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H_∞ Control with Preview and Delay

Yohei Kuroiwa and Hidenori Kimura

***Abstract*— An H_∞ control problem with simultaneous consideration of preview and delay is solved, where past signals and future signals are essential to describe behaviors of the control system. Solvability conditions and complete controller parameterization are derived in the frequency domain within the scope of finite dimensional computations, that is, infinite dimensional aspects attributed to preview and delay are transformed to equivalent finite dimensional problems. The two-degree-of-freedom version is also considered.**

I. INTRODUCTION

In designing control systems, it is sometimes reasonable to assume that the controller can make use of the finite preview information with respect to the reference signals beyond the present time, which can be used for enhancing a tracking performance, and improving a disturbance rejection. A preview control, in the context of tracking, amounts to tracking a delayed reference, and the benefit of preview has been studied by various authors [1][2].

In this paper, we first solve an H_∞ control problem for plants with both preview and delay using J-spectral factorization approach. The full information case, where control inputs and external inputs are delayed respectively, was discussed in the framework of semigroup theory in the time domain [3]. Our scenario to reach the solution in the frequency domains is based on J-spectral factorization approach [4]. An infinite dimensional J-spectral factorization corresponding to an irrational transfer function of preview part is eventually transformed to an equivalent finite dimensional J-spectral factorization by means of a partial fractional expansion of the irrational transfer function into an irrational stable part and a rational one like [5][6].

The following new facts are clarified: An irrationality of the derived model matching problem does depend on a difference between preview time and delay time, where delay and preview do not appear respectively. The solvability conditions and the complete parameterization of controller can be derived within the scope of finite dimensional computations. The designed controller is equipped with the separate structure between rational parts and infinite dimensional stable parts, which are attributed to preview and delay.

Next, we discuss the H_∞ preview tracking control problem in the two-degree-of-freedom setting. Essential differences from one-degree-of-freedom case lie in the internal controller structure attributed to the preview. In the one-degree-of-freedom case, the transfer function resulted from preview is involved in the feedback loop, while in the two-degree-of-freedom case, the effect of preview is realized as a pure feedforward action in the feedforward controller.

The Division of Optimization and Systems Theory, RoyalInstitute of Technology, SE 100 44 Stockholm, Sweden (e-mail: yohei@math.kth.se). Phone: 46 8 790 66 60

Biological Control Systems Laboratory, Bio-mimetic Control Centre, The Institute of Physical and Chemical Research, Shimo-shidami, Moriyama-ku, Nagoya 463-0003, Japan.

II. H_∞ CONTROL WITH PREVIEW AND DELAY

A. Notation

We begin with a brief description of the notation. Let $G(s)$ be a transfer function, where s denotes complex variable. Then, the conjugate of $G(s)$ is defined as

$$G^{\sim}(s) := G^T(-s).$$

$LFT(Linear\ Fractional\ Transformation)$ is defined as

$$LFT(\Phi; K) := \Phi_{11} + \Phi_{12}K(I - \Phi_{22}K)^{-1}\Phi_{21}.$$

To represent feedback interconnection, LFT is the major notation in the control theory area. In [7], the alternative representation of bilinear transformation, HM, which denotes $HoMographic\ Transformation$, is defined as

$$HM(\Phi; K) := (\Phi_{11}K + \Phi_{12})(\Phi_{21}K + \Phi_{22})^{-1}.$$

DHM, the dual notion of HM, is also defined as

$$DHM(\Phi; K) := -(\Phi_{11} - K\Phi_{21})^{-1}(\Phi_{12} - K\Phi_{22}).$$

HM and DHM satisfy the following cascade properties

$$\begin{aligned} HM(\Psi_1\Psi_2; K) &= HM(\Psi_1; HM(\Psi_2; K)), \\ DHM(\Psi_1\Psi_2; K) &= DHM(\Psi_2; DHM(\Psi_1; K)). \end{aligned}$$

The following notation is used to denote the dimension of the vector, for example, $dim(r) = n_r$.

B. Problem Formulation

Consider a feedback control system depicted in Fig.1, where the plant to be controlled is a linear time-invariant MIMO system with input delay, whose transfer function is given by

$$\begin{aligned} P &:= e^{-sh}P_r, \\ h &: delay\ time, \\ P_r &: strictly\ proper\ transfer\ function, \end{aligned}$$

and the transfer function of a feedback controller is denoted by K_b, where we suppress the variable s for simplicity.

The design objective is to find a feedback controller K_b satisfying the following specifications.

Control Objectives:

1. The closed loop system is internally stable.

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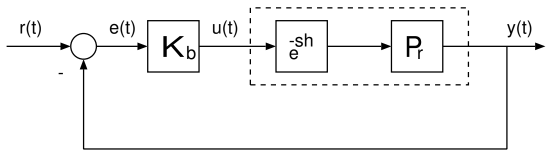

Fig. 1. Basic Feedback Scheme

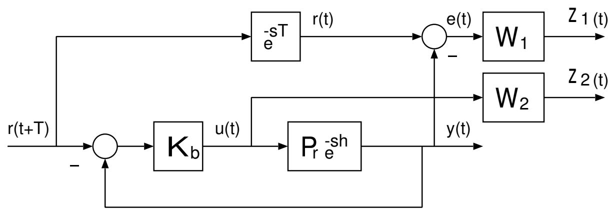

Fig. 2. Representation of Tracking with Preview

2. The control input u is adjusted not to be excessively large.
3. The plant output y tracks the reference signal r.

One of the main issues to be investigated in this paper is the use of noncausal actions for tracking. Preview control is a means of using the future information of the reference input for control, so that the feedback controller K_b can make use of reference inputs r up to time $t+T$ to reduce tracking errors at time t, where T denotes a preview time. In the context of tracking, it is identical to $tracking\ a\ delayed\ reference$. Fig.2 represents our control scheme.

The tracking error is defined as

$$e(t) := r(t) - y(t). \tag{1}$$

The Laplace transform of the preview tracking error $e(t)$ can be expressed as

$$e = (I - PK_b(I + PK_b)^{-1}e^{sT})r, \tag{2}$$
$$P = e^{-sh}P_r,$$

where transfer function is compatible dimension. In order to achieve the control objectives, we define the controlled output z as

$$\begin{aligned} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} &:= \begin{bmatrix} W_1 e \\ W_2 u \end{bmatrix} \\ &= \begin{bmatrix} W_1(I - PK_b(I + PK_b)^{-1}e^{sT}) \\ W_2K_b(I + PK_b)^{-1} \end{bmatrix} r, \end{aligned} \tag{3}$$

where W_1 and W_2 are weightings for penalizing the tracking error and input power, respectively. The assumptions are made through the paper as follows:

Assumptions:
1. W_1 and W_2 are stable transfer functions.
2. W_1 is a strictly proper transfer function.
3. The preview time T is longer than the delay one, i.e.,

$$T > h, \tag{4}$$

which is an essential presumption in this paper. In other words, the difference between preview time and delay time

$$\Delta := T - h > 0 \tag{5}$$

plays crucial roles in this paper.

Now, the H_∞ preview tracking control problem with delay is formulated as follows:

Problem:
For a given $\gamma > 0$, find K_b which internally stabilizes the closed loop system and satisfies

$$\left\| \begin{bmatrix} W_1(e^{-sT}I - PK_b(I + PK_b)^{-1}) \\ W_2K_b(I + PK_b)^{-1} \end{bmatrix} \right\|_\infty < \gamma, \tag{6}$$

$$P = e^{-sh}P_r. \tag{7}$$

This is a sub-optimal H_∞ control problem containing infinite dimensional transfer functions.

We can transform this problem to an equivalent finite dimensional one. First, a parameterization of stabilizing controller for systems with delay enables to transform the original problem (6) to a model matching problem, where the input delay is absorbed into the preview. Then, the model matching problem comprising irrational transfer function is shown to be solvable by a finite dimensional J-spectral factorization approach, through a partial fractional expansion of the irrational transfer function into an irrational stable part and a rational one.

C. Preliminaries

We shall introduce a partial fractional expansion of an irrational transfer function into a rational transfer function and an irrational stable one whose impulse response has a compact support. This is based on the fact that the input delay transfer function, $e^{-sh}P_r$, has only finite number of pole. A state-space realization of the plant is introduced to denote this factorization explicitly, and factorized such that

$$\begin{aligned} e^{-sh}P_r &:= e^{-sh}C(sI - A)^{-1}B \\ &= P_{r,u} - \Lambda, \end{aligned} \tag{8}$$

where

$$P_{r,u} := Ce^{-Ah}(sI - A)^{-1}B$$

is a rational part and

$$\Lambda := C(e^{-sh}I - e^{-Ah})(sI - A)^{-1}B$$

is an irrational stable one, respectively. This factorization plays an important role in this paper. A doubly coprime factorization of systems with input delay obtained in [8][9][10] and references therein enables to design a controller ignoring the delay. Assume that the doubly coprime factorization of $e^{-sh}P_r = NM^{-1} = \bar{M}^{-1}\bar{N}$ satisfies

$$\begin{bmatrix} \bar{X} & \bar{Y} \\ -\bar{N} & \bar{M} \end{bmatrix}\begin{bmatrix} M & -Y \\ N & X \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, \qquad (9)$$

where these matrices N, M, X, Y, etc., belong to H_∞ with compatible dimensions. It is well known that a parameterization of stabilizing controller K_b in the sense of in Fig.1 is given in terms of

$$K_b = (\bar{X} + Q\bar{N})^{-1}(\bar{Y} - Q\bar{M}), \;\; Q \in H_\infty^{n_u \times n_r}. \qquad (10)$$

The above coprime factorization and the parameterization of stabilizing controller can be derived using the coprime factorization of the rational part of plant $P_r = N_r M_r^{-1} = \bar{M}_r^{-1}\bar{N}_r$, that is, $N = N_r e^{-sh}$, $M = M_r$ and as follows:

Lemma 1[9][10]:
There exists a rational transfer function P_0 such that

$$\Sigma := P_0 - P_r e^{-sh} \in H_\infty^{n_y \times n_u} \qquad (11)$$

is an irrational stable transfer function whose impulse response has a compact support. A doubly coprime factorization of P given in (9) can be chosen such that

$$\begin{bmatrix} M & -Y \\ N & X \end{bmatrix} = \begin{bmatrix} I & 0 \\ -\Sigma & I \end{bmatrix}\begin{bmatrix} M_0 & -Y_0 \\ N_0 & X_0 \end{bmatrix} \qquad (12)$$

and

$$\begin{bmatrix} \bar{X} & \bar{Y} \\ -\bar{N} & \bar{M} \end{bmatrix} = \begin{bmatrix} \bar{X}_0 & \bar{Y}_0 \\ -\bar{N}_0 & \bar{M}_0 \end{bmatrix}\begin{bmatrix} I & 0 \\ \Sigma & I \end{bmatrix}, \qquad (13)$$

where M_0,N_0, etc., constitute a doubly coprime factorization of P_0.

Here we shall examine the structure of all stabilizing controllers of $e^{-sh}P_r$, which is parameterized by a free parameter $Q \in H_\infty^{n_u \times n_y}$. It is given by

$$\begin{aligned} K_b &= -DHM(\begin{bmatrix} \bar{X} & \bar{Y} \\ -\bar{N} & \bar{M} \end{bmatrix}; Q) \\ &= -DHM(\begin{bmatrix} \bar{X}_0 & \bar{Y}_0 \\ -\bar{N}_0 & \bar{M}_0 \end{bmatrix}\begin{bmatrix} I & 0 \\ \Sigma & I \end{bmatrix}; Q) \\ &= -DHM(\begin{bmatrix} I & 0 \\ \Sigma & I \end{bmatrix}; DHM(Z_r; Q)), \qquad (14) \end{aligned}$$

where Z_r is defined by

$$Z_r := \begin{bmatrix} \bar{X}_0 & \bar{Y}_0 \\ -\bar{N}_0 & \bar{M}_0 \end{bmatrix}.$$

Let define K_0 and T_r as follows:

$$K_0 := DHM(Z_r; Q), \qquad (15)$$

or

$$K_0 = LFT(T_r; Q), \qquad (16)$$

$$T_r := \begin{bmatrix} -\bar{X}_0^{-1}\bar{Y}_0 & \bar{X}_0^{-1} \\ \bar{M}_0 + \bar{N}_0\bar{X}_0^{-1}\bar{Y}_0 & -\bar{N}_0\bar{X}_0^{-1} \end{bmatrix},$$

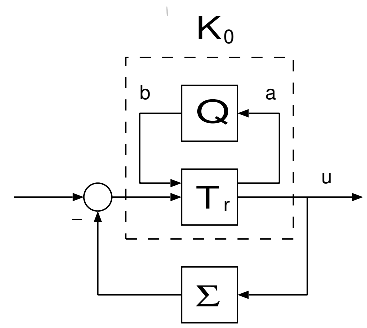

Fig. 3. Delay Free Controller Parameterization

and the equivalence of (15) and (16) is verified directly. Cascade property of DHM enables to represent the internal structure of a parameterization of stabilizing controller, explicitly. It has a structure divided into an infinite dimensional part and an rational one. In particular, we also rewrite (14) as

$$\begin{aligned} K_b &= -DHM(\begin{bmatrix} I & 0 \\ \Sigma & I \end{bmatrix}; K_0) \qquad (17) \\ &= -(I - \Sigma K_0)^{-1}K_0, \end{aligned}$$

which is shown it Fig.3. This structure is essentially identical to that of celebrated Smith predictor.

D. Solvability Conditions and Complete Parameterization of Controller

Due to (4), the sub-optimal H_∞ control problem (6) can be transformed to the following model matching problem by means of its delay free stabilizing controller parameterization (17).

Lemma 2:
The sub-optimal H_∞ control problem (6) is transformed to the following model matching problem with respect to $Q \in H_\infty^{n_u \times n_r}$,

$$\left\| \begin{bmatrix} e^{-s\Delta}W_1 - W_1 N_r \bar{Y}_0 \\ W_2 M_r \bar{Y}_0 \end{bmatrix} C_i^{\sim} + \begin{bmatrix} W_1 N_r \\ -W_2 M_r \end{bmatrix} Q C_0 \right\|_\infty < \gamma, \qquad (18)$$

where the components are the doubly coprime factors of $e^{-sh}P_r$ satisfying Bezout identity (9), furthermore, C_i and C_o are co-inner and co-outer factors of $\bar{M}_r$ satisfying $\bar{M}_r = C_o C_i$.

Proof: From (13) and (10), the sub-optimal H_∞ control problem (6) is directly transformed to the following model matching problem,

$$\left\| \begin{bmatrix} W_1(e^{-sT}I - e^{-sh}(N_r\bar{Y}_0 - N_r Q\bar{M}_r)) \\ W_2(M_r\bar{Y}_0 - M_r Q\bar{M}_r) \end{bmatrix} \right\|_\infty < \gamma.$$

■

In (18), $e^{-s\Delta}$ is the only irrationality in the whole model matching problem. It is interesting that the delay time h

and the preview time T do not enter the problem, but their difference $\Delta = T - h$ does.

For notational simplicity, we define A_b, B_b and $\hat{Q}$ as follows:

$$\begin{aligned} A_b &:= \begin{bmatrix} e^{-s\Delta}W_1 - W_1 N_r \bar{Y}_0 \\ W_2 M_r \bar{Y}_0 \end{bmatrix} C_i^{\sim}, \\ B_b &:= \begin{bmatrix} W_1 N_r \\ -W_2 M_r \end{bmatrix}, \\ \hat{Q} &:= QC_o. \end{aligned} \quad (19)$$

Then the model matching problem (18) is rewritten as

$$||A_b + B_b \hat{Q}||_\infty < \gamma. \quad (20)$$

The solvability conditions and a parameterization of $\hat{Q}$ are derived by a finite dimensional J-spectral factorization.

Let us introduce G_b, J_γ and $\hat{J}_\gamma$ as

$$\begin{aligned} G_b &:= \begin{bmatrix} B_b & A_b \\ 0 & I_{n_r} \end{bmatrix}, \\ J_\gamma &:= \begin{bmatrix} I_{n_{z_1}} & 0 & 0 \\ 0 & I_{n_{z_2}} & 0 \\ 0 & 0 & -\gamma^2 I_{n_r} \end{bmatrix}, \\ \hat{J}_\gamma &:= \begin{bmatrix} I_{n_u} & 0 \\ 0 & -\gamma^2 I_{n_r} \end{bmatrix}. \end{aligned}$$

Let a rational transfer function R_b and an irrational stable one Λ_b satisfy

$$e^{-s\Delta}\Pi^{-1} N_r^{\sim} W_1^{\sim} W_1 C_i^{\sim} = R_b - \Lambda_b, \quad (21)$$

where

$$\begin{aligned} \Pi &:= B_b^{\sim} B_b \\ &= N_r^{\sim} W_1^{\sim} W_1 N_r + M_r^{\sim} W_2^{\sim} W_2 M_r. \end{aligned}$$

Here we define Θ_b as

$$\begin{aligned} \Theta_b &:= \begin{bmatrix} I & 0 \\ -\Lambda_b^{\sim} & I \end{bmatrix} G_b^{\sim} J_\gamma G_b \begin{bmatrix} I & -\Lambda_b \\ 0 & I \end{bmatrix} \\ &= \begin{bmatrix} \Pi & \Theta_{b,12} \\ \Theta_{b,12}^{\sim} & \Theta_{b,22} \end{bmatrix}, \end{aligned}$$

where

$$\begin{aligned} \Theta_{b,12} &:= \Pi(R_b - \bar{Y}_0 C_i^{\sim}), \\ \Theta_{b,22} &:= \Theta_{12}^{\sim}(\Pi)^{-1}\Theta_{12} \\ &\quad + C_i W_1^{\sim}(I - W_1 N_r \Pi^{-1} N_r^{\sim} W_1^{\sim}) W_1 C_i^{\sim} \\ &\quad -\gamma^2 I. \end{aligned}$$

The point is that Θ_b is rational and proper, and the irrational factor $\begin{bmatrix} I & -\Lambda_b \\ 0 & I \end{bmatrix}$ is bistable. Moreover, the $(1,1)$ block of Θ_b is identical to that of $G_b^{\sim} J_\gamma G_b$. So the irrational J-spectral factorization problem is equivalent to finding a rational J spectral factor V_r such that

$$\Theta_b = V_r^{\sim} \hat{J}_\gamma V_r, \quad (22)$$

and the irrational J-spectral factor of $G^{\sim} J_\gamma G$ is obtained by

$$V = V_r \begin{bmatrix} I & \Lambda_b \\ 0 & I \end{bmatrix}. \quad (23)$$

We are now ready to show the main theorem of this paper.

Theorem 1:
There exists a stabilizing controller K_b satisfying (6), if and only if the rational transfer function Θ_b given by (22) has a J-spectral factorization

$$\Theta_b = V_r^{\sim} \hat{J}_\gamma V_r. \quad (24)$$

Then, all stabilizing controllers parameterized by a free parameter $U \in H_\infty^{n_u \times n_r}$ with $||U||_\infty < \gamma$ are given by

$$\begin{aligned} K_b &= -DHM(Z_r \begin{bmatrix} I & 0 \\ \Sigma & I \end{bmatrix}; \\ &\qquad HM(\begin{bmatrix} I & -\Lambda_b \\ 0 & I \end{bmatrix} V_r^{-1}; U) C_o^{-1}). \end{aligned} \quad (25)$$

Proof: The first half of the proof has already been done. To show the rest of the proof, assume that a V_r in (22) really exists. Then the set of all $\hat{Q} \in H_\infty^{n_u \times n_r}$ satisfying $||A_b + B_b \hat{Q}||_\infty < \gamma$ is given by

$$\begin{aligned} \hat{Q} &= HM(V^{-1}; U) \\ &= HM(\begin{bmatrix} I & -\Lambda_b \\ 0 & I \end{bmatrix}; HM(V_r^{-1}; U)). \end{aligned} \quad (26)$$

Combining (14), (19) and (26) yields (25). ■

Remark: The central solution of $\hat{Q}$ in (26), that is, $U = 0$, is given by

$$\begin{aligned} \hat{Q} &= \hat{Q}_r - \Lambda_b, \\ \hat{Q}_r &:= W_{r,12} W_{r,22}^{-1}, \end{aligned}$$

where $\hat{Q}_r$ is a finite dimensional transfer function, and V_r^{-1} is represented as

$$V_r^{-1} := \begin{bmatrix} W_{r,11} & W_{r,12} \\ W_{r,21} & W_{r,22} \end{bmatrix}.$$

The rational part of Q is rewritten by

$$HM(V_r^{-1}; U) = LFT(\Xi_r; U), \quad (27)$$

$$\Xi_r := \begin{bmatrix} W_{r,12} W_{r,22}^{-1} & W_{r,11} - W_{r,12} W_{r,22}^{-1} W_{r,21} \\ W_{r,22}^{-1} & -W_{r,22}^{-1} W_{r,21} \end{bmatrix},$$

so the block diagram of Q satisfying (20) is depicted in Fig.4. It is constructed by a parallel connection between a rational part, that is, finite dimensional transfer function Ξ_r and an infinite dimensional part Λ_b. The structure of designed controller finally obtained is shown in Fig.5, where the rational transfer functions, T_r and Ξ_r, are separated from the infinite dimensional transfer functions, Σ and Λ_b. A feedback interconnection of Σ, which plays a role of a predictor like Smith predictor, comes from a delay free

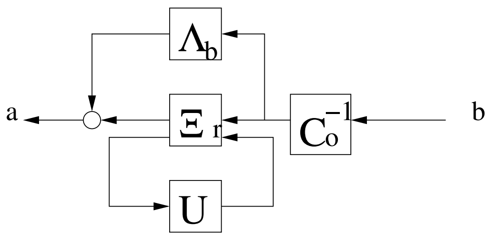

Fig. 4. The Block Diagram of Q

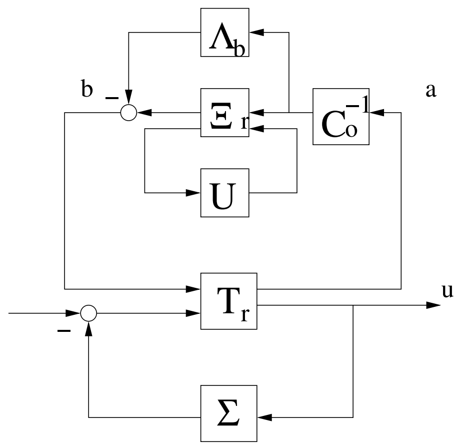

Fig. 5. The Complete Parameterization of Controller

controller parameterization. It possesses internal structure of parallel connection of Λ_b attributed to preview effects, which is also involved in a feedback loop interfering with Σ. On account of the existence of preview, the predictor can make use of future information.

III. Two Degree of Freedom Setting

In the two degree of control systems, the control input is generated by the reference input and the plant output. Fig.6 illustrates a standard form of the two degree of freedom controller with preview. The control input u is generated by

$$u = K_f e^{sT} r + K_b y,$$

where K_f and K_b denote feedforward controller and feedback controller, respectively. Since we shall intend to examine the design problem of feedforward controller in the H_∞ performance criteria, it is desirable that the controlled outputs are independent of the feedback controller [11]. Therefore, we adopt a two degree of freedom control scheme shown in Fig.7, and this two degree of freedom controller yields an appropriate closed loop transfer functions from the reference input r to the plant output y and to the control input u, where the feedback controller and the feedforward controller do not interfere with each other. These transfer functions are derived as

$$y = e^{-sh} N_r K_f e^{sT} r, \qquad u = M_r K_f e^{sT} r, \qquad (28)$$

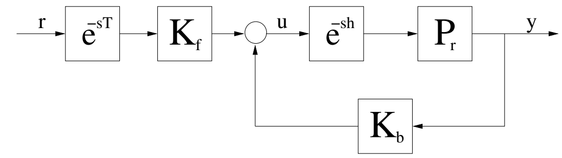

Fig. 6. Standard Two Degree of Freedom Control Scheme with Preview

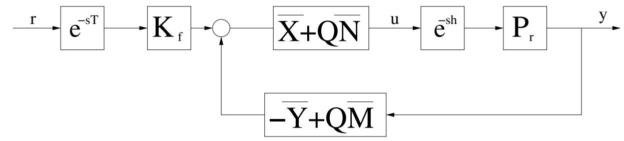

Fig. 7. Two Degree of Freedom Controller

where N_r and M_r satisfy $P_r = N_r M_r^{-1}$. Similar to (2), preview tracking error is defined as

$$\begin{aligned} e &= (I - e^{-sh} N_r K_f e^{sT}) r \\ &= (I - e^{s\Delta} N_r K_f) r, \end{aligned} \qquad (29)$$

where the unstable poles do not affect preview tracking errors. The controlled output is also defined as

$$\begin{aligned} z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} &:= \begin{bmatrix} W_1 e \\ W_2 u \end{bmatrix} \\ &= \begin{bmatrix} W_1(I - e^{-sh} N_r K_f) \\ W_2 M_r K_f \end{bmatrix} e^{sT} r, \end{aligned} \qquad (30)$$

where W_1 and W_2 are weightings for penalizing the tracking error and input power satisfying the assumptions shown in the previous sections.

Similar to (6), the H_∞ preview tracking problem with delay in the two degree of freedom setting is formulated as follows:

Problem:
For a given $\gamma > 0$, find a stable feedforward controller K_f such that

$$\left\| \begin{bmatrix} W_1(e^{-s\Delta} I - N_r K_f) \\ W_2 M_r K_f \end{bmatrix} \right\|_\infty < \gamma. \qquad (31)$$

For notational simplicity, we define A_f and B_f as

$$A_f := \begin{bmatrix} e^{-s\Delta} W_1 \\ 0 \end{bmatrix}, \qquad B_f := \begin{bmatrix} -W_1 N_r \\ W_2 M_r \end{bmatrix},$$

and the model matching problem (31) is rewritten as

$$\|A_f + B_f K_f\|_\infty < \gamma. \qquad (32)$$

As already mentioned, the partial fractional expansion techniques enable to transform the infinite dimensional problem to an equivalent finite dimensional one. Let a rational transfer function R_f and an irrational stable one Λ_f satisfy

$$e^{-s\Delta} \Pi^{-1} N_r^\sim W_1^\sim W_1 := R_f - \Lambda_f. \qquad (33)$$

Let us introduce G_f as

$$G_f := \begin{bmatrix} B_f & A_f \\ 0 & I \end{bmatrix}.$$

The solver of this model matching problem is analogous to Theorem 1. The solvability condition and a parameterization of K_f are derived in the context of a finite dimensional J-spectral factorization.

Theorem 2:
The model matching problem (32) is solvable, if and only if a bistable V_f, whose (1,1) block $V_{f,11}$ is bistable, exists such that

$$\Theta_f = V_f^{\sim} \hat{J} V_f, \tag{34}$$

where

$$\Theta_f := \begin{bmatrix} \Pi & -\Pi R_f \\ -R_f^{\sim}\Pi & \Theta_{f,22} \end{bmatrix},$$

$$\begin{aligned} \Theta_{f,22} &= R_f \Pi R_f^{\sim} + \\ &\quad W_1^{\sim}(I - W_1 N_r \Pi^{-1} N_r^{\sim} W_1^{\sim}) W_1 - \gamma^2 I. \end{aligned}$$

Then, all feedforward controller parameterized by a free parameter $U \in H_\infty^{n_u \times n_r}$ with $||U||_\infty < \gamma$ are given by

$$K_f = HM(\begin{bmatrix} I & -\Lambda_f \\ 0 & I \end{bmatrix}; HM(V_f^{-1}; U)). \tag{35}$$

$Proof$: The derivation is analogous to that of Theorem 1. The required infinite dimensional J-spectral factorization of $G_f^{\sim} J_\gamma G_f$ is transformed to an equivalent finite dimensional J-spectral factorization of Θ_f,

$$\Theta_f = \begin{bmatrix} I & 0 \\ -\Lambda_f^{\sim} & I \end{bmatrix} G_f^{\sim} J_\gamma G_f \begin{bmatrix} I & -\Lambda_f \\ 0 & I \end{bmatrix}, \tag{36}$$

which is a rational transfer function. As a result, the irrational J-spectral factor of $G_f^{\sim} J_\gamma G_f$ is obtained by $V_f \begin{bmatrix} I & \Lambda_f \\ 0 & I \end{bmatrix}$. Furthermore, the set of all K_f satisfying $||A_f + B_f K_f||_\infty < \gamma$ is given by (35) ■.

$Remark$: Let $\Xi_{r,f}$ satisfies

$$HM(V_f^{-1}; U) = LFT(\Xi_{r,f}; U),$$

then, the feedforward controller parameterization shown in Fig.8 contains a rational transfer function $\Xi_{r,f}$ and a parallel connection with a stable infinite dimensional transfer function Λ_f, which is attributed to the existence of a preview. In contrast with the one degree of freedom setting, where the infinite dimensional transfer function attributed to the existence of preview is involved in the feedback loop, preview effects are realized as a pure feedforward action in the feedforward controller.

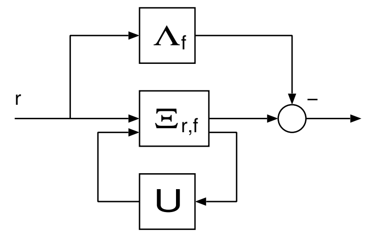

Fig. 8. The Parameterization of Feedforward Controller

IV. Conclusion

In this paper, the fundamental solvability conditions and complete parameterization of controller for an H_∞ control problem for systems with preview and delay have been obtained in the frequency domain. To transform the original problem to a model matching problem, a delay free stabilizing controller parameterization which enables to transform the problem to delay free case is introduced, and the infinite dimensional J-spectral factorization is eventually transformed to an equivalent finite dimensional J-spectral factorization. It should be noted that we have assumed the preview time longer than the delay time is essential to its solution, and its difference specify an irrationality of the model matching problem. The designed controller has a separate structure between rational parts and stable infinite dimensional parts. H_∞ control problem with preview and delay in the two degree of freedom setting is also investigated within the scope of finite dimensional manipulations.

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