

Systematic Configuration Procedure of LMI-Based Linear Anti-windup Synthesis

Dan Dai and Jingcheng Wang

Abstract—In this paper, a novel systematic configuration procedure in choosing parameters is presented for the synthesis of linear anti-windup scheme by revising the original goal of the unified anti-windup framework. The anti-windup controller is derived from the parameters, which are characterized by the sufficient conditions for the stability and performance objective as to protect the system from adverse effects in present of saturation and recover the energy deficit after saturation. Moreover, the entire synthesis is cast as sub-optimization problems over Linear Matrix Inequalities (LMIs) and the effectiveness of the result is shown via simulation examples with comparison to other existing anti-windup schemes.

I. INTRODUCTION

THE notion of ‘linear anti-windup design’ has played a very important role in the study of systems with actuator saturation. The first paradigm in this field was stated in [5], where all previous linear anti-windup designs were unified in a general framework, shown in special cases in terms of two matrix parameters as choices for left coprime factorizations of the linear controller. Then, a further result pointed out in [6], which developed sufficient conditions to guarantee stability for the general framework. Recently, more linear anti-windup schemes have been proposed, providing desirable stability properties as well as performance achievements (see e.g., [3,7,8,9,11]). Without going to the detail about these approaches, our work has focused on the original goal of the unified general framework, and obtained a series of encouraging results (see, [2]). In [2], by assuming one of the two parameters to be zero, stability condition and performance objectives are derived in the form of LMIs. These results give light to systematic procedures in choosing parameters for the synthesis of anti-windup controllers.

In this paper, based on the results in [2], the novel systematic configuration procedures in choosing parameters

are integrated, which is characterized by the sufficient conditions for stability and performance objectives as to protect the system from adverse effects in present of saturation and recover the energy deficit after saturation. Moreover, the anti-windup controller is derived by the parameters, and the entire synthesis is cast as a sub-optimization problem over LMIs.

Notation. The saturation nonlinearity is described as

$$\Phi = \text{diag}(\Phi_i)$$

$$\Phi_i(\sigma) = \begin{cases} \sigma(u_{\min}^i \leq \sigma \leq u_{\max}^i) \\ u_{\min}^i (\sigma \leq u_{\min}^i) \\ u_{\max}^i (\sigma \geq u_{\max}^i) \end{cases}$$

$$i=1, \dots, m; u_{\min} < 0, \text{ and } u_{\max} > 0$$

II. PROBLEM DEFINITION

Recall from [4] that all known linear anti-windup schemes can be unified as the modified controller:

$$\hat{K} = \begin{bmatrix} N(s) & R(s) \end{bmatrix} \quad (1)$$

where

$$N(s) = \begin{bmatrix} A_k - H_1 C_k & B_k - H_1 D_k \\ H_2 C_k & H_2 D_k \end{bmatrix}$$

$$R(s) = \begin{bmatrix} A_k - H_1 C_k & H_1 \\ H_2 C_k & I - H_2 \end{bmatrix} \quad (2)$$

$N(s)$ and $I-R(s)$ correspond to left coprime factors of the original linear controller $K(s)$ as:

$$K(s) = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} = (I - R(s))^{-1} N(s).$$

Insert the saturation function Φ in the controller as shown in Fig. 1.

When there is no saturation, ($\Phi=I$), controller will be the

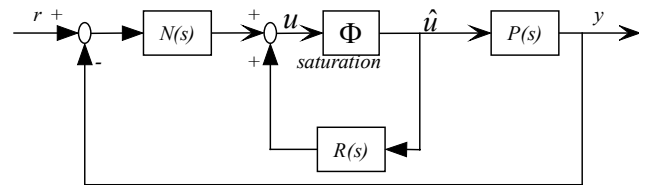


Fig. 1. The anti-windup controller structure linear controller. When saturation occurs, ($\Phi \neq I$), the

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feedback by linear controller does not work, and the anti-windup problem has deduced to select suitable parameter, H_1 and H_2 , in anti-windup forward controller $N(s)$ and dynamic compensator $R(s)$ to stabilize the closed-loop system and provide graceful performance degradation.

For further analysis, substitute Fig.1 with Fig.2 (a).

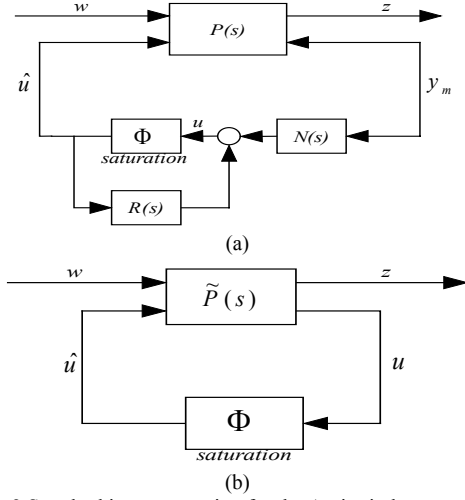


Fig.2 Standard interconnection for the Anti-windup structure

The Fig.2 (a) is equivalent to the Fig.2 (b). Then the interconnection of Fig. (b) can be concisely written as

$$\begin{aligned} \dot{x} &= Ax + B_w w + B_u \hat{u} \\ u &= H_2 C_u x + H_2 D_{uw} w + (I - H_2 D_{uu}) \hat{u} \\ z &= C_z x + D_{zw} w + D_{zu} \hat{u} \end{aligned} \quad (3)$$

Where A , B_w , $B_u \hat{u}$, C_u , D_{uw} , D_{uu} , C_z , D_{zw} , and $D_{zu} \hat{u}$ are matrices of suitable dimensions. In particular, H_1 acts independently on modifying the state equation via A , B_w , and $B_u \hat{u}$ and H_2 does that on the output equation.

III. LMI-BASED ANTI-WINDUP SYNTHESIS

A. Anti-windup performance

Firstly, consider a typical case that the system of Fig.1 is saturated by step disturbance and/or step reference input. Suppose that saturation starts at $t=0$, and the effects of the internal states on the response are denoted as $\tilde{P}(s)x_p(0)$ and $\tilde{K}(s)x_k(0)$. After the saturation occurs, the response of the difference between u^{\wedge} and u is represented by

$$\begin{aligned} u(s) - \hat{u}(s) &= N(s) \left\{ \frac{r_0}{s} - P(s)\hat{u}(s) + \tilde{P}(s)x_p(0) \right\} \\ &+ (R(s) - I)\hat{u}(s) + \tilde{K}(s)x_k(0) \end{aligned} \quad (4)$$

For the purpose of protecting the system from severe overshoot in plant output, the unconstrained signal u must be protected from increasing awfully. Since u^{\wedge} is bounded and other variables are finite, an ideal solution is $N(s)=\varepsilon$ and $R(s)=I-\varepsilon$ (if $\varepsilon \approx 0$), which satisfied $N(s)=(I-R(s))*K(s)$. So the performance objective is defined as the limitation of

the L_2 norm of the anti-windup forward controller $N(s)$ from its input, z , to its output, y_N

$$\sup_{\|z\|_2 \neq 0} \frac{\|y_N\|_2}{\|z\|_2} \quad (5)$$

Theorem 1. The anti-windup forward controller $N(s)$ in Fig.1 is asymptotic stabilizable and has a weighted induced ℓ_2 gain $\frac{\|y_N\|_2}{\|z\|_2} < \varepsilon_N$, which is a pre-defined upper bound

$1 \geq \varepsilon_N \geq 0$, if there exist matrix $Q=Q^T > 0$, such that the following LMI with respect to Q , Y , H_2 is satisfied:

$$\begin{bmatrix} A_k^T Q + Q A_k - C_k^T Y^T - Y C_k & Q B_k - Y D_k & C_k^T H_2^T \\ (Q B_k - Y D_k)^T & -\varepsilon_N I & D_k^T H_2^T \\ H_2 C_k & H_2 D_k & -\varepsilon_N I \end{bmatrix} < 0 \quad (6)$$

If the LMI is feasible, then $H_1=Q^{-1}Y$ and H_2 are the expected parameters to construct the anti-windup controller.

Proof. See the appendix.

In addition to protect the system from severe overshoot in plant output, the anti-windup controller is also required to guarantee the stability of the closed-loop system, and recover the energy deficit in present of saturation, so as to provide satisfied performance degradation. Therefore, by choosing the energy related form in ℓ_2 norm, the performance objective is defined as the weighted induced ℓ_2 norm from the exogenous input, w , to the deviation

between u^{\wedge} and u : $\sup_{\|w\|_2 \neq 0} \frac{\|u\|_2 - \|\hat{u}\|_2}{\|w\|_2}$.

Definition 1. (Memoryless time-varying nonlinearities)

Define the set N_{TV} of all allowable structured memoryless time-varying nonlinearities as follows:

$$\begin{aligned} N_{TV} &= \{N : \mathfrak{R}^{n_u} \times \mathfrak{R} \rightarrow \mathfrak{R}^{n_u} \mid N(0, t) = 0, \forall t \geq 0, \\ &N = \text{diag}\{N_1, N_2, \dots, N_{n_u}\}, N_i \in \text{sector}[0, 1]\}. \end{aligned}$$

Obviously, N_{TV} typically includes the saturation nonlinearity Φ presented by Notation.

Theorem 2. (ℓ_2 gain criterion). The anti-windup system (3) in Fig.1(b) is ℓ_2 stabilizable for all $N \in N_{TV}$ and has a weighted induced ℓ_2 gain $\frac{\|u\|_2 - \|\hat{u}\|_2}{\|w\|_2} \leq \sqrt{\lambda}$, which is a

pre-defined upper bound, if there exist $\delta > 0$, $P=P^T > 0$, with $W = \text{diag}(W_1, W_2, \dots, W_{n_u}) \in \mathfrak{R}^{n_u \times n_u} > 0$, and $X=WH_2$ such that the following LMI with respect to P , X , W , δ is satisfied:

$$\begin{bmatrix} A^T P + PA & PB_w & PB_{\hat{u}} + C_u^T X^T \\ B_w^T P & -\lambda W & D_{uw}^T X^T \\ B_{\hat{u}}^T P + XC_u & XD_{uw} & \delta I - XD_{\hat{u}\hat{u}} - D_{\hat{u}\hat{u}}^T X^T - W \\ XC_u & XD_{uw} & W - XD_{\hat{u}\hat{u}} \\ C_u^T X^T \\ D_{uw}^T X^T \\ W - D_{\hat{u}\hat{u}}^T X^T \\ -W \end{bmatrix} < 0 \quad (7)$$

The upper bound on the weighted ℓ_2 gain can be obtained from the definition of λ . $H_2 = W^{-1}X$ is the proposed solution to the anti-windup problem.

Proof. See the appendix.

B. Systematic Procedure in Anti-windup Synthesis

Although conditions in (6) and (7) give encouraging results to avoid adverse effects when saturation occurs, and pump necessary energy to recover the deficit after saturation, there are some limitations. Considering H_1 acting on the state equation in the closed-loop system in (3), it is difficult to arise LMI formulation for performance requirements in (7); on the other hand, the solutions in (6) only focus on improving the behavior of the anti-windup controller and lack of the consideration of the behavior of the entire closed-loop system. Therefore, a configuration procedure is presented to solve the problems and provide satisfied combination in (6) and (7).

Step 1. Solve the anti-windup controller condition

Given the linear controller $K(s)$ and plant with a proper smaller real scalar $1 \geq \varepsilon_N \geq 0$, determine a solution H_1 that satisfies condition (6).

Step 2. Modify the state equation of the closed-loop system

Given one parameter H_1 from step 1, modify A , B_w , and $B_{\hat{u}}$ in the state equation of the closed-loop system (3).

Step 3. Solve the stability and performance conditions in LMIs

Define the upper bound ℓ_2 gain $\sqrt{\lambda} \geq 0$, and determine a solution H_2 that satisfies condition (7), which guarantees desirable stability properties and anti-windup performance in the closed-loop system.

Step 4. Construct the anti-windup compensator

Given the parameters determined in step 1 and 3, construct the anti-windup forward controller $N(s)$ and feedback compensator $R(s)$ in the anti-windup controller as $\hat{K} = [N(s) \ R(s)]$.

Step 5. Validate this anti-windup controller

Given $N(s)$ and $R(s)$ determined in step 4, compute the scalar ε_N again, which is normally not identical with that pre-defined ε_N in step 1, and prove that it is properly small as expected in condition (4).

IV. COMPARISON TO OTHER ANTI-WINDUP METHODS

A. Example 1

Consider the following plant and linear controller taken from [7,10] as: $P(s) = \frac{10}{100s+1} \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$,

$$K(s) = \left(1 + \frac{1}{100s}\right) \begin{bmatrix} 2 & 2.5 \\ 1.5 & 2 \end{bmatrix}.$$

A set-point change of $[0.63 \ 0.79]^T$ is applied to the system at $t=0$, with a saturation limits of ± 2 on the controller output. To compare our method to other anti-windup methods such as the static compensator in [7] and the anti-windup IMC [4,10], the performance index is listed in Table.1.

TABLE I
PERFORMANCE INDEX INDUCED BY DIFFERENT ANTI-WINDUP STRUCTURES
1. UNCONSTRAINED; 2. CONVENTIONAL IMC; 3. SPECIAL IMC; 4. STATIC COMPENSATOR; 5. PROPOSED METHOD IN THIS PAPER

Method	$\ \hat{u}\ _2 / \ u\ _2 \begin{cases} input1 \\ input2 \end{cases}$	$\ e\ _2 \begin{cases} input1 \\ input2 \end{cases}$
1	44.4133/44.4133	6.3158
	34.6657/34.6657	7.9198
	41.9931/47.4444	33.6747
2	34.1193/35.2407	15.0941
	43.6209/49.8711	25.0804
3	34.0869/34.1251	31.1176
	41.9931/44.9290	7.0877
4	32.7767/32.7767	8.8853
	41.9931/42.0128	7.3850
5	32.7896/32.7896	8.6496

By setting $\varepsilon_N = \text{diag}(0.001, 0.001)$ and $\lambda = 0.003$

previously, parameters $H_1 = \begin{bmatrix} 8.0001 & -10.0001 \\ -6.0000 & 8.0001 \end{bmatrix}$

and $H_2 = \begin{bmatrix} 0.1866 & -0.2361 \\ -0.2021 & 0.2614 \end{bmatrix}$ are determined from (6)

and (7), respectively. Finally, compute the scalar ε_N again,

which is determined as $\varepsilon_N = \begin{bmatrix} 0.0125 & 0 \\ 0 & 0.0072 \end{bmatrix}$. It is small

enough and satisfied the condition (4) as expected.

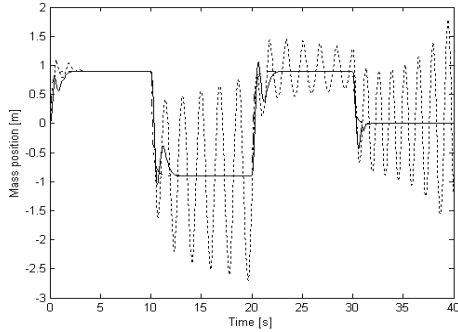
From the analysis of Table.1, the method in this paper is superior to IMC methods [4,10] obviously and its performance results are parallel to the static anti-windup compensator. However, the limitation of the static compensator synthesis technique is that the LMI constraints are not always feasible. Our method is more flexible to get feasible sub-optimal solutions from the conditions at the expense of the optimal solution. This will be proven in the following example where the linear static anti-windup

compensator is not feasible for it.

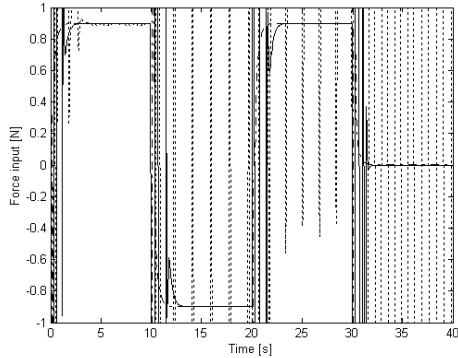
B. Example 2

The typical example as a damped mass-spring system is taken from [3]. By selecting $\varepsilon_N = 0.2$, $H_1 = [0.0052 \ 0.0001]^T$ is determined in condition (6). Then by setting $w=r$, $z=e$, and a smaller upper bound as $\lambda = 0.04$, parameter $H_2 = 1.9090 * 10^{-5}$ is determined from (7).

The response of unconstrained, constrained and anti-windup used method are shown in Fig.2. Our method performs satisfactorily, parallel to the method in [3, 11], and confirms the effectiveness of the systematic procedures to choose parameters.



(a) Output of the Plant



(b) Input of the Plant

Fig.3 Example plant response: dashed, unconstrained; dotted, constrained without anti-windup; solid, anti-windup method in this paper

V. CONCLUSION

Motivated partly from the ideas in unified anti-windup framework and extended the work in [2], the linear anti-windup paradigm proposed in this paper focuses on the development of systematic procedures to choose the two matrix parameters for the synthesis of anti-windup controller. And the performance of anti-windup synthesis is characterized by the ℓ_2 norm of the deviation between the controller output and plant input, which is related to the energy of the unconstrained signal and the constrained signal. Therefore, the proposed anti-windup controller can

protect the system from increasing awfully in the unconstrained signal (controller output), and recover the energy deficit in the constrained signal (plant input) in relation to the unconstrained control. Moreover, the performance objectives are shown to induce sub-optimal problem over LMIs. From the comparison between the proposed method and other existing anti-windup methods via simulation examples, our method shows graceful performance degradation, parallel to if not more effective than other schemes, in the presence of saturation, and confirms the success of the systematic procedures to choose parameters.

APPENDIX

Proof of Theorem 1. By applying a simple congruence transformation $\text{diag}\{ \varepsilon_N^{1/2} I, \varepsilon_N^{1/2} I, \varepsilon_N^{-1/2} I \}$ to (6), and defining $M = \varepsilon_N Q$, and a suitable small real scale $0 < \gamma < 1$, condition (6) is guaranteed if there exists a matrix $M > 0$, such that

$$\begin{bmatrix} (A_k - H_1 C_k)^T M + M(A_k - H_1 C_k) & M(B_k - H_1 D_k) \\ (B_k - H_1 D_k)^T M & -\varepsilon_N^2 (1 - \gamma) I \\ H_2 C_k & H_2 D_k \\ C_k^T H_2^T \\ D_k^T H_2^T \\ -I \end{bmatrix} < 0 \quad (A1)$$

It implies $(A_k - H_1 C_k)^T M + M(A_k - H_1 C_k) < 0$. Since $M > 0$, the forward controller $N(s)$ is asymptotically stable and $V(x) = x^T(t) M x(t)$ is a Lyapunov function of the system.

If $T > 0$ and the initial condition is assumed as $x_0 = 0$, get

$$\begin{aligned} J &= \int_0^T \|z(t)\|^2 dt - (1 - \gamma) \varepsilon_N^2 \int_0^T \|w(t)\|^2 dt \\ &= \int_0^T [z^T(t) z(t) dt - (1 - \gamma) \varepsilon_N^2 w^T(t) w(t) + \frac{d}{dt} V(x)] dt - V(x(T)) \\ &= \int_0^T \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} (H_2 C_k)^T \\ (H_2 D_k)^T \end{bmatrix} \begin{bmatrix} H_2 C_k & H_2 D_k \end{bmatrix} \\ &\quad + \begin{bmatrix} (A_k - H_1 C_k)^T M + M(A_k - H_1 C_k) & M(B_k - H_1 D_k) \\ (B_k - H_1 D_k)^T M & -\varepsilon_N^2 (1 - \gamma) I \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \\ &\quad - V(x(T)) \end{aligned}$$

If considering (A1), and using the initial condition, $T \rightarrow \infty$, get

$$\|z(t)\|^2 \leq (1 - \gamma) \varepsilon_N^2 \|w(t)\|^2 < \varepsilon_N^2 \|w(t)\|^2 \quad \square$$

Proof of Theorem 2.

Lemma 1 (Multiloop circle criterion in [2]). Let $\tilde{P}_{-11}(s)$ denote the transfer function relating $-\hat{u}$ to u in Fig.2(b). Then, the closed-loop in Fig.1 is ℓ_2 stabilizable for $\Phi \in \mathcal{N}_{TV}$ if

1. $\tilde{P}(s)$ in Fig.1(b) is asymptotically stable; and

2. there exist $W = \text{diag}(W_1, W_2, \dots, W_{n_u}) \in \mathfrak{R}^{n_u \times n_u} > 0$, with $P = P^T > 0$, $\delta > 0$, and $X = WH_2$, such that the following LMI with respect to P, X, δ is satisfied

$$\begin{bmatrix} A^T P + PA & -PB_{\hat{u}} - C_u^T X^T \\ -B_{\hat{u}}^T P - XC_u & \delta I - XD_{\hat{u}\hat{u}} - D_{\hat{u}\hat{u}}^T X^T \end{bmatrix} < 0 \quad (\text{B1})$$

Remark 1. Obviously, (B1) is guaranteed by the main result in (7).

Lemma 2. (Lyapunov stability criterion in [6]). Define Lyapunov function, which is equivalent to (B1) as:

$$V(x) = x^T P x + \int_0^t \hat{u}^T(\tau) \delta \hat{u}(\tau) d\tau + 2 \sum_{i=1}^{n_u} W_i \int_0^t (\hat{u}_i(\tau) u_i(\tau) - \hat{u}_i(\tau) \hat{u}_i(\tau)) d\tau \quad (\text{B2})$$

Then, the closed-loop system in Fig. 2(b) is L_2 stabilizable for all $N \in N_{TV}$, if there exist $P = P^T > 0$, $\delta > 0$, and $W = \text{diag}(W_1, W_2, \dots, W_{n_u}) \in \mathfrak{R}^{n_u \times n_u} > 0$, such that: $V(x) > 0$

for $x \neq 0$ and $V(0) = 0$; $V(x)$ satisfies $\frac{d}{dt} V(x) < 0$, for $x \neq 0$.

Remark 2. $V(x) > 0$ for $x \neq 0$ is guaranteed by the $P > 0$, $\delta > 0$, and $W > 0$; $V(0) = 0$ is obviously; $\frac{d}{dt} V(x) < 0$, for $x \neq 0$ is equivalent to the condition (B1), and thus guaranteed by (7).

Therefore, (7) guarantees the stability of the closed-loop system. Now, the main result about performance in condition (7) is stated. Condition (7) is equivalent to the existence of a matrix $W = \text{diag}(W_1, W_2, \dots, W_{n_u}) \in \mathfrak{R}^{n_u \times n_u} > 0$, with $\delta > 0$, $P = P^T > 0$, and λ such that

$$\frac{d}{dt} \left\{ x^T P x + \int_0^t \hat{u}^T \delta \hat{u} d\tau + 2 \sum_{i=1}^{n_u} W_i \int_0^t (\hat{u}_i u_i - \hat{u}_i \hat{u}_i) d\tau \right\} + W \left\{ u^T u - \hat{u}^T \hat{u} \right\} - \lambda w^T w < 0 \quad (\text{B3})$$

Integrate (B3) from 0 to t with the initial condition $x_0 = 0$, and get

$$V(x) + \sum_{i=1}^{n_u} W_i \int_0^t [(u_i u_i - \hat{u}_i \hat{u}_i) - \lambda w_i w_i] d\tau \leq 0$$

Since $V(x) > 0$, it implies $\frac{\|u\|_2^2 - \|\hat{u}\|_2^2}{\|w\|_2^2} \leq \lambda$.

$$\|u\|_2 \geq \|\hat{u}\|_2 \geq 0 \Leftrightarrow \frac{\|u\|_2 - \|\hat{u}\|_2}{\|w\|_2} \leq \sqrt{\frac{\|u\|_2^2 - \|\hat{u}\|_2^2}{\|w\|_2^2}} \leq \sqrt{\lambda}$$

which satisfies the performance objective.

Condition (B3) is guaranteed for all $(x, w, u) \neq 0$ if

$$J = \frac{d}{dt} \left\{ x^T P x + \int_0^t \hat{u}^T \delta \hat{u} d\tau + 2 \sum_{i=1}^{n_u} W_i \int_0^t (\hat{u}_i u_i - \hat{u}_i \hat{u}_i) d\tau \right\} + W \left\{ u^T u - \hat{u}^T \hat{u} \right\} - \lambda w^T w < 0$$

$$= \begin{bmatrix} x & w & \hat{u} \end{bmatrix} \begin{bmatrix} A^T P + PA & PB_w & PB_{\hat{u}} + C_u^T H_2^T W \\ B_w^T P & -\lambda W & D_{uw}^T H_2^T W \\ WH_2 C_u + B_{\hat{u}}^T P & WH_2 D_{uw} & \beta \end{bmatrix} \begin{bmatrix} x \\ w \\ \hat{u} \end{bmatrix} < 0$$

$$- \begin{bmatrix} C_u^T H_2^T \\ D_{uw}^T H_2^T \\ (I - H_2 D_{\hat{u}\hat{u}})^T \end{bmatrix} (-W^{-1})^{-1} \begin{bmatrix} C_u^T H_2^T \\ D_{uw}^T H_2^T \\ (I - H_2 D_{\hat{u}\hat{u}})^T \end{bmatrix}^T < 0$$

$$\beta = \delta I - WH_2 D_{\hat{u}\hat{u}} - D_{\hat{u}\hat{u}}^T H_2^T W - W$$

Because $J < 0$, the above matrix is negative definite, the equivalent condition is obtained as follows by applying Schur complement:

$$\begin{bmatrix} A^T P + PA & PB_w & PB_{\hat{u}} + C_u^T H_2^T W \\ B_w^T P & -\lambda W & D_{uw}^T H_2^T W \\ B_{\hat{u}}^T P + WH_2 C_u & WH_2 D_{uw} & \beta \\ H_2 C_u & H_2 D_{uw} & I - H_2 D_{\hat{u}\hat{u}} \end{bmatrix} < 0 \quad (\text{B4})$$

$$\begin{bmatrix} C_u^T H_2^T \\ D_{uw}^T H_2^T \\ I - D_{\hat{u}\hat{u}}^T H_2^T \\ -W^{-1} \end{bmatrix} < 0$$

By applying a simple congruence transformation $\text{diag}(I, I, I, W)$ to (B4) and defining $X = WH_2$, (7) will be obtained. \square

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