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A Multi-time Scales Model and Control for Hybrid Stochastic Production Systems with Quadratic Cost

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Abstract—**A hybrid stochastic production system corresponding to a situation where a basically deterministic production system, the fast subsystem, is subject to infrequent model disruptions occurring randomly, the slow subsystem is formulated with multi-time scales. One time scale is 1, the processing time scale, which is frequently and represents fast model. Another time scale is ε^{-1}, system failure time scale, which is infrequently and stands for slow model of the system. On different time scales, the long-run average cost function is decomposed into two sub-objective functions. Based on the two sub-objective functions, the optimal control policy of the system is gotten by using approximation numerical technique. Numerical experiments show its merit.**

Keywords—**Hybrid stochastic Production systems, Multi-time scales, Numerical technique**

I. INTRODUCTION

THE modeling and optimal control of stochastic production systems have been the objective of considerable investigations in Control Theory (see [1], [2] and [3] as samples of the literature on this topic). Typically, in these models, the stochastic jump process describes the evolution of the operational state of a flexible manufacturing shop, with jumps due to failures and repairs of the machines, whereas the deterministic state equations show the evolution of the surplus of products produced by the system. In most of these models, the jump Markov disturbances due to failures and repairs are assumed to be represented as a continuous-homogenous Markov chain with jump rate which is independent of state and control. In [4], a model is proposed where, for each machine of the systems, an additional state variable records the age of the machine and the failure rates are age dependent. And this model provides an example of a piecewise deterministic control systems (PDCS) with state dependent jump rates. In [5], a manufacturing system with control (production rate) dependent failure rates has been studied. Those efforts mentioned above are focused on flexible production systems, and more complex production systems are studied in [6], where a setup, which may involve setup time or setup cost or both, is required if products is to be switched from one product to another. Being a particularly difficult class of problem, modeling and optimization of PDCS are discussed by using hierarchical approach via singularly perturbation technique. (See [7], [8] and [9]). In those works, the original problem is decomposed into simpler problems, which turn out to be the limiting problems derived from averaging the given stochastic machine capacities, and the optimal control policy for original problem is constructed from the optimal control of the limiting problems. But the meaning of the singularly perturbation parameter in [7] is contrary to that in [8]. For relevant works dealing with production systems, we refer the readers to [10], [11], [12] and its references.

Being an elegant paradigm of hybrid stochastic control system, the modeling and approximation of optimal control of a failure-prone production system with quadratic cost are discussed, where the jump disturbances are state and control independent, and when the time scales of the stochastic and the deterministic parts are of different orders of magnitude. More precisely, The hybrid stochastic production system is composed of “fast model”, characterized by a continuous state variable and corresponding with the subsystem in the form of a controlled diffusion process, and “slow model”, characterized by a discrete variable and according with the subsystem in the form of a uncontrolled jump process. The hybrid stochastic production system we study in this paper is formulated as a long-run average cost stochastic control problem with quadratic cost in the form of a switching diffusion process with a hybrid state and a singularly

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perturbed generator. To illustrate the validity of this result we adopt the approximation numerical technique initiated by Kushner and Dupuis [13].

The paper is organized as follows. In section II, the hybrid stochastic production system is formulated precisely under consideration. Approximate optimal control via multi-time scales is proved in section III. Section IV illustrates the method on a numerical example, and Section V concludes the paper.

II. Control Model For Hybrid Stochastic Production Systems

A production line is treated as a set of failure-prone equipment in production systems, which can produce many types of products. In a certain degree, the optimal policy for this system is similar to that of a hybrid stochastic production system consisting of a single machine. Here, the production system consisting of a set of unreliable equipment can produce n different types of product P_i, i=1, … , n with only one at any given time. The equipment is subject to random failure and repairs.

The hybrid stochastic production system is described by using two types of state variables. One is a vector of variables continuously changing in $\mathbf{R}^n$, and another is the stochastic jump process taking values in a finite index state space $\boldsymbol{E}$ (defined later). Corresponding to any state $\alpha\in\boldsymbol{E}$, there exists a system of differential equations describing the dynamic of the continuously changing variables when the jump process is in the state α. The continuous state variables can be associated with the deterministic dynamics of the production system while the stochastic jump process represents the changes of its functional modes. A small parameter ε>0 is introduced below in such a way that continuous variables can have a finite (not tending to zero with ε) deviation on any time interval of the length ε while the probability for the jump process to change their value on such an interval is of the order $O(\varepsilon)$. Thus, continuous variables can be considered to be "fast" with respect to the rate of the occurrence of the jump.

For $t\geq 0$, let $x_i(t)\in\mathbf{R}^1=(-\infty,\ \infty)$, $u_i(t)\in\mathbf{R}^+=[0,\ \infty)$ and $z_i(t)\in\mathbf{R}^+$ denote the surplus, production rate, and the rate of demand for product P_i, i=1, … , n. X, U, and Z are used to denote vectors $[x_1(t), \ldots, x_n(t)]^T\in\mathbf{R}^n$, $[u_1(t), \ldots, u_n(t)]^T\in\mathbf{R}^{+n}$, and $[z_1(t), \ldots, z_n(t)]^T\in\mathbf{R}^{+n}$, respectively, where A^T denotes the transpose of a vector (or a matrix) A. $z_i(t)$ is considered as a constant in this paper. The inventory/shortage levels are described by the following dynamic differential equations:

$$dX(t)=f^{\alpha}(X(t),U(t))dt+\sigma d\omega(t),\quad X(0)=X\ , \tag{1}$$

where $\sigma=[\sigma_1, \sigma_2, \ldots, \sigma_n]^T$ is a given vector, and ($\omega(t)$, $t\geq 0$) is a standard $\mathbf{R}^n$-valued Wiener motion process, defined on a complete probability space ($\boldsymbol{\Omega}$, $\boldsymbol{\pounds}$, $\boldsymbol{P}$). Here, σ describes the random and uncontrolled product exhaustion by internal strife.

The unreliable equipment states can be classified as (i) operational, denoted by state 1; (ii) breakdown, denoted by state 0. Under operational state, any type of product can be produced; under breakdown state, nothing is produced. Let $\zeta(t)$ denote the state process of the equipment, and let $\boldsymbol{E}$={0, 1} be the state space of the process $\zeta(t)$, $\zeta(t)\in\boldsymbol{E}$. It is assumed that a discrete-state variable is "moving slowly" according to a continuous time stochastic jump process with jump rates $q_{\alpha\beta}$:

$$P[\zeta(t+dt)=\beta\mid\zeta(t)=\alpha]=\varepsilon q_{\alpha\beta}dt+o(dt)\,, \tag{2}$$

$$P[\zeta(t+dt)=\alpha\mid\zeta(t)=\alpha]=1+\varepsilon q_{\alpha\alpha}dt+o(dt)\ . \tag{3}$$

Where $\lim\limits_{dt\to 0} o(dt)/dt=0$, $q_{\alpha\alpha}=-\sum\limits_{\beta\neq\alpha}q_{\alpha\beta}$. $q_{\alpha\beta}$ is the jump rate of the process $\zeta(t)$ from state α to state β at time t. In expressions (2)-(3) the parameter ε is the time-scale ratio that will, eventually, be considered very small. And the production system can be described by the following differential equation precisely:

$$\begin{aligned} dX(t)&=f^{\alpha}(X(t),U(t))dt+\sigma d\omega(t)\\ &=\begin{cases}(U(t)-Z(t))dt+\sigma d\omega(t) & if\quad \alpha=1,\\ -Z(t)dt+\sigma d\omega(t) & if\quad \alpha=0.\end{cases}\end{aligned} \tag{4}$$

We are concerned with the optimal problem of finding a production control policy that minimizes the expected long-run average cost:

$$\begin{aligned} J(i,X,s,\alpha,\varepsilon)=\max_{U(\cdot)\in\pi}\ \liminf_{T\to\infty}\frac{1}{T}\bigg\{&\int_0^s G(X(t),0,1)dt\\ &+E\left(\int_s^T G(X(t),U(t),\zeta(t))dt\right)\bigg\}\ , \end{aligned} \tag{5}$$

where s denotes the remaining set up time of the system, and i denotes the initial set up state of the system. The decision variables are the rates of production $U(\cdot)$ over time. Let $G(X(t),\ U(t),\ \zeta(t))$ denote the instantaneous cost function of the surplus and repair. It is denoted by

$$\begin{aligned} &G(X(t),U(t),\zeta(t))=\\ &\qquad\sum_{i=1}^{n}c_i^+(x_i^+)^2+c_i^-x_i^-+c_r ind\{\zeta(t)=0\}. \end{aligned} \tag{6}$$

Positive surplus is supposed to incur a holding cost of c_i^+ per unit commodity per unit time, while the negative incur a holding cost of c_i^-, with $c_i^+>0$, $c_i^->0$. $x_i^+:=max(x_i,\ 0)$, $x_i^-:=max(-x_i,\ 0)$. Where c_r denotes cost parameter of repair, which is nonnegative constant. $ind\{\zeta(t)=\alpha\}$ is the indicator function of set $\{\zeta(t)=\alpha\}$

$$ind\{\zeta(t)=\alpha\}=\begin{cases}1 & if\quad \zeta(t)=\alpha;\\ 0 & otherwise.\end{cases} \tag{7}$$

Remark 1: The quadratic instantaneous cost function (6) is a useful cost approximation for the hybrid stochastic production systems, where products are perishable or may become obsolete, as well as systems with storage-space competition [14]. And the cost of repair in (6) ensures the system model more practical, which is ignored in most lectures on this topic.

According to the nature of this system, for $t \geq 0$, the production constraints are given as follows:

$$\begin{cases} 0 \leq u_i(t) \leq \zeta(t) r_i, & i = 1, 2, \cdots, n \\ u_j(t) = 0, & j \neq i. \end{cases} \tag{8}$$

$$\frac{r_i - z_i(t)}{q_{10}} \geq \frac{z_i(t)}{q_{01}}, \quad i = 1, 2, \cdots, n. \tag{9}$$

Where r_i denotes the maximum production rate of P_i. The constraint (9) is very appealing from an intuitive point of view. Note that $(q_{10})^{-1}$ and $(q_{10})^{-1}$ are mean sojourn time of the "slow model" in states 0 and 1 respectively.

Let $U(\alpha)$, a close subset of $\mathbf{R}^{+n}$, denote the production rate control constraints, $\forall\, \alpha \in \boldsymbol{E}$. Any measurable function $U(t)$ defined on $U(\alpha)$, for each $\alpha \in \boldsymbol{E}$, is called an admissible control. The set $\boldsymbol{\pi}=\{U(t)\colon t \geq 0\}$ is an admissible policy. The admissible control function $U(t)$ is supposed to be piecewise continuous in t and continuously differentiable with bounded partial derivatives in X. $U(t)$ is a feedback admissible control which can react to the current state. Feedback controls are of practical importance because they will adjust any unfavorable deviation of the state from the targeted position at any time and hence render a better performance, especially when uncertainties or disturbances are presented in the system.

Let $(X(t), \zeta(t))$ denote the system state at time t, and the space of the system state is $\mathbf{R}^n \times \boldsymbol{E}$. The problem is to find an admissible decision $U(\cdot) \in \boldsymbol{\pi}$ that minimizes $J(i, X, s, \zeta(t), \varepsilon)$, the expected long-run average cost, which is subject to Eq. (5), (8), (9).

III. Approximate Optimal Control Via Multi-time Scales

In this section, properties of the value function of the problem, the associated HJB (Hamilton-Jacobi-Bellman) equation, and an approximate optimal control policy on multi-time scales are considered.

Without losing generality, let $V(i, X, \alpha, \varepsilon)$ for $X \in \mathbf{R}^n$, $\alpha \in \boldsymbol{E}$, $s=0$, denote the value function of the problem, i.e.

$$V(i, X, \alpha, \varepsilon) = \inf_{U(\cdot) \in \pi} J(i, X, \alpha, 0, \varepsilon) \tag{10}$$

From (6), $G(X(t), U(t), \zeta(t))$ is locally Lipschitz and has at most polynomial growth, which agrees with the assumption A1 in [6]. The value function $V(i, X, \alpha, \varepsilon)$ is C^2 in x_i for each $\alpha \in \boldsymbol{E}$, and the following HJB equation holds. For any $\alpha \in \boldsymbol{E}$ and $i=1, 2, \ldots, n$

$$\begin{pmatrix} -Z(t)\dfrac{\partial V(i, X, 0, \varepsilon)}{\partial X} + \dfrac{\sigma^2}{2}\dfrac{\partial^2}{\partial X^2} V(i, X, 0, \varepsilon) \\ \min\limits_{U(\cdot) \in \pi}\{(U(t) - Z(t))\dfrac{\partial V(i, X, 1, \varepsilon)}{\partial X}\} + \dfrac{\sigma^2}{2}\dfrac{\partial^2}{\partial X^2} V(i, X, 1, \varepsilon) \end{pmatrix}$$
$$+ \varepsilon \begin{bmatrix} -q_{01} & q_{01} \\ q_{10} & -q_{10} \end{bmatrix} \begin{pmatrix} V(i, X, 0, \varepsilon) \\ V(i, X, 1, \varepsilon) \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} G(X(t), U(t), 0) \\ G(X(t), U(t), 1) \end{pmatrix}$$
$$= \begin{pmatrix} V(i, X, 0, \varepsilon) \\ V(i, X, 1, \varepsilon) \end{pmatrix} \tag{11}$$

According to the hybrid stochastic production system model formulated in section II, there are multi-time scales in the system. One time scale is 1, the processing time scale, which is frequency and represents fast model. The other time scale is ε^{-1}, system failure time scale, which is infrequency and stands for slow model of the system. Now the following notation is introduced as multi-time scales:

$$t_0 = t, \quad t_1 = \varepsilon t \tag{12}$$

Thus, (10) can be rewritten as the following by using the above stretched-out time scale

$$\begin{aligned} &V(i, X, \alpha, \varepsilon) \\ &= V_0(i, X(t, \varepsilon t), \alpha) + \varepsilon V_1(i, X(t, \varepsilon t), \alpha) + o(\varepsilon^2) \end{aligned} \tag{13}$$

Denote the operator D_n, $D_n = \dfrac{\partial}{\partial t_n}$, $n = 0, 1$, suppose some assumptions hold in [9], and the following HJB equations hold by comparing the each power coefficient of ε in the equation (11) via using this operator D_n

i. $$\min_{U(\cdot) \in \pi}\left[(U(t) - Z(t))\frac{\partial V_0(i, X, \alpha)}{\partial X} + G(X(t), U(t), \alpha) + \frac{1}{2}\sigma^2 \frac{\partial^2}{\partial X^2} V_0(i, X, \alpha)\right] - V_0(i, X, \alpha) = 0 \tag{14}$$

ii. $$\min_{U(\cdot) \in \pi}\left[(U(t) - Z(t))\frac{\partial V_1(i, X, \alpha)}{\partial X} + \sum_{\beta \neq \alpha} q_{\alpha\beta}(V_1(i, X, \beta) - V_1(i, X, \alpha)) + \frac{1}{2}\sigma^2 \frac{\partial^2}{\partial X^2} V_1(i, X, \alpha)\right] - V_1(i, X, \alpha) = 0 \tag{15}$$

iii. $V(i, X, \alpha, \varepsilon) \to V_0(i, X, \alpha)$ as $\varepsilon \to 0$ (16)

Remark 2: $V_0(i, X, \alpha)$ is the limiting problem commonly mentioned in [7], and asymptotic optimal controls for original problem (10) can be constructed from optimal or near-optimal controls for the limiting problem. Here, the meaning of ε is opposite to [7] and similar to [8].

Now a numerical technique is implemented to approximate the optimal control of the hybrid stochastic production system described above.

Let G_h be a grid of points in X with mesh h which defines the set of x-states for the approximate chain. Let ∂G_h^+ denote the reflecting boundary for the chain, which is disjoint from G_h. And the state space of the approximate chain will be $S_h = (G_h \cup \partial G_h^+) \times E$. Let e_i be the unit vector on the x_i coordinate. We approximate $V(\cdot,\cdot)$ by a function $V_h(\cdot,\cdot)$, and by replacing the first order partial derivative ($\frac{\partial}{\partial X}V(\cdot,\cdot)$) and the second order partial derivative ($\frac{\partial^2}{\partial X^2}V(\cdot,\cdot)$) of the value function by the following expressions (We refer the readers to [8] and [13] for details.). For convenience, the subscript of $V(\cdot,\cdot)$ are omitted in the following text:

First order:

$$\frac{\partial}{\partial x_i}V(i,X,\alpha) = \begin{cases} \frac{1}{h}[V_h(i,x_1,x_2,\cdots,x_i+h,\cdots,x_n,\alpha) \\ -V_h(i,x_1,x_2,\cdots,x_i,\cdots,x_n,\alpha)] \quad if\ \frac{dx_i}{dt} \ge 0; \\ \frac{1}{h}[V_h(i,x_1,x_2,\cdots,x_i,\cdots,x_n,\alpha) \\ -V_h(i,x_1,x_2,\cdots,x_i-h,\cdots,x_n,\alpha)] \quad otherwise. \end{cases} \tag{17}$$

Second order:

$$\frac{\partial^2}{\partial x_i^2}V(i,X,\alpha) = \frac{1}{h^2}[V_h(i,x_1,x_2,\cdots,x_i+h,\cdots,x_n,\alpha) + V_h(i,x_1,x_2,\cdots,x_i-h,\cdots,x_n,\alpha) - 2V_h(i,x_1,x_2,\cdots,x_i,\cdots,x_n,\alpha)] \tag{18}$$

Without losing generality, here we only demonstrate the transform of equation (15):

i. for $\frac{dx_i}{dt} \ge 0$

Regrouping terms in the equation (15) where the partial derivatives have been replaced by the (17) and (18), the following equation can be obtained:

$$[h^2 + \sum_{\beta\ne\alpha} q_{\alpha\beta}h^2 + \sigma^2 + h(U(t)-Z(t))]V_h(i,X,\alpha)$$
$$= \min_{U(\cdot)\in\pi}\left\{ + \sum_{\beta\ne\alpha} q_{\alpha\beta}h^2 V_h(i,X,\beta) + [\frac{1}{2}\sigma^2 + h((U(t)-Z(t))]V_h(i,\cdots,x_i+h,\cdots,\alpha) + \frac{1}{2}\sigma^2 V_h(i,\cdots,x_i-h,\cdots,\alpha) \right\} \tag{19}$$

ii. for $\frac{dx_i}{dt} < 0$, the corresponding equation holds.

For $\sum_{\beta} q_{\alpha\beta} = 0$ (20)

And define

$$Q_h(i,X,U,\alpha) = -q_{\alpha\alpha}h^2 + \sigma^2 + h\,|U(t)-Z(t)| + h^2 \tag{21}$$

$$P_h[(X,\alpha),(X+e_ih,\alpha)\,|\,U] = \frac{\frac{1}{2}\sigma^2 + h(U(t)-Z(t))^{\pm}}{Q_h(i,X,U,\alpha))} \tag{22}$$

$$P_h[(X,\alpha),(X,\beta)\,|\,U] = \frac{q_{\alpha\beta}h^2}{Q_h(i,X,U,\alpha)} \tag{23}$$

where $f^+(\cdot) = \max(0, f(\cdot))$ when $f(\cdot) \ge 0$, whereas $f^-(\cdot) = \max(0, -f(\cdot))$.

Define $P_h[(X,\alpha),(X',\alpha)\,|\,U] = 0$ for $X' \in \partial G_h^+$, and the following dynamic programming equation can be gotten

$$V_{0h}(i,X,\alpha) = \min_{U(\cdot)\in\pi}\left\{ \frac{h^2 G(X(t),U(t),\alpha)}{Q_h(i,X,U,\alpha)} + \sum_{X'\in G} \frac{Q_h(i,X,U,\alpha)}{Q_h(i,X,U,\alpha)+q_{\alpha\alpha}h^2} P[(X,\alpha),(X',\alpha)\,|\,U]V_{0h}(i,X',\alpha) \right\} \tag{24}$$

$$V_{1h}(i,X,\alpha) = \min_{U(\cdot)\in\pi}\left\{ \sum_{\beta\ne\alpha} P_h[(X,\alpha),(X,\beta)\,|\,U]V_{1h}(i,X,\beta) + \sum_{X'\in G} P[(X,\alpha),(X',\alpha)\,|\,U]V_{1h}(i,X',\alpha) \right\} \tag{25}$$

The solutions of the dynamic programming equations (24) and (25) can construct the solution of the original optimal control problem approximately.

IV. Numerical Experiments

The solution of the numerical technique for the hybrid stochastic production system is shown with an example including the following specifications: n=2, and r_1=r_2=1.0, z_1=0.3, z_2=0.4, $\alpha \in E$={0, 1}. The other parameters are shown in Tab. 1. And the simulation results are shown in Fig. 1.

Fig. 1.1.a and Fig. 1.1.b correspond to the control policy $u_1(t)$, $u_2(t)$ according with $x_1(0)$ = -1.5, $x_2(0)$=1.0 for $\zeta(0)$=1, respectively. Fig. 1.2.a corresponds to the tendency of the value function $V(\cdot,\cdot)$ according with $x_1(t)$, $x_2(t) \in$ [-1.5, 1.0] for $\zeta(0)$=1, and Fig. 1.2.b displays the tendency of the value function $V(\cdot,\cdot)$ when $x_1(t)$ varieties from –1.5 to 1.0,

whereas $x_2(t)$ is a constant, for $\zeta(0)=1$. Simulation shows that the optimal production control policy is of bang-bang control policy, and of hedging point policy. The hedging point is around zero, i.e. zero-inventory policy is the optimal control policy for the system. These numerical results illustrate and confirm the method developed above. And the numerical experiments show that the policy not only keeps the system run at the least cost but makes the production meet the demand perfectly. Moreover the policy makes the production satisfy the customers in sum and balances all types of the products, keeping the inventory in low level.

TABLE I. Parameters of The System

c_1^+	c_1^-	c_2^+	c_2^-	q_{10}	q_{01}	ε	σ_1	σ_2	r_1	r_2
0.5	3.0	1.0	3.0	0.1	0.2	0.02	0.5	0.5	1.0	1.0

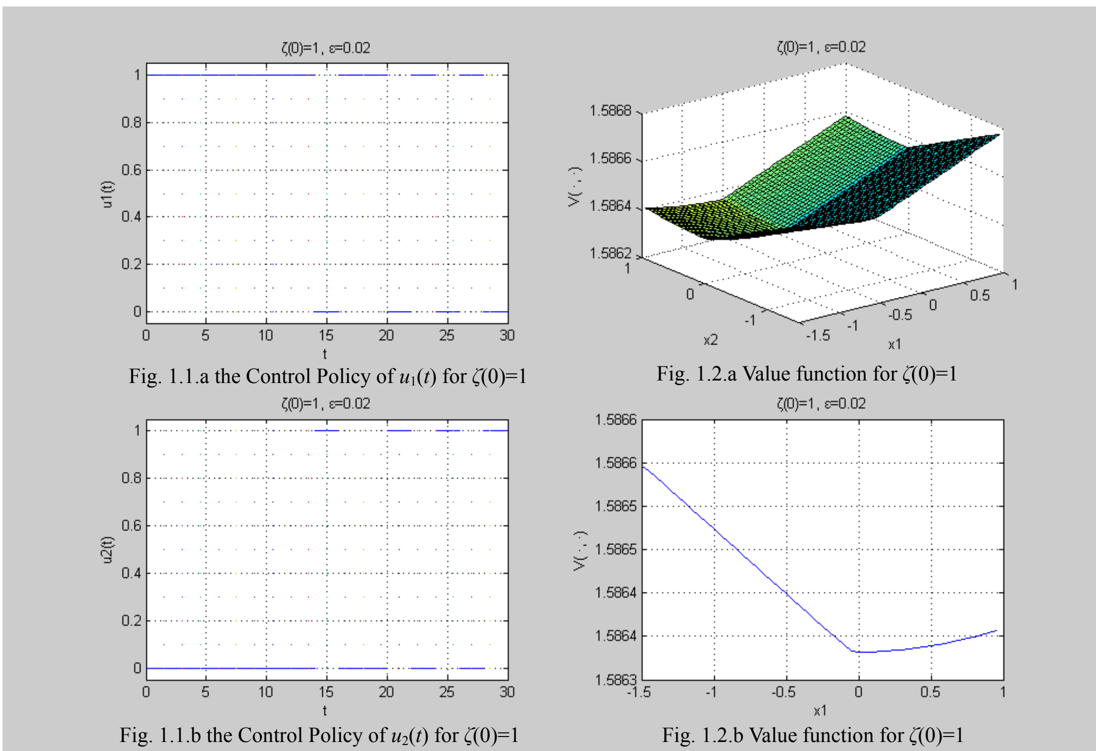

Fig. 1.1.a the Control Policy of $u_1(t)$ for $\zeta(0)=1$

Fig. 1.2.a Value function for $\zeta(0)=1$

Fig. 1.1.b the Control Policy of $u_2(t)$ for $\zeta(0)=1$

Fig. 1.2.b Value function for $\zeta(0)=1$

Fig. 1 the Simulation Results of the Numerical Experiments

V. CONCLUSIONS

The hybrid stochastic production system involving fast model and slow model, we studied in this paper, corresponds to a situation where a basically deterministic production system, the fast subsystem, is subject to infrequent model disruptions occurring randomly (i.e. machine failures process), the slow subsystem. The hybrid stochastic production system is formulated as a multi-time scales model and the value function of the system is decomposed over multi-time scales. Compared to the hierarchical production model with singularly perturbation proposed by [9] or [6], our modeling and optimization deal with the problem from a point of view of system engineering. It is encouraging that the numerical solution of the dynamic programming equations with multi-time scales characterizing the optimal control policy, has been obtained since a hybrid stochastic production example has been solved. The method of modeling and optimization can be extended to these systems involving hybrid state.

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