat{X}_P^- - \hat{X}_0^-}{\hat{R}^-} & \frac{\hat{Y}_P^- - \hat{Y}_0^-}{\hat{R}^-} & \frac{\hat{Z}_P^- - \hat{Z}_0^-}{\hat{R}^-} & -\frac{\hat{X}_P^- - \hat{X}_0^-}{\hat{R}^-} & -\frac{\hat{Y}_P^- - \hat{Y}_0^-}{\hat{R}^-} & -\frac{\hat{Z}_P^- - \hat{Z}_0^-}{\hat{R}^-} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} R \\ V \\ X_0 \\ Y_0 \\ Z_0 \\ X_P \\ Y_P \\ Z_P \end{bmatrix}
+
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\hat{R}^- \sin\hat{\gamma}_D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hat{R}^- \cos\hat{\gamma}_D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
B_1 & 0 & C_1 & D_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
B_2 & 0 & C_2 & D_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
B_3 & 0 & C_3 & D_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} \upsilon_{\gamma_D} \\ \upsilon_V \\ \upsilon_\gamma \\ \upsilon_H \\ \upsilon_{X_0} \\ \upsilon_{Y_0} \\ \upsilon_{Z_0} \\ \upsilon_{X_P} \\ \upsilon_{Y_P} \\ \upsilon_{Z_P} \\ \upsilon_R \end{bmatrix}
=
\begin{bmatrix} V_m \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ X_{0_m} \\ Y_{0_m} \\ Z_{0_m} \\ X_{P_m} \\ Y_{P_m} \\ Z_{P_m} \\ R_m \end{bmatrix}
\tag{24}
$$

estimation error covariance matrix. $\hat{X}^+$ provides the "best" estimate of the aircraft's position and velocity, and the position of the unknown ground object. The aiding of the initial aircraft position and the aircraft velocity are done in batch and thus occur at the completion of the bearings-only measurement sequence, at time T. The key aiding information comes in the form of an enhanced estimate of the aircraft's velocity.

4 CONCLUSIONS

A novel INS aiding method using passive, bearings-only measurements of an unknown ground object in three-dimensional space is obtained. The aiding concept is based on the relationships of the measured α' and β' angles between the aircraft's inertial velocity vector $\vec{V}$ and the body of the aircraft to the five angular navigation variables ψ, θ, ϕ, γ, H, and stadiametry. INS aiding using passive, bearings-only measurements of an unknown ground object, a.k.a optical flow measurement, creates a fully integrated and enhanced autonomous navigation system impervious to jamming, spoofing, or interference and without giving away the aircraft's presence.

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