FAULT DIAGNOSIS IN A CLASS OF NONLINEAR SYSTEMS USING IDENTIFICATION AND GLR TESTING

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Abstract: In a class of nonlinear systems, additive faults act as changes in the state-transition matrix. Each fault can be characterized by a trajectory in the parameter space. Identifying the parameters online and comparing them to pre-computed trajectories offers an approach to fault isolation. The distance between the trajectories and the observations is subjected to Generalized Likelihood Ratio (GLR) testing. As a key element in this procedure, the best parameter estimation technique has been selected by comparing some typical estimation methods. Simulation studies are included to support the theoretical conclusions.

1. INTRODUCTION

Most model based fault detection and diagnosis methods rely on the concept of analytical redundancy. Its main idea is the comparison of the measured outputs with the predictions computed from a mathematical model. The differences, expressed as residuals, are the indication of the presence of faults, as well as possible disturbances, noise and modeling errors. Approaches in this framework include parity relations, diagnostic observers, Kalman filters, parameter estimation etc (see e.g. Gertler, 1991).

Residuals are usually enhanced to support the isolation of faults. Any a priori information on the nature of faults may be utilized in residual enhancement. While in a linear system residuals may be designed to exhibit directional properties in response to a particular fault, in a nonlinear setting specific trajectories may be associated with faults.

In a class of nonlinear systems, faults appear as changes in the state transition matrix, leading to fault-specific trajectories in the space of the parameters of the characteristic polynomial. The comparison of the distance between the observed data and the parameter trajectories, pre-computed from of the fault model, offers an isolation scheme.

The actual parameters are obtained on-line, with some systems identification method. With the system subject to random noise, the comparison of distances needs to be done in a statistical framework. Relying on a Gaussian assumption, a Generalized Likelihood Ratio (GLR) approach has been developed which leads to easy χ^2 testing. Similar GLR ideas have been applied to linear systems in (Willsky and Jones, 1976), and (Gertler, 1998).

In this framework, the fault diagnosis performance highly depends on the accuracy of the parameter estimation. We will select the appropriate estimation technique by comparing the performance of some typical algorithms, including input-output and subspace identification methods.

Simulation studies on two artificial systems, one second order and one fourth order, will be presented. A rough picture of how the GLR-FDD procedure works will be shown on the first system, and then some more delicate issues will be discussed in connection with the second system.

2. PROBLEM FORMULATION

2.1 System Description

Consider a nonlinear state-space system:

$$\begin{cases} \mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\varphi(t) + \mathbf{\Psi}(t)\mathbf{p}(t) & (1a) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) & (1b) \end{cases}$$

where $\boldsymbol{\varphi}(t)=\boldsymbol{\varphi}[\mathbf{y}(t), \mathbf{u}(t)]$ and $\mathbf{\Psi}(t)= \mathbf{\Psi}[\mathbf{x}(t)]$. To make the problem more tractable, assume that

$$\mathbf{\Psi}(t) = [\boldsymbol{\psi}_1(t), \ldots, \boldsymbol{\psi}_k(t)] = [\mathbf{N}_1\mathbf{x}(t), \ldots, \mathbf{N}_k\mathbf{x}(t)]$$

so that Eq. (1a) becomes:

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\varphi(t) + \sum_{j=1}^{k} \mathbf{N}_j \mathbf{x}(t) p_j(t) \qquad (2)$$

This can be rewritten as

$$\begin{aligned} \mathbf{x}(t+1) &= \left[\mathbf{A} + \sum_{j=1}^{k} \mathbf{N}_j p_j(t) \right] \mathbf{x}(t) + \mathbf{B}\varphi(t) \\ &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}\varphi(t) \end{aligned} \qquad (3)$$

Thus the original nonlinear system has been recast as a linear time-varying system.

2.2 Fault-Specific Trajectory

Equation (3) indicates that the faults affect the state-transition matrix in a predictable, fault-specific way. Consider the characteristic polynomial of the system:

$$\begin{aligned} h^+(z) &= Det\left[z\mathbf{I} - \mathbf{A} - \sum_{j=1}^{k} \mathbf{N}_j p_j(t) \right] \\ &= z^n + h^1 z^{n-1} + \cdots + h^{n-1} z + h^n = (z-\xi_1)\cdots(z-\xi_n) \end{aligned} \qquad (4)$$

Clearly, the parameters h^1 , …, h^n and the poles ξ_1 , …, ξ_n depend on the presence/absence and size of the faults.

A certain type of fault, j, and its size, p_j, lead to a specific parameter vector $\mathbf{h}_j(p_j) = [h^1 , \ldots, h^n]^T$. Assuming one fault at a time, and with the matrices $\mathbf{N}_j$ known, a single trajectory of the parameters can be pre-computed for each fault, in terms of the fault-size, and "drawn" in the n-dimensional space. The isolation of the additive faults p_1, …, p_k will then require the estimation of the characteristic polynomial $h^+(z)$. The estimate represents a point in the parameter space. The distance of this point from each of the pre-computed trajectories is measured and the fault associated with the closest one is declared.

3. PARAMETER ESTIMATION

3.1 Estimation of the Characteristic Polynomial

Since any fault with a certain size corresponds to a specific point in the parameter space, the procedure relies on parameter estimation. The characteristic polynomial $h^+(z)$ is also the denominator of the system transfer function. The problem becomes the estimation of the denominator coefficients of the system transfer function. The estimate of the parameter vector can be obtained by any variant of the least squares approach.

A standard linear state-space model with output noise can be transferred into input-output form as

$$\mathbf{y}(t) = \mathbf{C}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\mathbf{u}(t) + \mathbf{v}(t) \tag{5}$$

In the simple, SISO case, this is

$$y(t) = \mathbf{c}^T (z\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}u(t) + v(t) \tag{6}$$

or re-written in transfer function format:

$$y(t) = \frac{G(z^{-1})}{H(z^{-1})}u(t) + v(t) \tag{7}$$

where v(t) represents the noise term. A direct polynomial form is written as:

$$y(t) = [1 - H(z^{-1})]y(t) + G(z^{-1})u(t) + H(z^{-1})v(t) \tag{8}$$

Let $\boldsymbol{\varphi}(t) = [-y(t-1) \ \cdots \ -y(t-n) \ \ u(t-1) \ \cdots \ u(t-m)]^T$, then the model becomes:

$$y(t) = \boldsymbol{\varphi}^T(t)\boldsymbol{\pi} + w(t) \tag{9}$$

where $\boldsymbol{\pi} = [h^1 \ \ldots \ h^n \ \ g^0 \ \ldots \ g^m]^T$ and the colored equation error term is:

$$w(t) = H(z^{-1})v(t) \tag{10}$$

Considering the Hankel matrix formed from N consecutive observations, the model becomes:

$$\mathbf{y}(t, N) = \boldsymbol{\Phi}(t, N)\boldsymbol{\pi} + \mathbf{w}(t, N) \tag{11}$$

Then the LS estimate of the parameter vector $\boldsymbol{\pi}$ is:

$$\hat{\boldsymbol{\pi}}_{LS} = \left[\boldsymbol{\Phi}(t, N)^T \boldsymbol{\Phi}(t, N)\right]^{-1} \boldsymbol{\Phi}(t, N)^T \mathbf{y}(t, N) \tag{12}$$

Retrieve the coefficients of the characteristic polynomial, $\hat{\mathbf{h}}_{LS}$, from $\hat{\boldsymbol{\pi}}_{LS}$

$$\hat{\mathbf{h}}_{LS} = [\hat{h}^1, \hat{h}^2, \cdots, \hat{h}^n]^T \tag{13}$$

In order to apply the GLR technique, the joint distribution of the parameter estimates must be derived.

3.2 Joint Distribution of the Parameter Estimates

As it is well known, if the equation error w(t) is zero-mean, Guassian white noise, the estimate for the model Eq.(9) follows the normal distribution:

$$\hat{\boldsymbol{\pi}}_{LS} \sim \mathcal{N}(\boldsymbol{\pi}, \mathbf{P}) \tag{14}$$

where $\mathbf{P} = \sigma_w^2 E\{\varphi(t)\varphi(t)^T\}^{-1}$, σ_w is the standard deviation of the noise, and $\boldsymbol{\pi}$ is the real parameter vector. Since we only need to consider the first n elements of the parameter vector $\hat{\boldsymbol{\pi}}_{LS}$, we retrieve the respective part of the variance matrix $\mathbf{P}$, namely, the variance matrix of $\hat{\mathbf{h}}_{LS}$, $\mathbf{M}$, which is the left-upper sub-matrix of $\mathbf{P}$**.** Thus, the joint distribution of $\hat{\mathbf{h}}_{LS}$ can be described as:

$$\hat{\mathbf{h}}_{LS} \sim \mathcal{N}(\mathbf{h}_0, \mathbf{M}) \tag{15}$$

where $\mathbf{h}_0$ is the real parameter vector**,** which is the first n elements of the real parameter vector, $\boldsymbol{\pi}$.

Note that we assumed that the equation error is a white Gaussian process. However, it is generally colored. A more accurate characterization of the distribution would be too complicated for real-time estimation. Therefore, we rely on the assumption of the Gaussian white noise, which is acceptable since the noise is relatively small in the real system. Thus, the density function of $\hat{\mathbf{h}}_{LS}(t)$ is:

$$f[\hat{\mathbf{h}}_{LS}(t), \mathbf{h}_j(p_j)] = K\exp\left\{-\frac{1}{2}\left[\hat{\mathbf{h}}_{LS}(t) - \mathbf{h}_j(p_j)\right]^T \mathbf{M}^{-1}\left[\hat{\mathbf{h}}_{LS}(t) - \mathbf{h}_j(p_j)\right]\right\} \tag{16}$$

where K is a constant, $\mathbf{h}_j(p_j(t))$ is the pre-computed vector for the j-th fault of size $p_j(t)$. We can define a simplified log-likelihood function as:

$$\log L[\hat{\mathbf{h}}_{LS}(t), \mathbf{h}_j(p_j)] = -\frac{1}{2}\left[\hat{\mathbf{h}}_{LS}(t) - \mathbf{h}_j(p_j)\right]^T \mathbf{M}^{-1}\left[\hat{\mathbf{h}}_{LS}(t) - \mathbf{h}_j(p_j)\right] \tag{17}$$

where we ignore the factor K since it cancels out in the likelihood ratio anyway.

Usually, the standard deviation of the noise is not available. However, we can estimate σ_w from the data as:

$$\hat{\sigma}_w^2 = \frac{\varepsilon^T(t, N)\varepsilon(t, N)}{N - \dim(\boldsymbol{\pi})} \tag{18}$$

where $\varepsilon(t, N)$ is the prediction error, and dim($\boldsymbol{\pi}$) is the dimension of vector $\boldsymbol{\pi}$.

4. GLR-FDD FRAMEWORK

4.1 Fault Detection

A simple testing can be constructed for the detection of the presence of faults (Gertler, 1998). The concept is to test whether the estimated parameter vectors are inside a hyper-ellipsoid in the hyperspace, which corresponds to a constant density contour of the parameter joint normal distribution. The contour is chosen according to a selected false alarm rate, α.

Re-write Eq. (17):

$$-2\log L[\hat{\mathbf{h}}_{LS}(t),\mathbf{h}_0]=\left[\hat{\mathbf{h}}_{LS}(t)-\mathbf{h}_0\right]^T\mathbf{M}_0^{-1}\left[\hat{\mathbf{h}}_{LS}(t)-\mathbf{h}_0\right] \quad (19)$$

which obeys the central χ^2 distribution with degree of freedom $\lambda = n \cdot N$. Here n is the order of the system and N is the number of observations. Note that here we only consider the single vector $\mathbf{h}_0$ and its variance matrix $\mathbf{M}_0$, which are from the fault-free situation.

4.2 Fault Isolation

A constrained GLR approach can be implemented (Basseville and Nikiforov, 1993; Gertler,1998) to measure the distances between the estimates by the windowed observations and the pre-computed parameter trajectory and then to decide which point at the trajectory is the closest to the estimated parameter vector $\hat{\mathbf{h}}_{LS}$. Based on this approach, we can not only determine the fault type but also estimate its size.

Define a set of fault hypotheses:

H^j: the j-th fault is present ($j = 1, \ldots, k$)

The isolation test is triggered once the detection test reports a fault. Then, the procedure can be described as:

(1) Pre-compute the trajectories $\mathbf{h}_j(p_j)$ ($j = 1,\ldots, k$);
(2) By applying the LS method to the windowed N observations, obtain $\hat{\mathbf{h}}_{LS}(t)$ and its joint distribution at time t;
(3) Compute the log-likelihood functions under each fault hypothesis.
(4) Find the minimal value (maximal density function) of the log-likelihood functions among various fault hypotheses and their sizes. This value indicates the estimated fault type and its size.

GLR testing can be simply written as:

$$[\hat{j}(t),\hat{p}_j(t)]=\arg\min_{p_j,j}\left\{\log L[\hat{\mathbf{h}}_{LS}(t),\mathbf{h}_j(p_j)]\right\} \quad (j=1,\ldots,k) \quad (20)$$

Generally, no closed-form solution exists for the fault size that corresponds to the minimum of the likelihood functions since $\mathbf{h}_j(p_j)$ implies the computation of the matrix determinant. Therefore, in practice, one has to rely on a numerical algorithm to obtain the optimal solution. In our research, a MATLAB built-in function, *fmin*, has been used. This minimization algorithm works without derivative computation (Brent, 1973) and employs a combination of golden-section search and successive parabolic interpolation, leading to fast convergence in reaching the minimal value in Eq. (20).

4.3 Enhanced LS Identification: RCLS

Since the noise term $w(t)$ in the model shown in Eq. (8) is a colored Gaussian noise, the standard LS estimation cannot lead to an unbiased result. Especially, with the growth of the model order, the estimation bias becomes significantly large even with very small noise. For the model in Eq. (8), an approach, called the Recursive Compensation Least Squares (RCLS, Fang and Xiao, 1988), is developed to overcome this bias problem.

In the RCLS approach, the noise term in Eq. (10) is split to two parts:

$$w(t)=H(z^{-1})v(t)=\mathbf{r}^T(t)\boldsymbol{\pi}+v(t) \quad (21)$$

where $\mathbf{r}(t)=[v(k-1),v(k-2),...,v(k-n),0,...,0]^T$.

Considering the LS shown in Eq. (12), we can derive:

$$E\{\hat{\boldsymbol{\pi}}_{LS}\}=\boldsymbol{\pi}+E\{\boldsymbol{\varphi}(t)\mathbf{r}^T(t)\}=\boldsymbol{\pi}-\sigma_w^2\mathbf{C}^{-1}\mathbf{D}\boldsymbol{\pi} \quad (22)$$

here $\mathbf{C}=E\{\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\}$, and $\mathbf{D}=\begin{bmatrix}\mathbf{I}_{n\times n} & \mathbf{0}_{n\times m}\\ \mathbf{0}_{m\times n} & \mathbf{0}_{m\times m}\end{bmatrix}$.

Apparently, the standard LS estimate, $\hat{\boldsymbol{\pi}}_{LS}$, is biased due to the term $\sigma_w^2\mathbf{C}^{-1}\mathbf{D}\boldsymbol{\pi}$. Thus, by considering a compensation term, we can eliminate the bias.

In each iteration of the RLS, the RCLS updates the LS result using a bias-compensated term:

$$\hat{\boldsymbol{\pi}}_{CLS}(t)=\hat{\boldsymbol{\pi}}_{LS}(t)+t\hat{\sigma}_w^2\mathbf{P}(t)\mathbf{D}\hat{\boldsymbol{\pi}}_{CLS}(t-1) \quad (23)$$

and re-evaluates the variance of the noise:

$$\hat{\sigma}_w^2(t)=\frac{\frac{1}{t}\sum_{i=1}^{t}\varepsilon_{LS}^2(i)}{1+\hat{\boldsymbol{\pi}}_{CLS}^T(t)\mathbf{D}\hat{\boldsymbol{\pi}}_{LS}(t)} \quad (24)$$

where $\mathbf{P}(t)=\left[\boldsymbol{\Phi}^T(t)\boldsymbol{\Phi}(t)\right]^{-1}$.

We can easily prove that the RCLS is unbiased. In order to introduce it into GLR framework, we need to derive its joint distribution.

The RCLS estimation can be expressed in one-step form:

$$\hat{\boldsymbol{\pi}}_{CLS}=\mathbf{G}\hat{\boldsymbol{\pi}}_{LS}=\mathbf{G}\left[\boldsymbol{\Phi}^T(t,N)\boldsymbol{\Phi}(t,N)\right]^{-1}\boldsymbol{\Phi}^T(t,N)\mathbf{y}(t,N) \quad (25)$$

$$\text{or } E\{\hat{\boldsymbol{\pi}}_{CLS}\}=E\{\mathbf{G}\left[\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\right]^{-1}\boldsymbol{\varphi}(t)y(t)\} \quad (26)$$

where $\mathbf{G}=\left[\mathbf{I}-\sigma_w^2\mathbf{C}^{-1}\mathbf{D}\right]^{-1}$. Then its variance is calculated as

$$\text{var}\{\hat{\boldsymbol{\pi}}_{CLS}\} = \text{var}\{\mathbf{G}\left[\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\right]^{-1}\boldsymbol{\varphi}(t)y(t)\}$$
$$= \text{var}\{\mathbf{G}\boldsymbol{\pi} + \mathbf{G}\left[\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\right]^{-1}\boldsymbol{\varphi}(t)w(t)\}$$

Since $\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{T}}(t)$ can be handled as a deterministic matrix (Ljung, 1999), we have:

$$\text{var}\{\hat{\boldsymbol{\pi}}_{CLS}\} = \sigma_w^2\mathbf{G}\left[\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\right]^{-1}\mathbf{G}^T$$

Thus the RCLS estimate follows the normal distribution as well:

$$\hat{\boldsymbol{\pi}}_{CLS} \sim \mathcal{N}(\boldsymbol{\pi}, \sigma_w^2\mathbf{G}\left[\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\right]^{-1}\mathbf{G}^T) \quad (27)$$

Note that there is only one term, σ_w^2, which is unknown; instead can we use its estimate, $\hat{\sigma}_w^2$. Similarly, we can retrieve the first *n* elements, $\hat{\mathbf{h}}_{CLS}$, and their variance matrix $\hat{\mathbf{M}}_{CLS}$. Thus, the joint distribution can be expressed as:

$$\hat{\mathbf{h}}_{CLS} \sim \mathcal{N}(\mathbf{h}_0, \hat{\mathbf{M}}_{CLS}) \quad (28)$$

Note that this result corresponds to Eq. (15) in the standard LS; then we can follow the same procedure.

4.4 Subspace Identification

In the past few years, a novel class of identification approaches, called S*ubspace Identification Methods* (SIMs), has been presented to avoid the priori parameterization and nonlinear optimization problem. SIMs (Overschee and DeMoor, 1994) refer to the state-space-based identification that determines the states of the system directly from the observation data, and allows the associated covariance matrices to be solved directly through linear LS. Their algorithms rely on some numerical techniques, especially singular value decomposition (SVD) plays a critical role in them. There are three main subspace algorithms: *numerical algorithms for subspace state space system identification* (N4SID, Overschee and DeMoor, 1994), *multivariable output-error state space* (MOESP, Verhaegen, 1994), and *canonical variate analysis* (CVA, Larimore, 1983). Furthermore, those algorithms have been placed in a unified framework, the only difference among them being the choice of weights (Ljung, 1999).

In order to compare the LS methods with the SIMs, we consider three typical subspace algorithms: MATLAB built-in function, *N4SID,* (Overschee and DeMoor, 1994), the instrumental-variables subspace algorithm (SIM-IV) contributed by Chou and Verhaegen (1997), and the CVA approach presented by Larimore (1990)

5. SIMULATION STUDIES

5.1 Scenario 1: Second-order LTI system

Let us start with a simple state-space model:

$$\mathbf{A} = \begin{bmatrix} 1.0 & -1.5 \\ 0.75 & -0.5 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ \mathbf{C} = [1 \quad 2];$$

Assume two types of faults and define their coefficient matrices as:

$$\mathbf{N}_1 = \begin{bmatrix} 0.25 & 0.0 \\ 0.05 & -0.15 \end{bmatrix},\ \mathbf{N}_2 = \begin{bmatrix} 0.01 & -0.1 \\ -0.05 & 0.20 \end{bmatrix}.$$

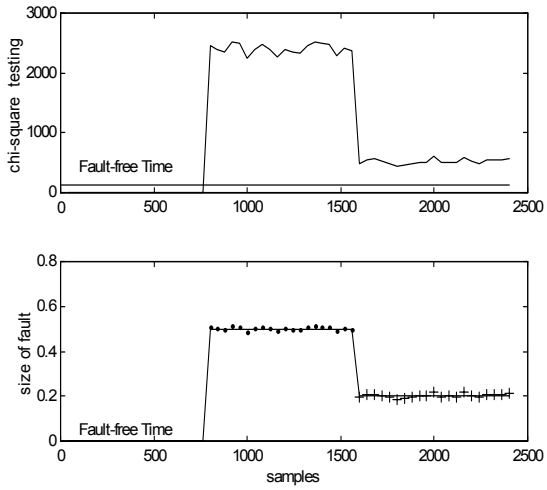

Figure 1: GLR-FDD for a 2nd order system

The size, p_j, (j = 1, 2) of the faults varies from 0.0 to 1.0. The standard deviation of measurement noise is 0.1. 2400 data points have been collected and each window contains 40 points. A process with the first 1/3 period without any fault, the second 1/3 period with fault 1 and size = 0.5, and the last 1/3 period with fault 2 and size = 0.2, was simulated. For the χ^2 test, the confidence level was set at 95%. Simulation results are shown in Figure 1.

Note that the upper plot in Figure 1 shows the result of the χ^2 testing and the lower plot exhibits the result of the fault diagnosis. Here the solid line in the lower plot represents the real fault size; the dots indicate the estimated fault type and size. If the result is fault 1, a symbol '·' is shown; if it is fault 2, the symbol '+' is drawn.

We can see that, in a simulated second order LTI system, the GLR-FDD framework can correctly detect the fault, determine the fault type, and estimate the fault size.

5.2 Scenario 2: Fourth-order LTI system

Here a more challenging model has been investigated. This is a fourth-order LTI system:

$$\mathbf{A} = diag([0.9 \quad 0.8 \quad 0.7 \quad 0.6]),$$
$$\mathbf{B} = [0.2 \quad -0.5 \quad 0.3 \quad -0.3]^T,$$
$$\mathbf{C} = [0.1 \quad 0.2 \quad 0.1 \quad 0.1].\ \mathbf{D} = 0.$$

Assume two types of faults and define their coefficient matrices as:

$$\mathbf{N}_1 = diag([-0.2 \quad -0.4 \quad 1.0 \quad 0.2]),$$

$$\mathbf{N}_2 = \begin{bmatrix} 0 & 0 & -0.3 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.1 \\ -0.3 & 0 & 0 & 0.8 \end{bmatrix}.$$

One of the key points of this GLR-FDD approach is to identify the parameters of the characteristic polynomial, **h**. The subsequent steps of this approach will be ineffective unless the result of the identification is reasonably close to the real parameters. With the growth of the system order, such as the fourth-order system here, the identification bias may become significantly large. Thus, in order to guarantee the effectiveness of the whole framework, we have to select an identification method as good as possible. Since identification is independent from the other procedures in this framework, we can consider this issue separately.

Of course, there are numerous identification methods after the exploration of several decades in this field. However, identification is not the main focus of this paper. Therefore, here we consider only five typical identification methods: CVA, SIM-IV and N4SID, which all belong to subspace identification; the standard LS, and the RCLS, which belong to the least squares family.

In the comparison of the five identification methods, we only consider the fault-free situation in the 4th order system. Different levels of the noise magnitude (standard deviations from 10e-6 to 10e0) have been taken into account; at each noise level, 200 pairs of data (input u and output y) have been generated and this simulation has been repeated 20 times to obtain an average.

The quality indicator of identification is the *mean square parameter estimation error* (MSPE), which is:

$$MSPE = \frac{1}{N}\sum_{i=1}^{N}\left(\left[\mathbf{h}-\hat{\mathbf{h}}(i)\right]^T\left[\mathbf{h}-\hat{\mathbf{h}}(i)\right]\right) \qquad (29)$$

where $\mathbf{h}$ is the real parameter vector, $\hat{\mathbf{h}}(i)$ is the i-th identification result, and N is the number of identification runs. As for the subspace identification methods, $\hat{\mathbf{h}}(i)$ has been indirectly obtained from the directly identified state-space model, by transformation.

Double-axis logarithmic scaling has been applied for clearer appearance. The comparison plot is shown in Figure 2. We can see that between –1 and 0 of the noise STD level, all the approaches behave similarly. However, the identification bias is too large to be useful for the FDD procedure. In the lower noise STD area (less than -3), the noise level is too small to make practical sense. Between –3 and –1, a reasonable range for the noise level, the RCLS method performs better than the other four.

In general, subspace identification provides the state-space model, which is not unique. Thus, only the eigenvalues of the state transition matrix, **A**, are regarded as the indicator

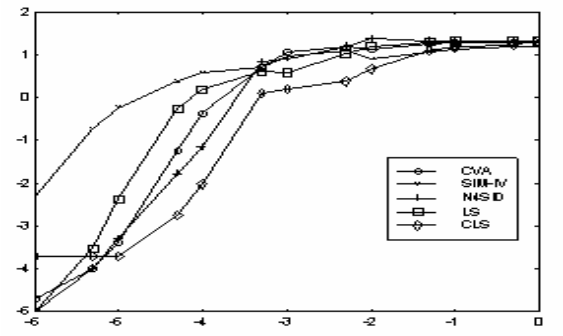

Figure 2: A full comparison among 5 identifications

of identification performance. However, they are less stable than the parameter vector of the polynomial model and sometimes are complex numbers. Therefore, the subspace identification method is not an appropriate choice to deal with the FDD problem presented in this paper. Thus, we only consider two LS approaches, the standard LS and the RCLS, as identification methods in the GLR-FDD framework.

The fault sizes, $p_j (j = 1, 2)$, vary from 0 to 1. The sequence does not contain any fault during its first ¼ part. Then fault 1 has been added with size = 0.8; the STD of the noise is 0.005. 4000 data points have been collected and 100 data were windowed. For the χ^2 test, the confidence level was set at 95%. The simulation result with the RCLS is shown in Figure 3. For comparison, the same GLR-FDD procedure but with the standard LS has been tested, its result is shown in Figure 4.

Note that this framework is able to detect the occurrence of the fault by χ^2 testing, moreover, it is able to determine the fault type and identify its size by the RCLS identification and the GLR technique. But if the standard LS identification is applied in this framework, the performance becomes worse. The algorithm fails to detect the fault and to identify its size although it gives the correct fault type.

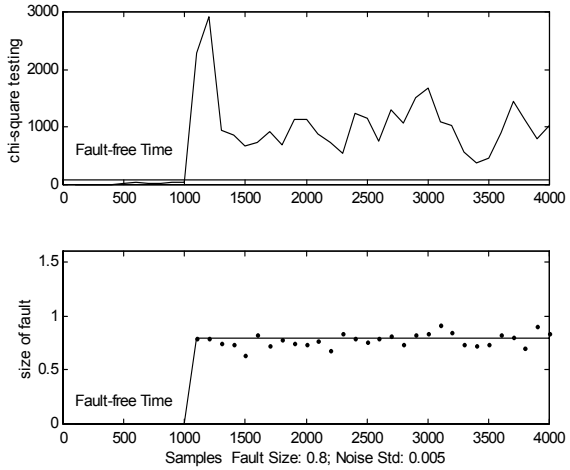

Figure 3: GLR-FDD with CLS for 4-th order system

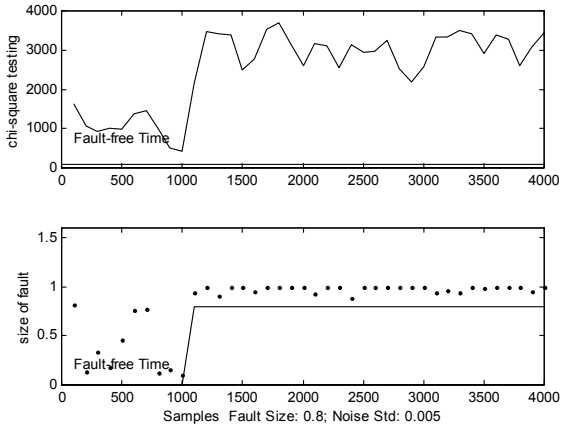

Figure 4: GLR-FDD with LS for 4-th order system

These results have also indicated that the identification method plays an important role in this GLR-FDD framework. More accurate identification will lead to more effective FDD performance. However, when the STD of noise is increased to 0.01, this procedure cannot give a meaningful FDD result with any identification method. The essential reason is that no identification technique is able to offer a reasonable estimate at that noise level. This can be observed apparently in Figure 2.

6. CONCLUSION

For a specific fault model shown in Eqs. (1) – (3), assuming that *a priori* information on the model and the faults is available, a statistical diagnostic algorithm has been constructed not only for fault detection but also for fault diagnosis. Certain system features, such as the parameter trajectory of the characteristic function or zero-pole trajectory, essentially provide a link to specific faults. We make use of this concept for the FDD investigation by measuring the statistical distance between the pre-computed trajectories and the observations.

A simple detection test is introduced which, in the usual way, converts the multidimensional Gaussian problem into a scalar χ^2 testing. For fault diagnosis, a Generalized Likelihood Ratio approach is developed. The conditional estimates of the parameter vector of the characteristic polynomial are computed, under the constraints imposed by the various fault hypotheses. This is followed by the computation of conditional likelihood functions for each hypothesis.

In our comparison study among the identification methods, the Subspace methods have not behaved better than the traditional LS methods in the 4-th order system. Moreover, an enhanced LS identification, the RCLS method, was found to possess outstanding identification ability. We have selected the RCLS also because the LS class methods have been well developed for several decades and they may offer more options. Moreover, the LS methods can directly give the transfer function parameter estimates, while the Subspace methods involve a detour through the state space model.

The joint distribution of the parameter estimates under the RCLS technique has then been derived for use in a GLR-FDD framework. Finally, the complete GLR-FDD framework, including χ^2 testing, RCLS identification and the GLR technique, has been developed. Simulation studies for a second order state space system and a 4-th order system have demonstrated the algorithm's ability in fault detection and diagnosis.

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