Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

MULTISENSOR TRACKING OF A MANEUVERING TARGET IN CLUTTER WITH ASYNCHRONOUS MEASUREMENTS USING IMMPDA FILTERING AND PARALLEL DETECTION FUSION

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ABSTRACT

We present a (suboptimal) filtering algorithm for tracking a highly maneuvering target in a cluttered environment using multiple sensors dealing with possibly asynchronous (time delayed) measurements. The filtering algorithm is developed by applying the basic interacting multiple model (IMM) approach, the probabilistic data association (PDA) technique, and asynchronous measurement updating to a state-augmented system for the target motion. A state augmentation approach is developed to estimate the time delay between local and remote sensors. A multisensor probabilistic data association filter is developed for parallel sensor processing for target tracking under clutter. The algorithm is illustrated via a highly maneuvering target tracking simulation example where two sensors, a radar and an infrared sensor, are used. Compared with an existing IMMPDA filtering algorithm designed for synchronous (no delay) sensor measurements processing, the proposed algorithm achieves considerable improvement in the accuracy of track estimation.

1. INTRODUCTION

We consider the problem of tracking a single maneuvering target in clutter. This class of problem has received considerable attention in the literature [1, 2, 3, 4, 9]. In target tracking systems measurements are typically collected in "scans" or "frames" and then transmitted to a processing center [5, 6]. Asynchronous (delayed) measurements arise in a multisensor central tracking system due to communication network delays, varying preprocessing times at the sensor platforms and possibly lack of sampling time synchronization among sensor platforms. One of the asynchronous measurement problems is that of out-of-sequence measurements (OOSM) where measurements at various sensors may arrive out-of-sequence (not in correct time order) at the central processor. OOSM has been considered using interacting multiple model (IMM) [6, 7, 8]. In this paper we do not consider OOSM but, instead, consider "in-sequence" measurements with a fixed but unknown relative time-delay among sensor measurements. Various sensor measurements are assumed to be at the same rate but not necessarily time synchronized. All measurements over one sampling interval (based on the local clock of the central processor) are collected at the central processor, attributed to one time instant and processed simultaneously. We exploit interacting multiple model (IMM) and probabilistic data association (PDA) techniques. It is assumed that a track has been formed (initiated) and the objective of this work is to investigate fixed-but-unknown relative time-delay (measurement timing mismatch) arising in a multisensor central tracking system.

In [6], fixed-lag smoothing techniques have been investigated using IMM algorithm combined with PDA filter in a multiple sensor scenario to propose a combined IMMM-SPDAF (interacting multiple model multiple sensor probabilistic data association filter). We exploit the basic structure of [1] in combination with a state-augmented approach to deal with the fixed-but-unknown relative time-delay. In [1] and [14] it is assumed that the sensors are collocated and (time) synchronized with the sampling rate. In contrast, the sensor collocation and (time) synchronization are no longer assumed in this paper. Also, unlike [1, 9, 12] which have used sequential updating of the state estimates with measurements (i.e., updating of the state estimates sequentially with measurements from different sensors), we use parallel updating of the state estimates with measurements (i.e., updating of the state estimates with all measurements at the same time). For linear systems, the two updating methods are algebraically equivalent but for nonlinear filtering, the parallel updating can yield better performance in spite of higher computational cost [4]. Ref. [14] uses parallel updating but has some errors: during data association, all measurements at the same time from different sensors are assumed to be either from clutter or from the target. The possibility that a measurement from sesnsor 1 may be from target while the measurement from sensor 2 may be clutter-induced (and vice-versa) is implicitly not allowed in [14] – this is clearly incorrect. Ref. [10] allows for such distinctions (hypotheses), however, it is limited to non-manevering targets. In this paper, we also extend the multisensor approach of [10] to maneuvering targets (see Step 4 in Sec. 4.).

The paper is organized as follows. Sec. 2 presents the problem formulation. Sec. 3 describes the state-augmented system approach. Sec. 4 describes the proposed IMMM-SPDAF algorithm for asynchronous measurements. Simulation results using the proposed algorithm for a realistic problem are given in Sec. 5.

2. PROBLEM FORMULATION

We assume that the target dynamics can be modeled by one of n hypothesized models. The model set is denoted as $\mathcal{M}_n := \{1, ..., n\}$ and there are total q sensors. The event that model m is in effect during the sampling period $(t_{k-1}, t_k]$ is denoted by M_k^m. For the mth hypothesized model (mode), the state dynamics and measurements, respectively, are modeled as

$$x_k = F_{k,k-1}^m x_{k-1} + G_{k,k-1}^m v_{k-1}^m, \tag{1}$$

and (local measurement model at the sensor l)

$$z_k^l = h^{m,l}(x_k) + w_k^{m,l} \quad \text{for } l = 1, ..., q, \tag{2}$$

where x_k is the system state at t_k and of dimension n_x, z_k^l is the (true) measurement vector (i.e., due to the target) at sensor l at t_k and of dimension n_{zl}, $F_{k,k-1}^m$ and $G_{k,k-1}^m$ are the system matrices when model m is in effect over the sampling period $(t_{k-1}, t_k]$, and $h^{m,l}$ is the nonlinear transformation of x_k to z_k^l $(l = 1, ..., q)$ for model m. A first-order linearized version of (2) is given by

$$z_k^l = H_k^{m,l} x_k + w_k^{m,l} \text{ for } l = 1, ..., q \tag{3}$$

where $H_k^{m,l}$ is the Jacobian matrix of $h^{m,l}$ evaluated at some value of the estimate of state x_k (see Sec. 3.). The process noise v_{k-1}^m and the measurement noise $w_k^{m,l}$ are mutually uncorrelated zero-mean white Gaussian processes with covariance matrices Q_{k-1}^m and $R_k^{m,l}$, respectively. At the initial time t_0, the initial conditions for the system state

This work was supported by the Office of Naval Research under Grant N00014-01-1-0971.

under each model m are assumed to be Gaussian random variables with the known mean $\bar{x}_0^m$ and the known covariance P_0^m. The probability of model m at t_0, $\mu_0^m = P\{M_0^m\}$, is also known. The switching from model M_{k-1}^i to model M_k^m is governed by a finite-state stationary Markov chain with known transition probabilities $p_{im} = P\{M_k^m | M_{k-1}^i\}$. Henceforth, time t_k will be denoted by k.

Assume that there is a fixed but unknown relative time delay d_k (modulo T=sampling interval) at sample time t_k between the local sensor clock and the central processor clock at sample time t_k. [This time delay could be due to unsynchronized clocks at the two locations or due to inherent delay due to congestion, insufficient bandwidth etc. in the communication link between the remote sensor platform and the central processor.] The measurements from sensor l are sent to the central processor where all measurements collected between local sampling interval $(t_{k-1}, t_k]$ are attributed to time t_k. The state dynamics and measurements reported from the remote sensor platform at time $t_{k_{dl}}$ (henceforth will be denoted by k_{dl}) to the center processor at time t_k can be modeled as

$$x_{k_{dl}} = F_{k_{dl},k-1}^m x_{k-1} + G_{k_{dl},k-1}^m v_{k-1}^m \tag{4}$$

and (measurement model of the sensor at the central processor)

$$z_k^l = h^{m,l}(x_{k_{dl}}) + w_k^{m,l} \tag{5}$$

where $t_{k_{dl}} = t_k - d_{kl}$ and d_{kl} is the time difference between the sampling time at the central processor and the measurement time at the local sensor (assume that $0 \le d_{kl} < T$, where T is sampling time), $x_{k_{dl}}$ is the system state at $t_{k_{dl}}$ and of dimension n_x, $F_{k_{dl},k-1}^m$ and $G_{k_{dl},k-1}^m$ are the system matrices when model m is in effect over the timing interval $(t_{k-1}, t_{k_{dl}}]$. See Sec. 5 for a concrete example.

3. STATE-AUGMENTED SYSTEM

Augment x_k in (1) to define the augmented state $\bar{x}_k$ as

$$\bar{x}_k' := [x_k', v_k^{m'}, x_{k-1}', v_{k-1}^{m'}] \tag{6}$$

where x_k' denotes the transpose of x_k. Assume that there is a fixed but unknown delay, d_{kl}, between the central processor and the remote sensor l platform. Using the above definitions (1), (6) and the measurement delay, d_{kl}, the augmented state equation may be written more compactly as

$$\bar{x}_k = \bar{F}_{k,k-1}^m \bar{x}_{k-1} + \bar{G}_{k,k-1}^m v_k^m \tag{7}$$

and

$$d_{kl} = d_{(k-1)l} + v_{k-1}^{dl} \tag{8}$$

where v_{k-1}^{dl} is a small processing noise assumed to be Gaussian noise with zero mean and (very) small but nonzero variance. Note that the process noise in (7) is v_k^m (at time k not at time $k-1$). Above equations (7) and (8) can also be absorbed into another augmented state $\tilde{x}_k$ as

$$\tilde{x}_k := \begin{bmatrix} \bar{x}_k \\ d_{kl} \end{bmatrix} = \tilde{F}_{k,k-1}^m \tilde{x}_{k-1} + \tilde{G}_{k,k-1}^m \tilde{v}_k^m \tag{9}$$

where

$$\tilde{v}_k^m := \begin{bmatrix} v_k^m \\ v_{k-1}^{dl} \end{bmatrix}, \tag{10}$$

and $\tilde{F}_{k,k-1}^m$ and $\tilde{G}_{k,k-1}^m$ are appropriately defined (see also Sec. 5., (41)-(46)). Using the augmented state (9), the counterparts to (2) and (5), respectively, are

$$z_k^l = h^{m,l}(\tilde{x}_k) + w_k^{m,l} = h^{m,l}([I, 0, 0, 0, 0]\tilde{x}_k) + w_k^{m,l}, \tag{11}$$

$$\begin{aligned} z_k^l &= h^{m,l}(\tilde{x}_{k_l}) + w_k^{m,l} \\ &= h^{m,l}([0, 0, F_{k_{dl},k-1}^m, G_{k_{dl},k-1}^m, 0]\tilde{x}_k) + w_k^{m,l} \end{aligned} \tag{12}$$

for measurements from local sensor and from remote sensor, respectively. To keep the notations and details to a bare minimum, *we will consider the case of two sensors only* and furthermore, we will assume that one of the sensors is either collocated with or is synchronized with the central processor, so that we will drop the subscript l from d_{kl}. For more than two sensors, we need to augment $\tilde{x}_k$ with additional d_k's (total $q-1$): in essence, these delays are relative to one of the sensors (reference sensor).

The following notations and definitions are used regarding the measurements at sensor l. Note that, in general, at any time some measurements may be due to clutter and some due to the target, i.e. there can be more than a single measurement at time k at sensor l. The measurement set (not yet validated) generated by sensor l at time k is denoted as $Z_k^l := \{z_k^{l(1)}, z_k^{l(2)}, ..., z_k^{l(m_l)}\}$ where m_l is the number of measurements generated by sensor l at time k. Variable $z_k^{l(i)}$ (i = 1,...,m_l) is the ith measurement within this set. The validated set of measurements of sensor l at time k will be denoted by Y_k^l, containing $\bar{m}_l$ ($\le m_l$) measurement vectors. The cumulative set of validated measurements from sensor l up to time k is denoted as $Z^{k(l)} := \{Y_1^l, Y_2^l, ..., Y_k^l\}$. The cumulative set of validated measurements from all sensors up to time k is denoted as $Z^k := \{Z^{k(1)}, Z^{k(2)}, ..., Z^{k(q)}\}$ where q is the number of sensors.

Our goal is to find the state estimate

$$\hat{\tilde{x}}_{k|k} := E\{\tilde{x}_k | Z^k\} \tag{13}$$

and the associated error covariance matrix

$$\tilde{P}_{k|k} := E\{[\tilde{x}_k - \hat{\tilde{x}}_{k|k}][\tilde{x}_k - \hat{\tilde{x}}_{k|k}]' | Z^k\} \tag{14}$$

where x_k' denotes the transpose of x_k.

4. IMMMSPDAF ALGORITHM FOR ASYNCHRONOUS MEASUREMENTS

We now modify the IMM/(J)PDA algorithms of [9] and [12] to apply to the multi-sensor asynchronous measurements system. We confine our attention to the case of 2 sensors; however, the algorithm can be easily adapted to the case of arbitrary q sensors. We will only briefly outline the basic steps in "one cycle" (i.e., processing needed to update for a new set of measurements) of the IMMMSPDA filter.

Assumed available: Given the state estimate $\hat{\tilde{x}}_{k-1|k-1}^m := E\{\tilde{x}_{k-1} | M_{k-1}^m, Z^{k-1}\}$, the associated covariance $\tilde{P}_{k-1|k-1}^m$, and the conditional mode probability μ_{k-1}^m $:= P[M_{k-1}^m | Z^{k-1}]$ at time $k-1$ for each mode $m \in \mathcal{M}_n$.

Step 1. Interaction − mixing of the estimate from the previous time ($\forall m \in \mathcal{M}_n$) : predicted mode probability:

$$\mu_k^{m-} := P[M_k^m | Z^{k-1}] = \sum_i p_{im} \mu_{k-1}^i. \tag{15}$$

mixing probability:

$$\mu^{i|m} := P[M_{k-1}^i | M_k^m, Z^{k-1}] = p_{im} \mu_{k-1}^i / \mu_k^{m-}. \tag{16}$$

mixed estimate:

$$\hat{\tilde{x}}_{k-1|k-1}^{0m} := E\{\tilde{x}_{k-1} | M_k^m, Z^{k-1}\} = \sum_i \hat{\tilde{x}}_{k-1|k-1}^i \mu^{i|m}. \tag{17}$$

covariance of the mixed estimate: $\tilde{P}_{k-1|k-1}^{0m} :=$

$$E\{[\tilde{x}_{k-1} - \hat{\tilde{x}}_{k-1|k-1}^{0m}][\tilde{x}_{k-1} - \hat{\tilde{x}}_{k-1|k-1}^{0m}]' | M_k^m, Z^{k-1}\}$$

$$= \sum_i \{\tilde{P}^i_{k-1|k-1} + [\hat{\tilde{x}}^i_{k-1|k-1} - \hat{\tilde{x}}^{0m}_{k-1|k-1}]$$

$$\times [\hat{\tilde{x}}^i_{k-1|k-1} - \hat{\tilde{x}}^{0m}_{k-1|k-1}]'\}\mu^{i|m}. \quad (18)$$

Step 2. Predicted state and measurements for sensors 1 and 2 ($\forall m \in \mathcal{M}_n$) : state prediction:

$$\hat{\tilde{x}}^m_{k|k-1} := E\{\tilde{x}_k | M^m_k, Z^{k-1}\} = \tilde{F}^m_{k-1}\hat{\tilde{x}}^{0m}_{k-1|k-1}. \quad (19)$$

state prediction error covariance:

$$\tilde{P}^m_{k|k-1} := E\{[\tilde{x}_k - \hat{\tilde{x}}^m_{k|k-1}][\tilde{x}_k - \hat{\tilde{x}}^m_{k|k-1}]' | M^m_k, Z^{k-1}\}$$

$$= \tilde{F}^m_{k-1}\tilde{P}^{0m}_{k-1|k-1}\tilde{F}^{m'}_{k-1} + \tilde{G}^m_{k-1}Q^m_{k-1}\tilde{G}^{m'}_{k-1}. \quad (20)$$

The mode-conditioned predicted measurement for sensor l is

$$\hat{z_k}^{m,l} := h^{m,l}(\hat{\tilde{x}}^m_{k|k-1}). \quad (21)$$

Using the linearized version (3), the covariance of the mode-conditioned residual

$$\nu^{m,l(i)}_k := z^{l(i)}_k - \hat{z}^{m,l}_k$$

is given by (assume q=2, the case of 2 sensors)

$$S^{m,1}_k := E\{\nu^{m,1(i)}_k \nu^{m,1(i)'}_k | M^m_k, Z^{k-1}\}$$

$$= \tilde{H}^{m,1}_k \tilde{P}^m_{k|k-1}\tilde{H}^{m,1'}_k + R^{m,1}_k, \quad (22)$$

$$S^{m,2}_k := E\{\nu^{m,2(i)}_k \nu^{m,2(i)'}_k | M^m_k, Z^{k-1}\}$$

$$= \tilde{H}^{m,2}_k \tilde{P}^m_{k|k-1}\tilde{H}^{m,2'}_k + R^{m,2}_k \quad (23)$$

where $\tilde{H}^{m,l}_k$ is the first order derivative (Jacobian matrix) of $h^{m,l}(.)$ evaluated at the state prediction $\hat{\tilde{x}}^m_{k|k-1}$ (see (21)). Note that (22) and (23) assume that $z^{l(i)}_k$ originates from the target.

As mentioned earlier, since our approach also deals with multiple simultaneous measurements [10, 11] arising from two separate sensors that are tracking a single target through a common surveillance region, a method for fusion of multiple measurements has to be devised. In order to do this, now the combined covariance S^m_k of the mode-conditioned residual obtained from (22) and (23) also needs to be considered as

$$S^m_k := \begin{bmatrix} \tilde{H}^{m,1}_k \\ \tilde{H}^{m,2}_k \end{bmatrix} \tilde{P}^m_{k|k-1} \begin{bmatrix} \tilde{H}^{m,1'}_k & \tilde{H}^{m,2'}_k \end{bmatrix} +$$

$$\begin{bmatrix} R^{m,1}_k & 0 \\ 0 & R^{m,2}_k \end{bmatrix}. \quad (24)$$

Step 3. Measurement validation for sensors 1 and 2 ($\forall m \in \mathcal{M}_n$) : There is uncertainty regarding the measurements' origins. Therefore, we perform validation for each target separately. One sets up a validation gate for sensor l centered at the mode-conditioned predicted measurement, $\hat{z}^{m,l}_k$. Let ($|A| = \det(A)$) $m_a := \arg\left\{\max_{m\in\mathcal{M}_n} \left|S^{m,l}_k\right|\right\}$. Then measurement $z^{l(i)}_k$ (i=1,2,...,m_l) is validated if and only if

$$[z^{l(i)}_k - \hat{z}^{m_a,l}_k]'[S^{m_a,l}_k]^{-1}[z^{l(i)}_k - \hat{z}^{m_a,l}_k] < \gamma \quad (25)$$

where γ is an appropriate threshold. The volume of the validation region with the threshold γ is $V^l_k := c_{n_{zl}}\gamma^{n_{zl}/2}|S^{m_a,l}_k|^{1/2}$ where n_{zl} is the dimension of the measurement and $c_{n_{zl}}$ is the volume of the unit hypersphere of this dimension ($c_1 = 2$, $c_2 = \pi$, $c_3 = 4\pi/3$, etc.). Choice of γ is discussed in more detail in [4, Sec. 2.3.2]. After performing the validation for each target separately, we deal with all the validated data for measurement fusion.

Step 4. State estimation with validated measurement from sensors 1 and 2 ($\forall m \in \mathcal{M}_n$) : From among all the raw measurements from sensor l at time k, i.e. $Z^l_k := \{z^{l(1)}_k, z^{l(2)}_k, ..., z^{l(m_l)}_k\}$, define the set of validated measurement for sensor l at time k as $Y^l_k := \{y^{l(1)}_k, y^{l(2)}_k, ..., y^{l(\bar{m}_l)}_k\}$ where $\bar{m}_l$ is total number of validated measurement for sensor l at time k and $y^{l(i)}_k := z^{l(l_i)}_k$ where $1 \leq l_1 < l_2 < ... < l_{\bar{m}_l} \leq m_l$ when $\bar{m}_l \neq 0$. Define the association events (hypotheses) $\theta^{i,j}_k$ as follows (here we follow [10])

- $\theta^{0,0}_k$: none of the measurements in Y^1_k or Y^2_k is target originated.
- $\theta^{0,j}_k$: only $y^{2(j)}_k$ in Y^2_k is a target measurement, all other measurements in Y^1_k or Y^2_k are clutter, $i = 0$, $j = 1, ..., \bar{m}_2$.
- $\theta^{i,0}_k$: only $y^{1(i)}_k$ in Y^1_k is a target measurement, all other measurements in Y^1_k or Y^2_k are clutter, $i = 1, ..., \bar{m}_1$, $j = 0$.
- $\theta^{i,j}_k$: $y^{1(i)}_k$ and $y^{2(j)}_k$ in Y^1_k and Y^2_k, respectively, are target measurements, all other measurements are clutter, $i = 1, ..., \bar{m}_1$, $j = 1, ..., \bar{m}_2$.

Therefore, there are a total of $\bar{m}_1\bar{m}_2+\bar{m}_1+\bar{m}_2+1$ possible association hypotheses. Define the mode-conditioned association event probabilities as

$$\beta^{m,i,j}_k := P\{\theta^{i,j}_k | M^m_k, Y^1_k, Y^2_k, Z^{k-1}\}. \quad (26)$$

Exploiting the diffuse model for clutter in [1, 4], it turns out that

$$\beta^{m,0,0}_k = C\frac{(1-P_{D_1}P_{G_1})(1-P_{D_2}P_{G_2})}{(V^1_k)^{\bar{m}_1}(V^2_k)^{\bar{m}_2}}, \quad i = 0,\ j = 0$$

$$\beta^{m,0,j}_k = C\frac{P_{D_2}(1-P_{D_1}P_{G_1})\mathcal{N}\left[\nu^{m,2(j)}_k; 0, S^{m,2}_k\right]}{(V^2_k)^{\bar{m}_2-1}\bar{m}_2},$$

$$i = 0, \quad j = 1, ..., \bar{m}_2$$

$$\beta^{m,i,0}_k = C\frac{P_{D_1}(1-P_{D_2}P_{G_2})\mathcal{N}\left[\nu^{m,1(i)}_k; 0, S^{m,1}_k\right]}{(V^1_k)^{\bar{m}_1-1}\bar{m}_1},$$

$$i = 1, ..., \bar{m}_1, \quad j = 0$$

$$\beta^{m,i,j}_k = C\frac{\mathcal{N}\left[\nu^{m,1(i)}_k; 0, S^{m,1}_k\right]\mathcal{N}\left[\nu^{m,2(j)}_k; 0, S^{m,2}_k\right]P_{D_1}P_{D_2}}{\bar{m}_1\bar{m}_2(V^1_k)^{\bar{m}_1-1}(V^2_k)^{\bar{m}_2-1}},$$

$$i = 1, ..., \bar{m}_1, \quad j = 1, ..., \bar{m}_2 \quad (27)$$

where P_{D_1} and P_{D_2} are the detection probabilities that the sensors 1 and 2 detect the target, respectively, P_{G_1} and P_{G_1} are probabilities the target is in the validation region observed from sensors 1 and 2, respectively, C is a normalization constant such that $\sum_{i=0}^{\bar{m}_1}\sum_{j=0}^{\bar{m}_2}\beta^{m,i,j}_k = 1\ \forall m$ and

$$\mathcal{N}[x; y, P] := |2\pi P|^{-1/2}\exp\left[-\frac{1}{2}(x-y)'P^{-1}(x-y)\right].$$

Define the mode-conditioned innovations $\nu_k^{m,i,j}$ as

$$\nu_k^{m,0,0} = \begin{bmatrix} 0_{n_{z1\times 1}} \\ 0_{n_{z2\times 1}} \end{bmatrix}, \quad i=0,\ j=0$$

$$\nu_k^{m,0,j} = \begin{bmatrix} 0_{n_{z1\times 1}} \\ \nu_k^{m,2(j)} \end{bmatrix}, \quad i=0,\ j=1,...,\bar{m}_2$$

$$\nu_k^{m,i,0} = \begin{bmatrix} \nu_k^{m,1(i)} \\ 0_{n_{z2\times 1}} \end{bmatrix}, \quad i=1,...,\bar{m}_1,\ j=0$$

$$\nu_k^{m,i,j} = \begin{bmatrix} \nu_k^{m,1(i)} \\ \nu_k^{m,2(j)} \end{bmatrix}, \quad i=1,...,\bar{m}_1,\ j=1,...,\bar{m}_2. \tag{28}$$

The likelihood function for each mode m is

$$\Lambda_k^m$$

$$:= p\left[Y_k^1, Y_k^2 | M_k^m, Z^{k-1}\right] = \sum_{i=0}^{\bar{m}_1}\sum_{j=0}^{\bar{m}_2} p\left[Y_k^1, Y_k^2, \theta_k^{i,j} | M_k^m, Z^{k-1}\right] \tag{29}$$

where

$$p\left[Y_k^1, Y_k^2, \theta_k^{i,j} | M_k^m, Z^{k-1}\right] = p\left[Y_k^1, Y_k^2 | M_k^m, \theta_k^{i,j}, Z^{k-1}\right] P[\theta_k^{i,j}]$$

$$= \begin{cases} \frac{(1-P_{D_1}P_{G_1})(1-P_{D_2}P_{G_2})}{[V_k^1]^{\bar{m}_1}[V_k^2]^{\bar{m}_2}}, \quad i=0,\ j=0 \\ \frac{(1-P_{D_1}P_{G_1})(P_{D_2}P_{G_2})/\bar{m}_2}{P_{G_2}[V_k^2]^{\bar{m}_2-1}} \times \mathcal{N}\left[\nu_k^{m,2(j)}; 0, S_k^{m,2}\right], \\ \quad i=0,\ j=1,...,\bar{m}_2 \\ \frac{(P_{D_1}P_{G_1})(1-P_{D_2}P_{G_2})/\bar{m}_1}{P_{G_1}[V_k^1]^{\bar{m}_1-1}} \times \mathcal{N}\left[\nu_k^{m,1(i)}; 0, S_k^{m,1}\right], \\ \quad i=1,...,\bar{m}_1,\ j=0 \\ \frac{(P_{D_1}P_{G_1})(P_{D_2}P_{G_2})/(\bar{m}_1\bar{m}_2)}{P_{G_1}[V_k^1]^{\bar{m}_1-1}P_{G_2}[V_k^2]^{\bar{m}_2-1}} \times \mathcal{N}\left[\nu_k^{m,i,j}; 0, S_k^m\right], \\ \quad i=1,...,\bar{m}_1,\ j=1,...,\bar{m}_2. \end{cases} \tag{30}$$

Using $\hat{\tilde{x}}_{k|k-1}^m$ (from (19)) and its covariance $\tilde{P}_{k|k-1}^m$ (from (20)), one computes the partial update $\hat{\tilde{x}}_{k|k}^m$ and its covariance $\tilde{P}_{k|k}^m$ according to the standard PDAF [1], except that the augmented state is conditioned on $\theta_k^{i,j}$ with data fusion from sensors 1 and 2. Define the combined mode-conditioned innovations

$$\nu_k^m := \sum_{i=0}^{\bar{m}_1}\sum_{j=0}^{\bar{m}_2} \beta_k^{m,i,j}\nu_k^{m,i,j}. \tag{31}$$

Therefore, partial update of the state estimate

$$\hat{\tilde{x}}_{k|k}^{m,i,j} := E\left\{x_k | \theta_k^{i,j}, M_k^m, Z^{k-1}, Y_k^1, Y_k^2\right\}$$

$$= \hat{\tilde{x}}_{k|k-1}^m + W_k^{m,i,j}\nu_k^{m,i,j} \tag{32}$$

where Kalman gains, $W_k^{m,i,j}$, are computed as

$$\begin{aligned} W_k^{m,0,0} &= 0, \qquad \text{for} \quad i=0,\ j=0 \\ W_k^{m,i,0} &= \tilde{P}_{k|k-1}^m[\tilde{H}_k^{m,1'}[S_k^{m,1}]^{-1} \ \ 0], \text{ for } i\neq 0,\ j=0 \\ W_k^{m,0,j} &= \tilde{P}_{k|k-1}^m[0 \ \ \tilde{H}_k^{m,2'}[S_k^{m,2}]^{-1}], \text{ for } i=0,\ j\neq 0 \\ W_k^{m,i,j} &= \tilde{P}_{k|k-1}^m\tilde{H}_k^{m'}[S_k^m]^{-1}, \text{ for } i\neq 0,\ j\neq 0, \end{aligned} \tag{33}$$

and $\tilde{H}_k^{m'} = \left[\tilde{H}_k^{m,1'} \ \ \tilde{H}_k^{m,2'}\right]$. Therefore, mode-conditioned update of the state estimate

$$\hat{\tilde{x}}_{k|k}^m := E\left\{x_k | M_k^m, Z^{k-1}, Y_k^1, Y_k^2\right\} = \sum_{i=0}^{\bar{m}_1}\sum_{j=0}^{\bar{m}_2} \beta_k^{m,i,j}\hat{\tilde{x}}_{k|k-1}^{m,i,j} \tag{34}$$

and covariance of $\hat{\tilde{x}}_{k|k}^m$, $\tilde{P}_{k|k}^m :=$

$$\begin{aligned} &\tilde{P}_{k|k-1}^m - \sum_{i=0,\ (i,j)\neq(0,0)}^{\bar{m}_1}\sum_{j=0}^{\bar{m}_2} \beta_k^{m,i,j}W_k^{m,i,j}S_k^{m,i,j}W_k^{m,i,j'} \\ &\quad + \sum_{i=0}^{\bar{m}_1}\sum_{j=0}^{\bar{m}_2} \beta_k^{m,i,j}W_k^{m,i,j}\nu_k^{m,i,j}\nu_k^{m,i,j'}W_k^{m,i,j'} \\ &- \left[\sum_{i=0}^{\bar{m}_1}\sum_{j=0}^{\bar{m}_2} \beta_k^{m,i,j}W_k^{m,i,j}\nu^{m,i,j}\right]\left[\sum_{i=0}^{\bar{m}_1}\sum_{j=0}^{\bar{m}_2} \beta_k^{m,i,j}W_k^{m,i,j}\nu^{m,i,j}\right]'. \end{aligned} \tag{35}$$

Step 5. Update of mode probabilities ($\forall m \in \mathcal{M}_n$) :

$$\mu_k^m := P\left[M_k^m | Z^k\right] = \frac{1}{C}\mu_k^{m-}\Lambda_k^m \tag{36}$$

where C is such that $\sum_m \mu_k^m = 1$.

Step 6. Combination of the mode-conditioned estimates ($\forall m \in \mathcal{M}_n$) : The final augmented state estimate update at time k is given by

$$\hat{\tilde{x}}_{k|k} = \sum\nolimits_m \hat{\tilde{x}}_{k|k}^m\mu_k^m \tag{37}$$

and its covariance is given by

$$\tilde{P}_{k|k} = \sum\nolimits_m \left\{\tilde{P}_{k|k}^m + \left[\hat{\tilde{x}}_{k|k}^m - \hat{\tilde{x}}_{k|k}\right]\left[\hat{\tilde{x}}_{k|k}^m - \hat{\tilde{x}}_{k|k}\right]'\right\}\mu_k^m. \tag{38}$$

From the final augmented state (see (37)), the state filtered vector $\hat{x}_{k|k}$ and the state smoothing vector $\hat{x}_{k-1|k}$ can be easily obtained.

5. SIMULATION EXAMPLE

The following example of tracking a highly maneuvering target in clutter is considered. The target starts at location [21689 10840 40] in Cartesian coordinates in meters. The initial velocity (in m/s) is [-8.3 -399.9 0] and the target stays at constant altitude with a constant speed of 400 m/s. Its trajectory is a straight line with constant velocity between 0 and 20s, a coordinated turn (0.15 rad/s) with constant acceleration of 60 m/s^2 between 20 and 35s, a straight line with constant velocity between 35 and 55s, a coordinated turn (0.1 rad/s) with constant acceleration of 40 m/s^2 between 55 and 70s, and a straight line with constant velocity between 70 and 90s. The target motion models are patterned and modified after [1]. In each mode the target dynamics are modeled in Cartesian coordinates as

$$\tilde{x}_k = \tilde{F}_{k,k-1}^m\tilde{x}_{k-1} + \tilde{G}_{k,k-1}^m\tilde{v}_k^m, \tag{39}$$

$$\tilde{x}_{k_d} = \tilde{F}_{k_d,k-1}^m\tilde{x}_{k-1} + \tilde{G}_{k_d,k-1}^m\tilde{v}_k^m \tag{40}$$

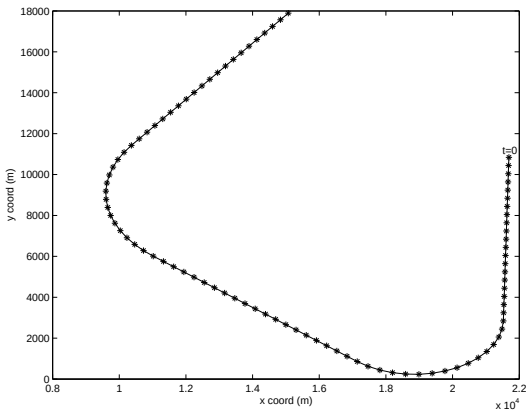

Figure 1. True trajectory (xy position) of the maneuvering target.

where the augmented state of the target consists of position, velocity, acceleration, and the process noise in each of the three Cartesian coordinates (x, y, and z) at t_k and t_{k-1} as well as the delay time d_k at t_k. Thus both $\tilde{x}_k$ and $\tilde{x}_{k_d}$ are of dimension 25 ($n_x = 25$). Three maneuver models are considered in the following discussion. The system matrices $\tilde{F}_{k,k-1}$, $\tilde{G}_{k,k-1}$, $\tilde{F}_{k_d,k-1}$ and $\tilde{G}_{k_d,k-1}$ are defined as

$$\tilde{F}^m_{k,k-1} = \begin{bmatrix} \bar{F}^m_{k,k-1} & 0 \\ 0 & I \end{bmatrix}, \ \tilde{G}^m_{k,k-1} = \begin{bmatrix} \bar{G}^m_{k,k-1} & 0 \\ 0 & I \end{bmatrix} \quad (41)$$

$$\tilde{F}^m_{k_d,k-1} = \begin{bmatrix} \bar{F}^m_{k_d,k-1} & 0 \\ 0 & I \end{bmatrix}, \ \tilde{G}^m_{k_d,k-1} = \begin{bmatrix} \bar{G}^m_{k_d,k-1} & 0 \\ 0 & I \end{bmatrix} \quad (42)$$

where

$$\bar{F}^m_{k,k-1} = \begin{bmatrix} F^m_{k,k-1} & G^m_{k,k-1} \\ 0 & 0 \end{bmatrix}, \ \bar{G}^m_{k,k-1} = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad (43)$$

$$\bar{F}^m_{k_d,k-1} = \begin{bmatrix} F^m_{k_d,k-1} & G^m_{k_d,k-1} \\ 0 & 0 \end{bmatrix}, \ \bar{G}^m_{k_d,k-1} = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad (44)$$

$F^m_{k,k-1}$ =block-diag$\{F^m, F^m, F^m\}$, $G^m_{k,k-1}$ =block-diag$\{G^m, G^m, G^m\}$, $F^m_{k_d,k-1}$ =block-diag$\{F^m_d, F^m_d, F^m_d\}$, $G^m_{k_d,k-1}$ =block-diag$\{G^m_d, G^m_d, G^m_d\}$.

Model 1. Nearly constant velocity model with zero mean perturbation in acceleration:

$$F^1 = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ G^1 = \begin{bmatrix} \frac{T^2}{2} \\ T \\ 0 \end{bmatrix}, \quad (45)$$

$$F^1_d = \begin{bmatrix} 1 & (T-d_k) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ G^1_d = \begin{bmatrix} \frac{(T-d_k)^2}{2} \\ (T-d_k) \\ 0 \end{bmatrix} \quad (46)$$

where T is the sampling period. The standard deviation of the process noise of M^1 is 5 m/s^2 (as in [1]).

Model 2. Wiener process acceleration (nearly constant acceleration motion)

$$F^2 = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \ G^2 = \begin{bmatrix} \frac{T^2}{2} \\ T \\ 1 \end{bmatrix}, \quad (47)$$

$$F^2_d = \begin{bmatrix} 1 & (T-d_k) & \frac{(T-d_k)^2}{2} \\ 0 & 1 & (T-d_k)) \\ 0 & 0 & 1 \end{bmatrix}, \ G^2_d = \begin{bmatrix} \frac{(T-d_k)^2}{2} \\ (T-d_k) \\ 1 \end{bmatrix} \quad (48)$$

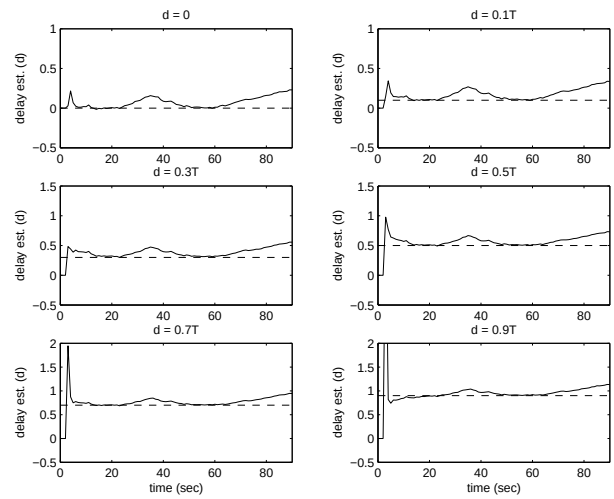

Figure 2. Estimation of delay (given unknown but fixed timing mismatch between two separated sensors) based on 100 Monte Carlo runs (read left to right, top to bottom). (a) d = 0. (b) d = 0.1T. (c) d = 0.3T. (d) d = 0.5T. (e) d = 0.7T. (f) d = 0.9T. (T = sampling rate). Solid curves: estimate averaged over 100 runs; dashed curves: true values.

The standard deviation of the process noise of M^2 is 7.5 m/s^2 (as in [1]).

Model 3. Wiener process acceleration (model with large acceleration increments, for the onset and termination of maneuvers), with $F^3 = F^2$, $G^3 = G^2$, $F^3_d = F^2_d$ and $G^3_d = G^2_d$. The standard deviation of the process noise of M^3 is 40 m/s^2 (as in [1]).

The initial model probabilities are $\mu^1_0 = 0.8$, $\mu^2_0 = 0.1$ and $\mu^3_0 = 0.1$. The mode switching probability matrix is given by (as in [1])

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.0 & 0.2 \\ 0.0 & 0.8 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}. \quad (49)$$

The Sensors: Two sensors are used to obtain the measurements. Sensors 1 and 2 are located at $[x_1, y_1, z_1]$=[-4000 4000 0] m and $[x_2, y_2, z_2]$=[5000 0 0] m, respectively, and the central processor is collocated with sensor 1 platform (we assume that there is no time delay between sensor 1 and central processor and there is fixed but unknown time delay between sensor 2 and central processor). The measurements from sensor l for model m are $z^l_k = h^{m,l}(x_k) + w^{m,l}_k$ for $l = 1$ and 2, reflecting range and azimuth angle for sensor 1 (radar) and azimuth and elevation angles for sensor 2 (infrared). The range, azimuth, and elevation angle transformations, respectively, would be given by $r_l = \{(x-x_l)^2 + (y-y_l)^2 + (z-z_l)^2\}^{1/2}$, $a_l = \tan^{-1}[(y-y_l)/(x-x_l)]$, $e_l = \tan^{-1}[(z-z_l)/\{(x-x_l)^2 + (y-y_l)^2\}^{1/2}]$, respectively, if the sensor l were located at $[x_l \ y_l \ z_l]$. As we see from (1), (2), (4) and (5), the measurements obtained from sensors 1 and 2 can be expressed as

$$z^1_k = h^1([I, 0, 0, 0, 0]\tilde{x}_k) + w^1_k \quad (50)$$

$$z^2_k = h^2([0, 0, F^m_{k_d,k-1}, G^m_{k_d,k-1}, 0]\tilde{x}_k) + w^2_k. \quad (51)$$

The measurement noise $w^{m,l}_k$ for sensor l is assumed to be zero-mean white Gaussian with known covariances, $R^1 =$

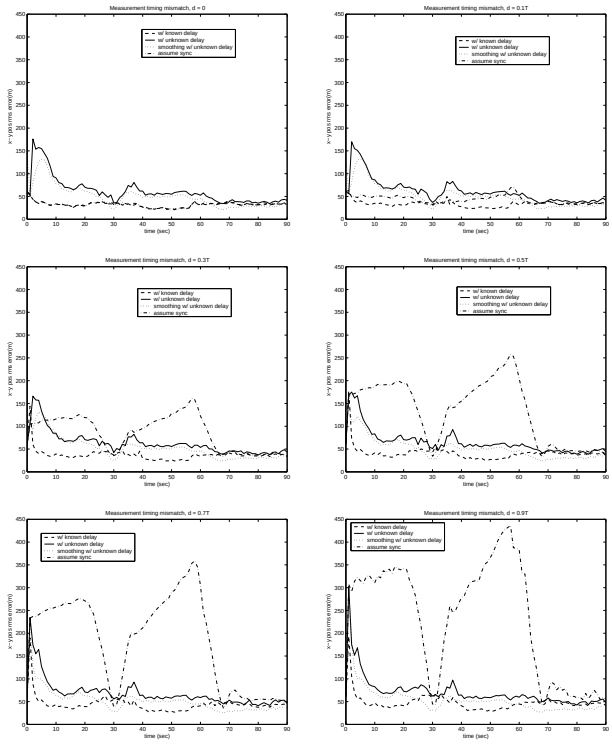

Figure 3. RMSE in position using IMMMSPDAF under various scenarios of known delay, estimated delay and ignoring delay, for various delay values (read left to right, top to bottom). (a) d = 0. (b) d = 0.1T. (c) d = 0.3T. (d) d = 0.5T. (e) d = 0.7T. (f) d = 0.9T. (T = sampling rate). Solid curves: unknown delay, proposed approach with filtered state estimate; dotted curves: unknown delay, proposed approach with one-sample fixed-lag smoothed state estimate; dashed curves: IMMMSPDAF with known delay; dot-dash curves: IMMMSPDAF with delay assumed to be zero.

$\text{diag}[q_r, q_{a1}] = \text{diag}[400\text{m}^2, 49\text{mrad}^2]$ with q_r and q_{a1} denoting the variances for the radar range and azimuth measurement noises, respectively, and $R^2 = \text{diag}[q_{a2}, q_e] = \text{diag}[4\text{mrad}^2, 4\text{mrad}^2]$ with q_{a2} and q_e denoting the variances for the infrared sensor's azimuth and elevation measurement noises, respectively. The sampling interval was T=1s and it was assumed that the probability of detection P_D=1 for both sensors.

The Clutter: For generating false measurements in simulations, the clutter was assumed to be Poisson distributed with expected number of $\lambda_1 = 13\times10^{-6}$/m-mrad for sensor 1 and $\lambda_2 = 7\times10^{-4}/\text{m}^2$-mrad for sensor 2 [1, case 1]. These statistics were used for generating the clutter in all simulations. However, a nonparametric clutter model was used for implementing all the algorithms for target tracking.

Other Parameters: The gates for setting up the validation regions for both the sensors were based on the threshold γ=16. With the measurement vector of dimension 2, this leads to a gate probability P_G=0.997.

Simulation Results: The results were obtained from 100 Monte Carlo runs. Fig. 1 shows the true trajectory of the target. Fig. 2 shows the delay estimates (given unknown but fixed timing mismatch between the two sensors) based on 100 Monte Carlo runs. Fig. 3 shows a comparison among the performances (root mean-square error in target positions) of the proposed IMMMSPDAF algorithm dealing with asynchronous measurements with unknown but fixed d_k, with known d, and the standard IMMMSPDAF algorithm with the assumption that d=0 always applies. It is seen from Fig. 3 that when the unknown but fixed timing mismatch d_k is more than one fifth of the sampling time, the performance improvement is significant compared with the standard IMMMSPDAF algorithm that ignores the time-delay d (i.e. assumes it to be zero).

6. CONCLUSIONS

We investigated an IMMMSPDAF algorithm with asynchronous measurements (there is an unknown but fixed timing mismatch between sensor platforms) for tracking a highly maneuvering target in clutter. The proposed algorithm was illustrated via a simulation example where it outperformed a standard IMMMSPDAF algorithm that ignored the possible timing mismatch (especially when the possible timing mismatch is more than one fifth of the sampling interval).

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