Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Gradient Algorithm for Structural Parameter Estimation and Nonlinear Restoring Forces

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***Abstract*—Civil structures undergo hysteresis cycles due to cracking or yielding when subjected to severe strong motions. Seismic instrumentation, structural monitoring and system identification techniques have been used in the past years to assess damage. The present research proposes a gradient algorithm to simultaneously identify the combined driving and nonlinear restoring forces, and structural parameters, such as damping and stiffness in a single-degree-of-freedom structure. Simulations are carried out using the El Centro (1940) and the Mexico City (1985) seismic records. Encouraging results are obtained for the identification of the parameters and the estimation of the driving and restoring forces.**

I. INTRODUCTION

STRONG motions may cause damage in civil structures by cracking or yielding its resistant elements. Damage in a structure is associated to energy dissipation through hysteresis. The area enclosed by a hysteresis cycle represents the energy dissipation and gives evidence of the time variant relation of the stiffness [4]. Lately, several buildings have been instrumented in order to monitor their structural health. The identification of such systems has been the focus of researchers in the areas of civil and automatic control engineering. Parametric identification of civil structures subjected to hysteresis is widely used for structural health monitoring applications.

In [3] a very thorough literature review of the identification and control techniques in civil structures is presented. A statistical technique for structural health monitoring considering the typical vibration signature of mechanical systems is proposed in [12]. A Least Squares method for the parametric estimation of linear and nonlinear structures is given in [9]. An orthogonal algorithm for parameter estimation, based on the NARMAX model is proposed in [5]. A recursive Least Squares algorithm with a combined parametric and non-parametric estimation to identify the system is presented in [8]. A Gauss Newton method for a two-stage iterative Least Squares algorithm is proposed in [15]. A spectral method for structural system identification is presented in [11]. A non-parametric identification approach for hysteretic systems considering the Duhem operator is presented in [10]. A neural network to on-line estimate the restoring force in nonlinear structures is considered in [6]. The estimation of the parameters using a three-stage method with a sequential regression analysis, Gauss Newton optimization and Least Squares with an extended Kalman filter is reviewed in [7]. An examination of the use of the ERA-OKID algorithm to estimate linear parameters of structures with mild-nonlinearities is presented in [2].

It could be noted that several methods have appeared in the last couple of decades. Nevertheless, few are on-line algorithms and their complexity is considerable. It is worth noting that on-line algorithms would be necessary for closed loop control of structures. The present research proposes an on-line algorithm, analytically simple and computationally easy to implement on an instrumented structure. The simulations consider the Bouc-Wen differential hysteresis model [14]. Two features of this model are that it has the ability to represent several hysteresis shapes and it correlates well with laboratory tests.

In this paper, we propose a gradient algorithm that identifies linear structural parameters such as damping and stiffness of a single-degree-of-freedom system (SDOF), where a mass estimate is known before hand. To estimate the mass of a SDOF is relatively easy in civil structures because the volumetric weight of the materials and the imposed loads are derived from field experience. Aside from identifying the structural parameters, the algorithm simultaneously estimates a variable that combines both, the driving and the nonlinear restoring forces (combined forces, CF) acting on the system. The seismic records of El Centro

Manuscript received September 11, 2003.

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(1940) and Mexico City at SCT station (1985) are used in the simulations to verify the effectiveness of the algorithm. The identification of the elastic parameters of the structure and the CF are presented. It is important to note that if the SDOF does not undergo hysteresis, the estimate of the CF will be directly the driving force.

II. Bouc-Wen Hysteresis Model

One of the most popular differential hysteresis models for civil structures is the Bouc-Wen model [14]. Equation (1) shows the mathematical model for a SDOF with two simultaneous differential equations. The first equation represents the motion of the system and the second represents the rate of change of the nonlinear restoring force.

$$\begin{aligned} &m\ddot{x} + c\dot{x} + \alpha kx + (1-\alpha)kz = f(t) \\ &\dot{z} = \frac{A\dot{x} - \nu\left(\beta|\dot{x}||z|^{n-1}z + \gamma\dot{x}|z|^{n}\right)}{\eta} \end{aligned} \tag{1}$$

The SDOF has the properties of mass m, damping c, initial stiffness k and the driving force $f(t)$ as a function of time. Variables x, $\dot{x}$ and $\ddot{x}$ represent the displacement, velocity and acceleration of the mass respectively. Variable z is the nonlinear time-dependent restoring force. A, ν, β, γ, η, and n are parameters which control the shape of the hysteresis loops and system degradation. It could be noted that variables A, α, η, and k control the initial tangent stiffness. Variables β, γ, and n control the transition between the initial stiffness to yielding. Variables ν and η were introduced in order to allow for the formulation of various degrading systems [1].

III. Gradient Algorithm

The proposed algorithm simultaneously identifies the damping and stiffness parameters as well as the CF of the SDOF, when the mass and the initial stiffness to yielding ratio α are known. The CF estimate includes the driving force and the nonlinear restoring force, thus, if the driving force is previously known, it is possible to estimate the restoring force of the system. Let us consider the following relationship (linear in the parameters),

$$y = \phi^T\theta + d \tag{2}$$

We want to put the motion of the system in (1) in this form. If from (1), the equation of motion is divided by the mass, (3) is obtained.

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{\alpha k}{m}x + (1-\alpha)\frac{k}{m}z = \frac{f(t)}{m} \tag{3}$$

If the driving and nonlinear restoring forces are associated, (4) is obtained.

$$\ddot{x} + \left(\frac{c}{m}\right)\dot{x} + \left(\frac{\alpha k}{m}\right)x = \left[\left(\frac{f(t)}{m}\right) - \left(\frac{(1-\alpha)k}{m}z\right)\right] \tag{4}$$

Note that the right hand term of (4) will be considered as d in the proposed algorithm. Rewriting (4) to have the form of (2), (5) is obtained.

$$\begin{aligned} &\ddot{x} = \begin{bmatrix} -x & -\dot{x} \end{bmatrix} \begin{bmatrix} \alpha\frac{k}{m} \\ \frac{c}{m} \end{bmatrix} + d \\ &y = \ddot{x} \end{aligned} \tag{5}$$

It could also be noted that in (5) the regression vector (6) and the parameter vector (7) are:

$$\phi = \begin{bmatrix} -x \\ -\dot{x} \end{bmatrix} \tag{6}$$

$$\theta = \begin{bmatrix} \alpha\frac{k}{m} \\ \frac{c}{m} \end{bmatrix} \tag{7}$$

Now, the estimated plant is of the form:

$$\hat{y} = \phi^T\hat{\theta} + \hat{d} \tag{8}$$

In (8) $\hat{y}$ is the estimated acceleration, ϕ is the regression vector with the state, $\hat{\theta}$ is the identified linear parameters vector and $\hat{d}$ is the estimated CF. The proposed gradient algorithm is:

$$e = \hat{y} - y \tag{9}$$

$$\dot{\hat{\theta}} = -\Gamma\phi e \tag{10}$$

$$\dot{\hat{d}} = -a\left(\hat{d} + \tau e\right) \tag{11}$$

Where Γ, a, and τ are positive constants and e is the estimation error.

IV. CONVERGENCE ANALYSIS

The following equalities are introduced for the convergence analysis of the proposed algorithm.

$$e = \hat{y} - y = \phi^T \tilde{\theta} + \tilde{d} \tag{12}$$

$$\tilde{\theta} = \hat{\theta} - \theta \tag{13}$$

$$\tilde{d} = \hat{d} - d \tag{14}$$

Also, the following bounds for the CF are assumed:

$$\begin{aligned} |d| &\le D \\ |\dot{d}| &\le E \end{aligned} \tag{15}$$

The candidate function for the convergence analysis is presented in (16).

$$V = \tfrac{1}{2}\tilde{\theta}^T\tilde{\theta} + \tfrac{1}{2}\tilde{d}^2 \tag{16}$$

Since (16) is represented in quadratic terms of the parameters and d, function V is definite positive. The rate of change with respect to time of (16) is represented by (17) as:

$$\dot{V} = \tilde{\theta}^T\dot{\tilde{\theta}} + \tilde{d}\dot{\tilde{d}} \tag{17}$$

Substituting (13) and (14) in (17) and regrouping some of the variables, the following equations are derived:

$$\dot{V} = -\tilde{\theta}^T\Gamma\phi e + \tilde{d}\dot{\tilde{d}} \tag{18}$$

$$\dot{V} = -\Gamma e\left(e - \tilde{d}\right) + \tilde{d}\dot{\tilde{d}} \tag{19}$$

$$\dot{V} = -\Gamma e^2 + \Gamma e\tilde{d} + \tilde{d}\dot{\tilde{d}} \tag{20}$$

$$\dot{V} = -\Gamma e^2 + \Gamma e\tilde{d} + \tilde{d}\left[-a\hat{d} - a\tau e - \dot{d}\right] \tag{21}$$

$$\dot{V} = -\Gamma e^2 + \Gamma e\tilde{d} - a\tilde{d}\hat{d} - a\tilde{d}\tau e - \tilde{d}\dot{d} \tag{22}$$

$$\dot{V} = -\Gamma e^2 - a\tilde{d}\left(\hat{d} + \tau e\right) + \tilde{d}\left(\Gamma e - \dot{d}\right) \tag{23}$$

$$\dot{V} = -\Gamma e^2 - a\tilde{d}\left(\tilde{d} + d + \tau e\right) + \tilde{d}\left(\Gamma e - \dot{d}\right) \tag{24}$$

Regrouping and introducing the absolute value in some of the terms, the following inequalities are obtained:

$$\begin{aligned} \dot{V} \le & -\Gamma e^2 - a\tilde{d}^2 + a\left|\tilde{d}\right|\left(|d| + \tau|e|\right) \\ & + \left|\tilde{d}\right|\left(\Gamma|e| + |\dot{d}|\right) \end{aligned} \tag{25}$$

$$\begin{aligned} \dot{V} \le & -\Gamma e^2 - a\tilde{d}^2 + \left|\tilde{d}\right|\left(aD + a\tau|e| + \Gamma|e| + E\right) \\ & \le 0 \end{aligned} \tag{26}$$

Completing squares in (26) it can be shown that

$$\dot{V} \le -\frac{1}{2}\Gamma e^2 - \frac{1}{2}a\tilde{d}^2 + \left|\tilde{d}\right|\left(aD + E\right) \le 0 \tag{27}$$

From (27) it is concluded that $\tilde{\theta}$ and $\tilde{d}$ remain bounded if (28) is fulfilled.

$$\left|\tilde{d}\right| \ge 2\left(D + \frac{E}{a}\right) \tag{28}$$

Thus, the convergence analysis shows that, in fact, the algorithm leads to stability and the error in CF is bounded by (28). In [1] it is shown that the restoring force z is bounded. From this result it is not difficult to conclude that all the states in (1) and the time derivative $\dot{z}$ are bounded. Then, assuming that the driving force is continuously differentiable with bounded first time derivative, we conclude that the term CF and its first time derivative are bounded. In this way, conditions (15) are satisfied.

V. SIMULATIONS

The SDOF was subjected to seismic excitations from the El Centro (1940) and Mexico City at SCT station (1985) records. Fig. 1 shows the acceleration record of El Centro and Fig. 2 shows the SCT record, both applied to the base of the SDOF.

The Mexico City record at SCT was held for 180 seconds. For the case of the simulations presented here, a reduced version of the record containing only the intense part of the earthquake motion was considered in order to reduce the simulation time. Fig. 3 shows the reduced version of the SCT record.

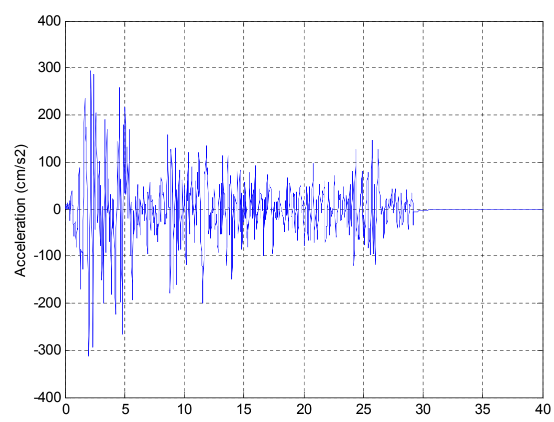

Fig. 1 Seismic record of El Centro, 1940.

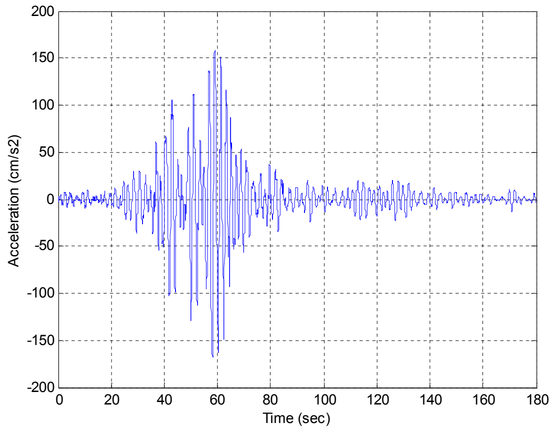

Fig. 2 Seismic record of SCT, 1985.

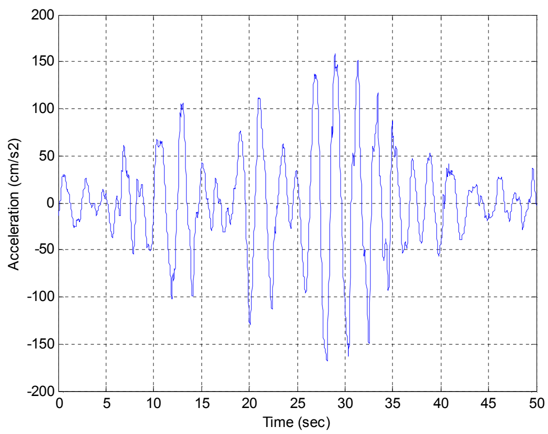

Fig. 3 Reduced version of the SCT record.

The SDOF was simulated using the following parameters: mass m = 9.14 kg s^2 / cm; stiffness k = 6472 kg / cm and damping c = 2.661 kg s / cm. Simulations were performed under Matlab Simulink [13]. Also, 5% white noise has been added to the theoretical acceleration in all the simulations. The earthquake record of El Centro was scaled 3.5 times and the SCT record was scaled 5.0 times. In this way, the structure undergoes hysteresis. Table I shows the constant values considered in the algorithm for the simulations.

TABLE I
CONSTANT VALUES IN THE SIMULATIONS

	a	τ	Γ=diag(•)
El Centro	100	12	$\begin{bmatrix}0.06\\0.000099\end{bmatrix}$
SCT	100	12	$\begin{bmatrix}0.90\\0.00085\end{bmatrix}$

Next, the simulation with the El Centro record is presented.

Fig. 4 shows the comparison between the theoretical (blue line) and the estimated (red line) CF. The calculated error, as the theoretical minus the estimated acceleration, is shown later in Fig. 5. The error is small as noted with respect to the scale of the CF. Fig. 6 shows the estimation of the stiffness k with respect to time.

It is easy to calculate the mass of the system if the loads are known before hand and if a typical value of $\alpha = 1/21$ is considered [1]. Therefore, parameters k and c are easy to derive.

Fig. 7 shows the estimation for damping with respect to time. Once the structural parameters are estimated, and for the case of these simulations in which the driving force is known, it is easy to estimate the nonlinear restoring force.

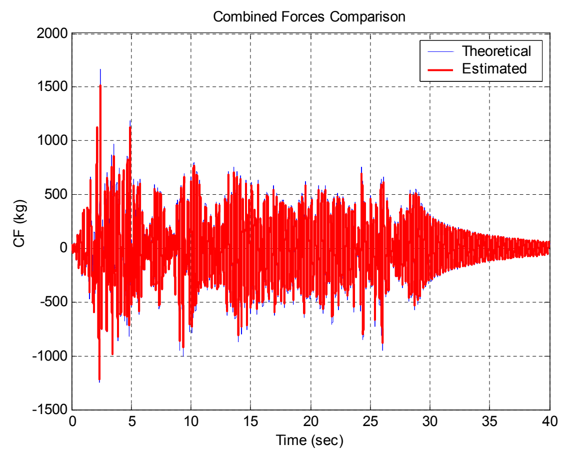

Fig. 4 CF comparison.

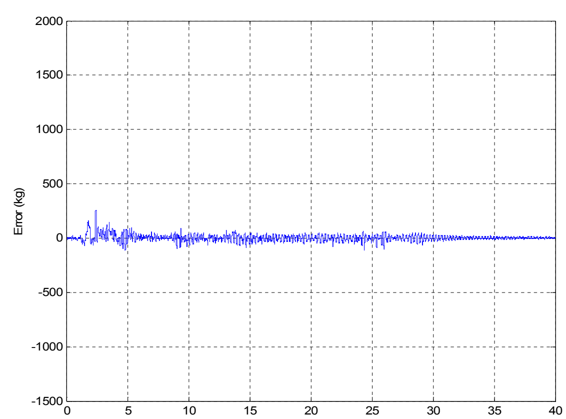

Fig. 5 CF estimation error.

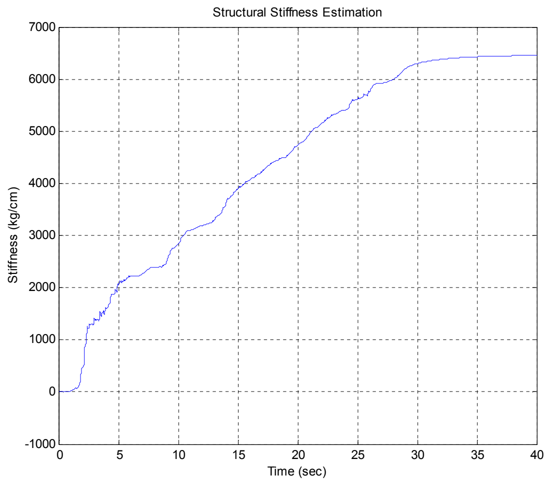

Fig. 6 Stiffness k estimation.

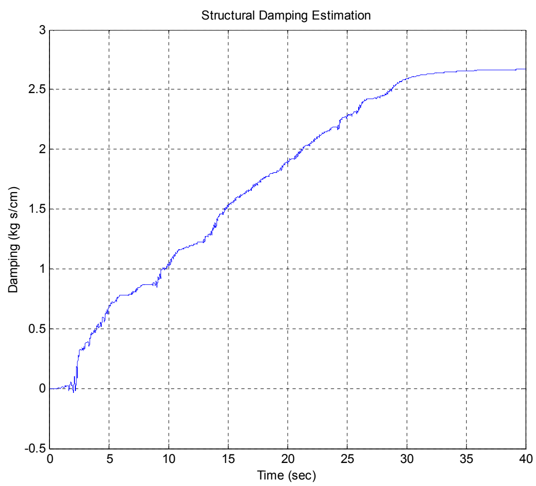

Fig. 7 Damping c estimation.

Fig. 8 shows the comparison between the theoretical (blue line) and the estimated (red line) hysteresis.

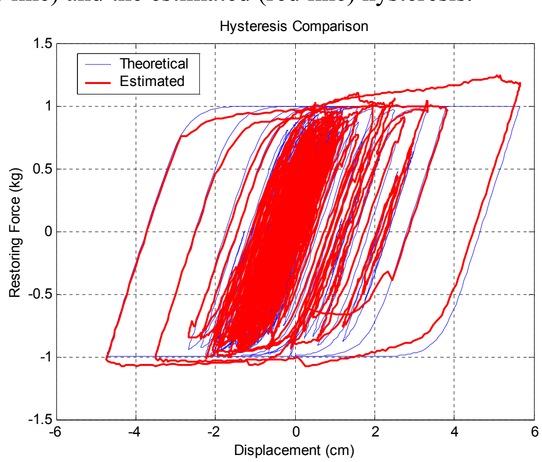

Fig. 8 Hysteresis comparison.

Next, the simulation of the SDOF subjected to the SCT strong motion record is presented. Fig. 9 shows the comparison between the theoretical (blue line) and the estimated (red line) CF. The calculated error is shown in Fig. 10. Fig. 11 shows the estimation of the stiffness k. In Fig. 12, the estimation of damping c is observed, and Fig. 13 shows the comparison between the theoretical (blue line) and estimated (red line) hysteresis.

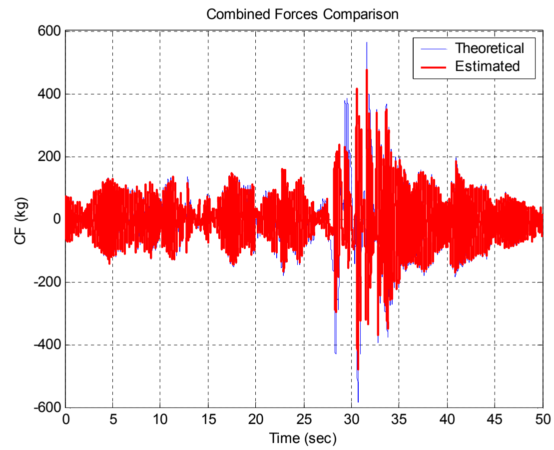

Fig. 9 CF comparison.

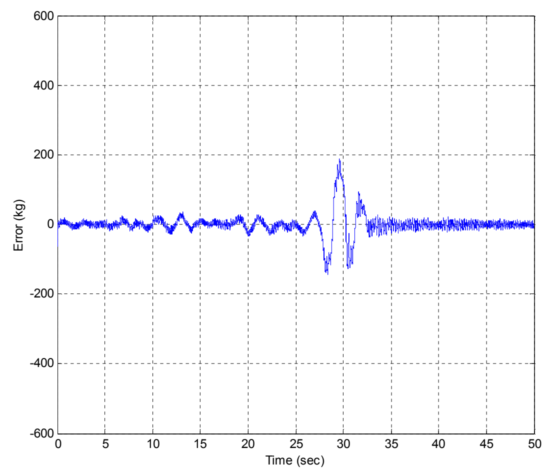

Fig. 10 CF estimation error.

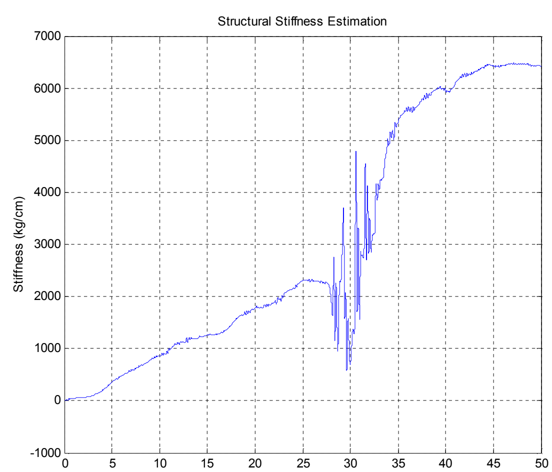

Fig. 11 Stiffness k estimation.

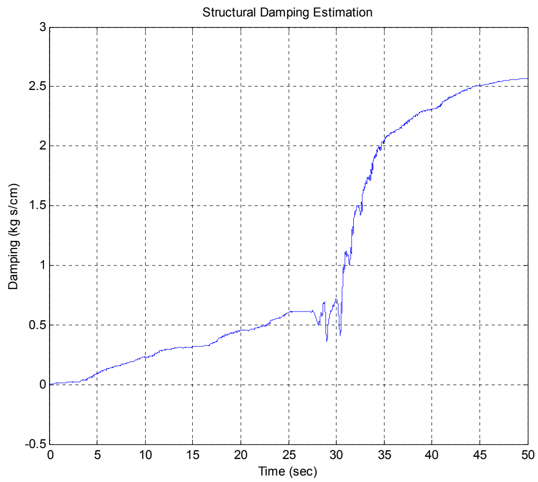

Fig. 12 Damping c estimation.

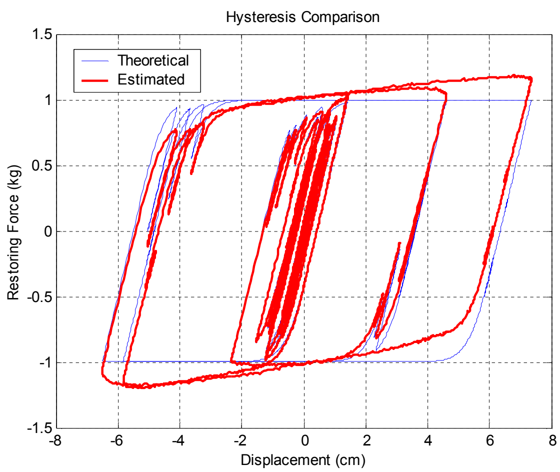

Fig. 13 Hysteresis comparison.

VI. Conclusion

The use of seismic instrumentation for structural identification has been the focus of recent research. The present research proposes a new gradient algorithm to on-line identify the linear parameters of a SDOF, and to estimate the combined effect of the driving and the nonlinear restoring forces (combined forces, CF).

A convergence and stability analysis of the algorithm was performed. Noised simulations with two seismic records were performed. Slow convergence of the parameters is observed. Nevertheless, the algorithm proved to be a fine tool to identify the parameters and the CF.

The authors are currently working on the extension to multi-degree-of-freedom (MDOF) systems.

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