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Adaptive Controller Design and Disturbance Attenuation for SISO Linear Systems with Noisy Output Measurements and Partly Measured Disturbances

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***Abstract*— In this paper, we present robust adaptive controller design for SISO linear systems with noisy output measurements and partly measured disturbances. Using the worst-case analysis approach, we formulate the robust adaptive control problem as a nonlinear H^∞-optimal control problem under imperfect state measurements, and solve it using game theory. The design paradigm is the same as that in [1] with the only difference being the treatment of the measured disturbances. The same results as those in [1] are achieved. In addition, when the relative degrees from the measured disturbances to the output are no less than that from the control input, the controllers designed achieve the *zero* disturbance attenuation level with respect to the measured disturbance inputs. The asymptotic tracking objective is achieved even if the measured disturbance is only uniformly bounded, without requiring it to be of £nite energy. This strong robustness property is then illustrated by a numerical example.**

***Index Terms*— Nonlinear H^∞ control; cost-to-come function; integrator backstepping; adaptive control; measured disturbances.**

I. INTRODUCTION

The design of adaptive controllers has been an important research topic since $1970s$. The classical adaptive control design is based on the certainty equivalence principle [2], which has been shown to be effective for linear systems with or without stochastic disturbance inputs [3], [4]. Using this approach, the controller is designed as if the unknown parameters are known, in implementation these unknown parameters are substituted by their on-line estimates, which are generated through a variety of identi£ers, as long as the estimates satisfy certain properties independent of the controller. This approach leads to structurally simple adaptive controllers. Yet, early designs based on this approach has been shown to be nonrobust [5], [6] when the system is subject to exogenous disturbance inputs and unmodeled dynamics. This approach has also failed to generalize to nonlinear systems with severe nonlinearities. This has motivated the study of robust adaptive control in 1980s and 1990s, and nonlinear adaptive control in 1990s.

Nonlinear adaptive control attracted a lot of research attention in 1990s after the celebrated characterization of feedback linearizable or partially feedback linearizable systems [7]. A breakthrough is achieved when the integrator backstepping methodology was introduced [8] to systematically design adaptive controllers for parametric strict-feedback and parametric pure-feedback nonlinear systems. This has led to an period of intense research into the topic of nonlinear adaptive control when a large volume of results ¤ourished, see the book [9] for a complete list of references. More recently, this approach has been generalized to systems with unknown sign of the high frequency gain. This nonlinear design methodology has also been applied to linear systems [9] to compare performance with that of the certainty equivalence approach. As to be expected, a systematically designed nonlinear adaptive control law leads to better closed-loop performance than that for the certainty equivalence based design when the system is free of disturbance. Yet this approach has also been shown to be nonrobust when the system is subject to exogenous disturbance inputs.

Robust adaptive control has been an important research topic in late 1980s and early 1990s. This research leads to various modi£cation techniques in adaptive control design in order to render the closed-loop systems robust [10]. Despite their successes, they fell short of directly addressing the disturbance attenuation property of the closed-loop system.

The objectives of robust adaptive control are to improve transient response, to accommodate unmodeled dynamics, and to reject exogenous disturbance inputs. These objectives are the same as those that motivate the study of the H^∞-optimal control problem, where these objectives are ful£lled by studying the disturbance attenuation property of the system. The game-theoretic approach to H^∞-optimal control [11] offers the most promising tool to generalize the results to nonlinear systems [12]–[14]. These observations and results motivated the worst-case analysis approach to adaptive control, where the adaptive control problem is formulated as a nonlinear H^∞-optimal control problem under imperfect state measurements. The unknown parameter vector is viewed as part of the expanded state vector, and the measures of transient response, disturbance attenuation, and asymptotic tracking are all incorporated into a single game-theoretic cost function. The cost-to-come function analysis [13] is applied to obtain the state estimator for the expanded state vector, which results in an on-line parameter identi£er and a state estimator for the original system. This step converts the nonlinear H^∞-optimal control problem under imperfect state measurements into one under full information measurements. This full information measurement problem is then solved for a suboptimal solution using the integrator backstepping methodology. This design paradigm has been applied to worst-case parameter identi£cation problems, which has led to new classes of parametrized identi£ers for linear and nonlinear systems. It has also been applied to adaptive control problems [1], [15], [16], which has led to new classes of parametrized robust adaptive controllers for linear and nonlinear systems.

In this paper, we further generalize the worst-case analysis based approach to linear systems with partly measured disturbance inputs. We assume that the linear system admits a known upper bound for its dynamic order, a known relative degree, a known sign of high-frequency gain with a nonvanishing bound away from 0,

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a strictly minimum phase transfer function from the control input to the output, and some other assumptions, which are the same as [1]. The adaptive control design follows the same paradigm as discussed above, which leads to two classes of parametrized controllers in closed form with the following robustness properties. The close-loop system admits a guaranteed disturbance attenuation level with respect to the exogenous disturbance inputs, where the ultimate attenuation lower bound is equal to the noise intensity in the measurement channel. The closed-loop system is totally stable with respect to the disturbance input and the initial condition. Furthermore, it achieves asymptotic tracking of the reference trajectory for all uniformly bounded disturbance inputs that are of bounded energy. These results are the same as those of [1]. In addition, with proper scaling, the controller achieves any positive attenuation level with respect to the measured disturbance inputs. When the output has relative degrees for the measured disturbances that are greater than or equal to that for the control input, the controllers designed achieve *zero* disturbance attenuation level with respect to the measured disturbances. This does not mean that we achieve decoupling from the measured disturbances. Therefore, when the unmeasured disturbance is $\mathcal{L}_2 \cap \mathcal{L}_\infty$, the tracking error asymptotically converges to zero for any measured disturbance that is uniformly bounded.

The balance of the paper is organized as follows. In Section II, we present the formulation of the adaptive control problem and discuss the general solution methodology. In Section III, we obtain parameter identi£er and state estimator using the cost-to-come function analysis. Then we derive the adaptive control law in Section IV, and present the main result on the robustness of the closed-loop system. The theoretical results are illustrated by one numerical example in Section V. The paper ends with some conclusions in Section VI.

II. Problem Formulation

We consider the adaptive control problem for single-input-single-output (SISO) linear systems.

Assumption 1: The linear system is known to be at most n dimensional, $n \in \mathbb{N}$. □

By adding additional dynamics if necessary, we consider the following true system dynamics:

$$\dot{\grave{x}} = \grave{A}\grave{x} + \grave{B}u + \grave{D}_1\grave{w} + \grave{D}_2\breve{w}; \quad \grave{x}(0) = \grave{x}_0 \tag{1a}$$

$$y = \grave{C}\grave{x} + \grave{E}\grave{w} \tag{1b}$$

where $\grave{x} \in \mathbb{R}^n$ is the state vector; $u \in \mathbb{R}$ is the scalar control input; $y \in \mathbb{R}$ is the scalar system output; $\grave{w} \in \mathbb{R}^{\grave{q}}$ is the unmeasured disturbance input vector, $\grave{q} \in \mathbb{N}$; $\breve{w} \in \mathbb{R}^{\breve{q}}$ is the measured disturbance input vector, $\breve{q} \in \mathbb{N}$; all input and output signals y, u, $\breve{w}$, and $\grave{w}$ are continuous; and the matrices $\grave{A}$, $\grave{B}$, $\grave{C}$, $\grave{D}_1$, $\grave{D}_2$, and $\grave{E}$ are of the appropriate dimensions, generally unknown. The transfer function from u to y is $H(s) = \grave{C}(sI - \grave{A})^{-1}\grave{B}$.

Assumption 2: $(\grave{A}, \grave{C})$ is observable. The transfer function $H(s)$ is known to have relative degree $r \in \mathbb{N}$, and is strictly minimum phase. The uncontrollable part (with respect to u) of (1) is stable in the sense of Lyapunov. Any uncontrollable mode correponding to an eigenvalue of the matrix $\grave{A}$ on the $j\omega$-axis is uncontrollable from $\grave{w}$ and $\breve{w}$. □

As discussed in [1], there exists a state diffeomorphism $x = \grave{T}\grave{x}$ and a disturbance transformation $w = \grave{M}\grave{w}$, such that system (1) can be transformed into the following form in the x cooridinates

$$\dot{x} = Ax + (y\bar{A}_{211} + u\bar{A}_{212} + \sum_{j=1}^{\breve{q}} \breve{w}_j \bar{A}_{213j})\theta + Dw + \breve{D}\breve{w} \tag{2a}$$

$$y = Cx + Ew \tag{2b}$$

where θ is the σ-dimensional vector of unknown parameters of the systems, $\sigma \in \mathbb{N}$; the matrices A, $\bar{A}_{211}$, $\bar{A}_{212}$, $\bar{A}_{2131}$, $\cdots$, $\bar{A}_{213\breve{q}}$, D, $\breve{D}$, C, and E are known and have the following structures, $A = (a_{ij})_{n\times n}; a_{i,i+1} = 1, a_{ij} = 0$, for $1 \le i \le r-1$ and $i+2 \le j \le n$; $\bar{A}_{212} = \begin{bmatrix} \mathbf{0}_{\sigma\times(r-1)} & \bar{A}'_{2120} & \bar{A}'_{212r} \end{bmatrix}'$, $\bar{A}_{2120} = \begin{bmatrix} 1 & \mathbf{0}_{1\times(\sigma-1)} \end{bmatrix}$, $C = \begin{bmatrix} 1 & \mathbf{0}_{1\times(n-1)} \end{bmatrix}$, and $x(0) = x_0$. We will design adaptive controller using system (2), which is called the design model.

Assumption 3: The matrices D and E are such that $EE' > 0$. De£ne $\zeta := 1/(EE')^{\frac{1}{2}}$ and $L := DE'$. □

Because of the structure of A and $\bar{A}_{212}$, the £rst element of the parameter vector θ, denoted by b_0, is the high frequency gain of the transfer function $H(s)$. We partition parameter vector θ into $\theta = \begin{bmatrix} b_0 & \theta'_s \end{bmatrix}'$, where θ_s is a $(\sigma-1)$-dimensional vector.

Assumption 4: The sign of b_0 is known, and W.L.O.G., assume $b_0 > 0$. There exists a known smooth nonnegative radially-unbounded strictly convex function $P(\bar{\theta})$, such that the true value $\theta \in \Theta := \{\bar{\theta} : P(\bar{\theta}) \le 1\}$. Moreover, $\forall \bar{\theta} \in \Theta$, $\bar{b}_0 > 0$. □

Assumption 4 delineated the *a priori* convex compact set where the parameter vector θ lies in.

The control law is generated by $u(t) = \mu(t, y_{[0,t]}, \breve{w}_{[0,t]})$, where $\mu : [0,\infty) \times \mathcal{C} \times \mathcal{C} \to \mathbb{R}$. We denote the class of these admissible controllers by $\mathcal{M}$.

The control objective is to design a robust adaptive controller for (1), such that $Cx(t)$ tracks a reference signal $y_d(t)$ while rejecting the uncertainty quadruple $(x_0, \theta, \grave{w}_{[0,\infty)}, \breve{w}_{[0,\infty)}) \in \grave{\mathcal{W}} := \mathbb{R}^n \times \Theta \times \mathcal{C} \times \mathcal{C}$, which comprises the initial state, the true values of unknown parameter vector, the unmeasured disturbance waveform, and the measured disturbance waveform, and keeping all signals in the closed-loop system uniformly bounded.

Assumption 5: The reference trajectory, y_d, is r times continuously differentiable. The signal y_d and the £rst r derivatives of y_d are bounded and available for feedback. □

For design purposes, instead of attenuating the effect of $\grave{w}$ and $\breve{w}$, we design the adaptive controller to attenuate the effect of w and $\breve{w}$. This is done to allow our design paradigm to be carried out. This will result in a guaranteed attenuation level with respect to $\grave{w}$ and $\breve{w}$ as well, see [1] for a discussion. We take the uncertainty quadruple $(x_0, \theta, w_{[0,\infty)}, \breve{w}_{[0,\infty)})$ to belong to the set $\mathcal{W} = \mathbb{R}^n \times \Theta \times \mathcal{C} \times \mathcal{C}$.

De£nition 1: A controller μ is said to achieve *disturbance attenuation level* γ if there exist $l(t, \theta, x, y_{[0,t]}, \breve{w}_{[0,t]}) \ge 0$ and $l_0(\check{x}_0, \check{\theta}_0) \ge 0$ such that

$$\sup_{(x_0,\theta,\grave{w}_{[0,\infty)},\breve{w}_{[0,\infty)})\in\grave{\mathcal{W}}} J_{\gamma t} \le 0; \qquad \forall t \ge 0 \tag{3}$$

where

$$J_{\gamma t} := \int_0^t ((x_1 - y_d)^2 + l(\tau, \theta, x(\tau), y_{[0,\tau]}, \breve{w}_{[0,\tau]}) - \gamma^2|w|^2 - \gamma^2|\breve{w}|^2)d\tau - \gamma^2|\theta - \check{\theta}_0|^2_{Q_0} - \gamma^2|x_0 - \check{x}_0|^2_{\Pi_0^{-1}} - l_0(\check{x}_0, \check{\theta}_0)$$

$\check{\theta}_0 \in \Theta$ is the initial guess of θ; $Q_0 > 0$ is the weighting matrix, quantifying the level of con£dence in the estimate $\check{\theta}_0$; $\check{x}_0$ is the initial guess of x_0; and $\Pi_0^{-1} > 0$ is the quadratic weighting matrix, quantifying the level of con£dence in the estimate $\check{x}_0$; and $|z|_Q$ denotes $z^T Qz$ for any vector z and any symmetric matrix Q.

The following notation will be used throughout this paper. $\check{x}$ denotes the estimate of x; $\check{\theta}$ denotes the estimate of θ; $e_{j,i}$ denotes a j-dimensional column vector, all of its elements are 0, except its ith row is 1, such as $e_{4,2} = [0,1,0,0]'$.

De£ne $\xi := [\theta', x']'$. Note that $\dot{\theta} = 0$, we have the following expanded dynamics for system (2)

$$\begin{aligned}\dot{\xi} &= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ y\bar{A}_{211} + u\bar{A}_{212} + \sum_{i=1}^{\check{q}} \bar{A}_{213i}\check{w}_i & A \end{bmatrix}\xi \\ &\quad + \begin{bmatrix} \mathbf{0} \\ D \end{bmatrix} w + \begin{bmatrix} \mathbf{0} \\ \check{D} \end{bmatrix} \check{w} := \bar{A}\xi + \bar{D}w + \bar{\check{D}}\check{w} \quad (4a)\\ y &= \begin{bmatrix} \mathbf{0} & C \end{bmatrix}\xi + Ew := \bar{C}\xi + Ew \quad (4b)\end{aligned}$$

The worst-case optimization of the cost function (3) can be carried out in two steps as depicted in the following inequality.

$$\sup_{(x_0,\theta,\dot{w},\check{w})\in\check{\mathcal{W}}} J_{\gamma t} \le \sup_{y,\check{w}} \sup_{(x_0,\theta,w)|y,\check{w}} J_{\gamma t} \quad (5)$$

The inner supremum operator will be carried out £rst. It is the identi£cation design step, to be discussed in Section III. Succinctly stated, in this step, we calculate the maximum cost that is consistent with the given measurement waveform.

The outer supremum operator will be carried out second. It is the controller design step, to be discussed in Section IV. In this step we use a backstepping procedure to design the control input u.

This completes the formulation of the robust adaptive control problem. We turn to the identi£cation design in the next section.

III. Design of a Worst-case Identifier

In this section, we present the identi£cation design for the adaptive control problem formulated. In this step, the measurement waveform $y_{[0,\infty)}$ and $\check{w}_{[0,\infty)}$ are assumed to be known. Since the control input is a causal function of y and $\check{w}$, then it is known. This calculation uses the cost-to-come function methodology. Set function l in (3) to $|\xi-\hat{\xi}|^2_{\bar{Q}} + \check{l}$, where $\hat{\xi}$ is the worst-case estimate for the expanded state ξ, $\hat{\xi} = [\hat{\theta}', \hat{x}']'$, $\bar{Q}$ is a matrix-valued weighting function to be introduced later, and $\check{l}$ is a function to be introduced in Section IV, which is a constant in this section. By the cost-to-come function analysis of [1], we have

$$\begin{aligned}\dot{\bar{\Sigma}} =& (\bar{A} - \zeta^2\bar{L}\bar{C})\bar{\Sigma} + \bar{\Sigma}(\bar{A} - \zeta^2\bar{L}\bar{C})' + \gamma^{-2}\bar{D}\bar{D}' - \gamma^{-2}\zeta^2\bar{L}\bar{L}' \\ &-\bar{\Sigma}(\gamma^2\zeta^2\bar{C}'\bar{C} - \bar{C}'\bar{C} - \bar{Q})\bar{\Sigma};\quad \bar{\Sigma}(0) = \gamma^{-2}\begin{bmatrix} Q_0^{-1} & \mathbf{0} \\ \mathbf{0} & \Pi_0 \end{bmatrix} \quad (6a)\\ \dot{\check{\xi}} =& (\bar{A} + \bar{\Sigma}(\bar{C}'\bar{C} + \bar{Q}))\check{\xi} + \zeta^2(\gamma^2\bar{\Sigma}\bar{C}' + \bar{L})(y - \bar{C}\check{\xi}) + \bar{\check{D}}\check{w} \\ &-\bar{\Sigma}(\bar{C}'y_d + \bar{Q}\hat{\xi});\quad \check{\xi}(0) = \begin{bmatrix} \check{\theta}_0' & \check{x}_0' \end{bmatrix}' \quad (6b)\end{aligned}$$

where $\bar{L}$ is de£ned as $\bar{L} = \begin{bmatrix} \mathbf{0}_{1\times\sigma} & L' \end{bmatrix}'$. Then

$$\begin{aligned}J_{\gamma t} =& \int_0^t (|\bar{C}\check{\xi} - y_d|^2 + |\hat{\xi} - \check{\xi}|^2_{\bar{Q}} + \check{l} - \gamma^2\zeta^2|y - \bar{C}\check{\xi}|^2 \\ &-\gamma^2|w - w_*|^2)d\tau - |\xi(t) - \check{\xi}(t)|^2_{\bar{\Sigma}^{-1}(t)} - l_0 \quad (7)\end{aligned}$$

where w_* is the worst-case disturbance, given by

$$w_* = \zeta^2E'(y - \bar{C}\xi) + \gamma^{-2}(I - \zeta^2E'E)\bar{D}'\bar{\Sigma}^{-1}(\xi - \check{\xi}) \quad (8)$$

The following derivation for the identi£er closely resembles that in [1]. Partition $\bar{\Sigma}$ as $\bar{\Sigma} = \begin{bmatrix} \Sigma & \bar{\Sigma}_{12} \\ \bar{\Sigma}_{21} & \bar{\Sigma}_{22} \end{bmatrix}$ and introduce $\Phi := \bar{\Sigma}_{21}\Sigma^{-1}$ and $\Pi := \gamma^2(\bar{\Sigma}_{22} - \bar{\Sigma}_{21}\Sigma^{-1}\bar{\Sigma}_{12})$. Σ and Φ satisfy (12a), (12c), and Π satis£es (10) with proper initialization.

We make the following assumption on the weighting matrix $\bar{Q}$,

Assumption 6: The weighting matrix $\bar{Q}$ is given by

$$\bar{Q} = \bar{\Sigma}^{-1}\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Delta \end{bmatrix}\bar{\Sigma}^{-1} + \begin{bmatrix} \epsilon\Phi'C'(\gamma^2\zeta^2 - 1)C\Phi & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

where $\Delta > 0$ is $n \times n$ dimensional, and ϵ is de£ned by $\epsilon(\tau) := \mathrm{Tr}(\Sigma^{-1}(\tau))/K_c$, $K_c \ge \gamma^2\mathrm{Tr}(Q_0)$ is a constant, $\forall\tau \ge 0$. □

The matrix Σ will play the role of worst-case covariance matrix of the parameter estimation error. Assumption 6 guarantees that Σ is uniformly bounded from above and below.

Lemma 1: Consider the dynamic equation (12a) for the covariance matrix Σ. Let Assumption 6 hold and $\gamma \ge \zeta^{-1}$. Then, the matrix Σ is uniformly upper and lower bounded as follows: $K_c^{-1}I_\sigma \le \Sigma(\tau) \le \Sigma(0) = \gamma^{-2}Q_0^{-1}$; $\gamma^2\mathrm{Tr}(Q_0) \le \mathrm{Tr}(\Sigma^{-1}(\tau)) \le K_c$; $\forall\tau \in [0,t]$.

Proof: See [17] for details. ■

To avoid the inversion of Σ on-line, we de£ne $s_\Sigma(\tau) := 1/\mathrm{Tr}(\Sigma^{-1}(\tau))$, and its time derivative is given by

$$\dot{s}_\Sigma = -s_\Sigma^2(\gamma^2\zeta^2 - 1)(1-\epsilon)C\Phi\Phi'C';\ s_\Sigma(0) = \frac{1}{\gamma^2\mathrm{Tr}(Q_0)} \quad (9)$$

Then, $\epsilon(\tau) = K_c^{-1}s_\Sigma^{-1}(\tau)$.

¿From Assumption 6 and (12a), we need $\gamma \ge \zeta^{-1}$. This means the quantity ζ^{-1} is the ultimate lower bound on the achievable performance level for the adaptive system, using the design method proposed in this paper.

Assumption 7: If the matrix $A - \zeta^2LC$ is Hurwitz, then the desired disturbance attenuation level $\gamma \ge \zeta^{-1}$. Otherwise, the desired disturbance attenuation level $\gamma > \zeta^{-1}$. □

Assumption 8: The matrix Π_0 is chosen as the unique positive de£nite solution to the algebraic Riccati equation:

$$\begin{aligned}&(A - \zeta^2LC)\Pi + \Pi(A - \zeta^2LC)' - \Pi C'(\zeta^2 - \gamma^{-2})C\Pi \\ &+DD' - \zeta^2LL' + \gamma^2\Delta = 0 \quad (10)\end{aligned}$$

□

Then, Π satis£es (10) and is a constant matrix.

To guarantee the boundness of parameter estimates without persistently exciting signals, we introduce soft projection design by using the *a priori* information that $\theta \in \Theta$.

De£ne $\rho := \min_{\theta_s} P(0, \theta_s)$ and $\Theta_o := \{\bar{\theta} : P(\bar{\theta}) < \frac{1+\rho}{2}\}$. By Assumption 4, we have $1 < \rho < \infty$.

Add the term $-\bar{\Sigma}\begin{bmatrix} \left(P_r(\check{\theta})\right)' & \mathbf{0}_{1\times n} \end{bmatrix}'$ to the right-hand side of the dynamics (6b), where

$$P_r(\check{\theta}) := \begin{cases} \dfrac{\exp\left(\frac{1}{1-P(\check{\theta})}\right)}{\left(\frac{1+\rho}{2} - P(\check{\theta})\right)^3}\left(\frac{\partial P}{\partial\theta}(\check{\theta})\right)' & \forall\theta \in \Theta_o\backslash\Theta \\ 0 & \forall\theta \in \Theta \end{cases} \quad (11)$$

Partition $\check{\xi}$ into $(\check{\theta}', \check{x}')'$ to obtain the dynamics for the identi£er.

$$\begin{aligned}\dot{\Sigma} &= (\epsilon - 1)\Sigma\Phi'C'(\gamma^2\zeta^2 - 1)C\Phi\Sigma;\quad \Sigma(0) = \gamma^{-2}Q_0^{-1} \quad (12a)\\ A_f &= A - \zeta^2LC - \Pi C'C(\zeta^2 - \gamma^{-2}) \quad (12b)\\ \dot{\Phi} &= A_f\Phi + y\bar{A}_{211} + u\bar{A}_{212} + \sum_{i=1}^{\check{q}}\check{w}_i\bar{A}_{213i};\ \Phi(0) = \mathbf{0} \quad (12c)\\ \dot{\check{\theta}} &= -\Sigma P_r(\check{\theta}) - \Sigma\Phi'C'(y_d - C\check{x}) - \begin{bmatrix} \Sigma & \Sigma\Phi' \end{bmatrix}\bar{Q}\xi_c \\ &\quad +\gamma^2\zeta^2\Sigma\Phi'C'(y - C\check{x});\quad \check{\theta}(0) = \check{\theta}_0 \quad (12d)\end{aligned}$$

$$\dot{\check{x}} = -\Phi\Sigma P_r(\check{\theta}) + A\check{x} - \left[\begin{array}{cc} \Phi\Sigma & \frac{1}{\gamma^2}\Pi + \Phi\Sigma\Phi' \end{array} \right] \bar{Q}\xi_c - (\gamma^{-2} \cdot\Pi + \Phi\Sigma\Phi')C'(y_d - C\check{x}) + (y\bar{A}_{211} + u\bar{A}_{212} + \sum_{j=1}^{\check{q}} \check{w}_j \bar{A}_{213j})\check{\theta} + \zeta^2(\Pi C' + \gamma^2\Phi\Sigma\Phi'C' + L)(y - C\check{x}) + \check{D}\check{w};\ \check{x}(0) = \check{x}_0 \quad (12e)$$

where $\xi_c = \hat{\xi} - \check{\xi}$.

Introduce the value function

$$W(t, \xi(t), \check{\xi}(t), \bar{\Sigma}(t)) = |\xi(t) - \check{\xi}(t)|^2_{\bar{\Sigma}^{-1}(t)} = |\theta - \check{\theta}(t)|^2_{\Sigma^{-1}(t)} + \gamma^2|x(t) - \check{x}(t) - \Phi(t)(\theta - \check{\theta}(t))|^2_{\Pi^{-1}}$$

whose time derivative is given by

$$\begin{aligned}\dot{W} = &-|x_1 - y_d|^2 - \gamma^4|x - \hat{x} - \Phi(\theta - \hat{\theta})|^2_{\Pi^{-1}\Delta\Pi^{-1}} \\ &-\epsilon(\gamma^2\zeta^2 - 1)|\theta - \hat{\theta}|^2_{\Phi'C'C\Phi} + \gamma^2|w|^2 + |C\check{x} - y_d|^2 + |\xi_c|^2_{\bar{Q}} \\ &-\gamma^2\zeta^2|y - C\check{x}|^2 - \gamma^2|w - w*|^2 + 2(\theta - \check{\theta})'P_r(\check{\theta}) \end{aligned} \quad (13)$$

We note that the last term in $\dot{W}$ is nonpositive, zero on the set Θ and approaches $-\infty$ as $\check{\theta}$ approaches the boundary of the set Θ_o, which guarantees the boundness of $\check{\theta}$.

This completes the identi£cation design step.

IV. Control Design and Main Result

In this section, we describe the controller design for the uncertain system under consideration. Based on the cost function (3), the controller design is to guarantee that the following supremum is less than or equal to zero for all measurement waveforms,

$$\sup_{(x_0,\theta,\dot{w}_{[0,\infty)},\check{w}_{[0,\infty)})\in\hat{\mathcal{W}}} J_{\gamma t} \leq \sup_{y_{[0,\infty)},\check{w}_{[0,\infty)}} \left\{ \int_0^t \left(|C\check{x} - y_d|^2 + |\xi_c|^2_{\bar{Q}} + \check{l} - \gamma^2|\check{w}|^2 - \gamma^2\zeta^2|y - C\check{x}|^2 \right) d\tau - l_0(\check{x}_0, \check{\theta}_0) \right\} \quad (14)$$

where function $\check{l}$ is part of the weighting function l to be designed in this step.

By equation (14), we observe that the cost function is expressed in term of the states of the estimator we derived, whose dynamics are driven by y, u, $\check{w}$, and $\hat{\xi}$, which are signals we either measure or can construct. This is then a nonlinear H^∞-optimal control problem under full information measurements. Instead of considering y and $\check{w}$ as the maximizing variable, we can equivalently deal with the transformed variable: $v := \left[\begin{array}{cc} \zeta(y - C\check{x}) & \check{w}' \end{array} \right]'$.

Introduce the matrix $M_f := \left[\begin{array}{cccc} A_f^{n-1}p_n & \cdots & A_f p_n & p_n \end{array} \right]$, where p_n is a n-dimensional vector such that the pair (A_f, p_n) is controllable. Then the following $(\check{q}+2)n$-dimensional pre£ltering system for y, u, and $\check{w}$ generates the Φ online:

$$\begin{aligned} \dot{\eta} &= A_f\eta + p_n y; \qquad \eta(0) = 0 && (15a) \\ \dot{\lambda} &= A_f\lambda + p_n u; \qquad \lambda(0) = 0 && (15b) \\ \dot{\eta}_{\check{w}_i} &= A_f\eta_{\check{w}_i} + p_n\check{w}_i;\ \eta_{\check{w}_i}(0) = 0,\ i = 1, \cdots, \check{q} && (15c) \end{aligned}$$

For ease of the ensuing study, we will separate Φ as the sum of several matrices,

$$\begin{aligned} \Phi &= \Phi_u + \Phi_y + \Phi_{\check{w}} && (16a) \\ \Phi_y &= \left[\begin{array}{cccc} A_f^{n-1}\eta & \cdots & A_f\eta & \eta \end{array} \right] M_f^{-1}\bar{A}_{211} && (16b) \\ \dot{\Phi}_u &= A_f\Phi_u + u\bar{A}_{212}; \qquad \Phi_u(0) = \mathbf{0}_{n\times\sigma} && (16c) \\ \dot{\Phi}_{\check{w}} &= A_f\Phi_{\check{w}} + \sum_{j=1}^{\check{q}} \check{w}_j\bar{A}_{213j}; \qquad \Phi_{\check{w}}(0) = \mathbf{0}_{n\times\sigma} && (16d) \end{aligned}$$

Substituting y and $\check{w}$ by v in the equations (9), (12), (15), and (16), we obtain the dynamics for control design. The variables to be designed at this stage include u and ξ_c. Note that Σ, Π, s_Σ, and $\check{\theta}$ are always bounded by the design in Section III. $\Phi_{\check{w}}$ is bounded by its dynamic structure. Since Φ_u is explicitly driven by control u, it can not be stabilized in conjunction with $\check{x}$ in the backstepping design. We will assume it is bounded and prove later it is indeed so under the derived control law. We observe that the matrix A_f has the same structure as the matrix A, then, we apply the integrator backstepping methodology [8] to stabilize the variables η, $\check{x}_1$, $\cdots$, $\check{x}_r$. Since there is a nonnegative de£nite weighting on ξ_c in the cost function (14), we can not use integrator backstepping to design feedback law for ξ_c. Hence, we set $\xi_c = 0$ in the backstepping procedure. After the completion of the backstepping procedure, we will then optimize the choice of ξ_c based on the value function obtained. To stabilize η, we introduce variable η_d, which satis£es $\dot{\eta}_d = A_f\eta_d + p_n y_d$ with initial condition $\eta_d(0) = 0$, and is the reference trajectory for η to track. Choosing value function $V_0 := |\eta - \eta_d|^2_Z$, where Z is the solution to an algebraic Riccati equation. Treating $\check{x}_1$ as the virtual control input, we complete the step 0 with the virtual control law $\alpha_0 = 0$, which will guarantee the $\dot{V}_0 \leq 0$ under $\check{x}_1 = y_d$. At step 1, we introduce $z_1 := \check{x}_1 - y_d$, and choose value function $V_1 = V_0 + \frac{1}{2}z_1^2$. Treating $\check{x}_2$ as the virtual control input, we end the step 1 with the virtual control law α_1, which guarantees $\dot{V}_1 \leq 0$ under $\check{x}_2 = y_d^{(1)} + \alpha_1$, where $y_d^{(j)}$ denotes the jth order derivative of y_d. De£ne the variable $z_2 = \check{x}_2 - y_d^{(1)} - \alpha_2$ for step 2. Repeating the backstepping procedure until step r, the control input u will appear in the dynamic of $\dot{z}_r$. Using the similar procedure as previous steps, we can derive the robust adaptive controller μ such that $\dot{V}_r \leq 0$ under $u := \mu$. Later, we will prove the control law μ will guarantee the uniform boundedness of the closed-loop system states and the asymptotic convergence of tracking error. For detailed equations of the backstepping design, see the full version [17].

For the closed-loop adaptive nonlinear system, we have the following value function,

$$U := V_r + W = |\theta - \check{\theta}|^2_{\Sigma^{-1}} + \gamma^2|x - \check{x} - \Phi(\theta - \check{\theta})|^2_{\Pi^{-1}} + |\tilde{\eta}|^2_Z + \frac{1}{2}\sum_{j=1}^{r}(\check{x}_j - y_d^{(j-1)} - \alpha_{j-1})^2$$

where $\tilde{\eta} = \eta - \eta_d$.

The time derivative of this function is given by

$$\begin{aligned}\dot{U} = &-|x_1 - y_d|^2 - \gamma^4\left|x - \hat{x} - \Phi(\theta - \hat{\theta})\right|^2_{\Pi^{-1}\Delta\Pi^{-1}} \\ &-\epsilon(\gamma^2\zeta^2 - 1)\left|\theta - \hat{\theta}\right|^2_{\Phi'C'C\Phi} + 2(\theta - \check{\theta})'P_r(\check{\theta}) \\ &+\left|\xi_c + \frac{1}{2}\bar{\varsigma}_r\right|^2_{\bar{Q}} - \frac{1}{4}|\bar{\varsigma}_r|^2_{\bar{Q}} - |\tilde{\eta}|^2_Y - \sum_{j=1}^{r}\bar{\beta}_j z_j^2 \\ &+\gamma^2|w|^2 + \gamma^2|\check{w}|^2 - \gamma^2|w - w_*|^2 - \gamma^2|v - \bar{\nu}_r|^2\end{aligned}$$

where $\bar{\beta}_i$'s are design functions chosen by designer in backstepping procedure; $\bar{\varsigma}_r$ and $\bar{\nu}_r$ are terms derived in the backstepping procedure. See [17] for the detailed information.

Then, the optimal choice for the variable $\hat{\xi}$ is $\hat{\xi}_* = \check{\xi} - \frac{1}{2}\bar{\varsigma}_r$, which yields that the closed-loop system is dissipative with storage function U and supply rate $-|x_1 - y_d|^2 + \gamma^2|w|^2 + \gamma^2|\check{w}|^2$.

Furthermore, the worst case disturbance with respect to the value function U is given by

$$\begin{aligned} w_{opt} &= \zeta E' e'_{\check{q}+1,1}\bar{\nu}_r + \gamma^{-2}(I-\zeta^2 E'E)\bar{D}'\bar{\Sigma}^{-1}(\xi-\check{\xi}) \\ &\quad +\zeta^2 E'C(\check{x}-x) && \text{(17a)} \\ \check{w}_{opt} &= K\bar{\nu}_r && \text{(17b)} \end{aligned}$$

where K is de£ned by $\begin{bmatrix} e_{\check{q}+1,1} & K' \end{bmatrix} = I_{\check{q}+1}$.

Next, we summerize in the following theorem the strong robustness property of the closed-loop system, whose proof (see [17] for details) is omitted due to page limitation.

Theorem 1: Consider the robust adaptive control problem formulated in Section II with Assumptions 1—8 holding. The robust adaptive controller μ with either the optimal choice $\hat{\xi}_*$ or the suboptimal choice $\hat{\xi} = \check{\xi}$ achieve the following strong robustness properties for the closed-loop system.

1) The controller μ achieves disturbance attenuation level γ for any uncertainty quadruple $(x_0, \theta, \grave{w}_{[0,\infty)}, \check{w}_{[0,\infty)}) \in \grave{\mathcal{W}}$.
2) Given a $c_w > 0$, there exists a constant $c_c > 0$ and a compact set $\Theta_c \subset \Theta_o$, such that for any uncertainty quadruple $(x_0, \theta, \grave{w}_{[0,\infty)}, \check{w}_{[0,\infty)}) \in \grave{\mathcal{W}}$ with $|x_0| \le c_w$; $|\grave{w}(t)| \le c_w$; $|\check{w}(t)| \le c_w$; $\forall t \in [0,\infty)$, all closed-loop state variables are bounded as follows, $\forall t \in [0,\infty)$, $|x(t)| \le c_c$; $|\check{x}(t)| \le c_c$; $\check{\theta}(t) \in \Theta_c$; $|\eta(t)| \le c_c$; $|\eta_d(t)| \le c_c$; $|\lambda(t)| \le c_c$; $\frac{1}{K_c} I_\sigma \le \Sigma(t) \le \frac{1}{\gamma^2} Q_0^{-1}$; $\frac{1}{K_c} \le s_\Sigma(t) \le \frac{1}{\gamma^2 \mathrm{Tr}(Q_0)}$; $|\eta_{\check{w}_i}(t)| \le c_c$, $i = 1, 2, \cdots, \check{q}$.
3) $\forall (x_0, \theta, \grave{w}_{[0,\infty)}, \check{w}_{[0,\infty)}) \in \grave{\mathcal{W}}$ with $\grave{w}_{[0,\infty)} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\check{w}_{[0,\infty)} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, then, $\lim_{t\to\infty}(x_1(t) - y_d(t)) = 0$.

Based on Theorem 1, we note that the above controller μ can achieve disturbance attenuation level γ with respect to w and $\check{w}$. Then, the time derivative of value function U satis£es:

$$\dot{U} \le -|x_1 - y_d|^2 + \gamma^2|w|^2 + \gamma^2|\check{w}|^2$$

Consider $\gamma^2|\check{w}|^2 = \check{\gamma}^2\left|\frac{\gamma}{\check{\gamma}}\check{w}\right|^2$, we can achieve arbitrary disturbance attenuation level $\check{\gamma}$ with respect to $\check{w}$ by multipling a scalar to $\check{w}$. However, $\check{\gamma}$ have to be positive. To achieve disturbance attenuation level 0 with respect to $\check{w}$, we need the following assumption.

Assumption 9: The transfer function from $\check{w}_i$ to y has relative degree greater than or equal to r, $i = 1, \cdots, \check{q}$. □

De£nition 2: A controller μ is said to achieve *disturbance attenuation level γ with respect to w and disturbance attenuation level 0 with respect to $\check{w}$*, if there exist $l(t, \theta, x, y_{[0,t]}, \check{w}_{[0,t]}) \ge 0$ and $l_0(\check{x}_0, \check{\theta}_0) \ge 0$ such that

$$\sup_{(x_0,\theta,\grave{w}_{[0,\infty)},\check{w}_{[0,\infty)})\in\grave{\mathcal{W}}} J_{\gamma,0t} \le 0; \qquad \forall t \ge 0. \qquad (18)$$

where

$$\begin{aligned} J_{\gamma,0t} := \int_0^t &((x_1 - y_d)^2 + l(\tau, \theta, x(\tau), y_{[0,\tau]}, \check{w}_{[0,\tau]}) \\ &-\gamma^2|w|^2)d\tau - \gamma^2|\theta - \check{\theta}_0|^2_{Q_0} - \gamma^2|x_0 - \check{x}_0|^2_{\Pi_0^{-1}} - l_0(\check{x}_0, \check{\theta}_0) \end{aligned}$$

We have the following results for this objective. For the proof of this corollary, which is omitted due to page limitation, see [17] for details.

Corollary 1: Consider the robust adaptive control problem formulated in Section II, under the assumptions of Theorem 1 and Assumption 9, the results of Theorem 1 hold for the control law μ with either the optimal policy $\hat{\xi}_*$ or the suboptimal policy $\hat{\xi} = \check{\xi}$. In addition,

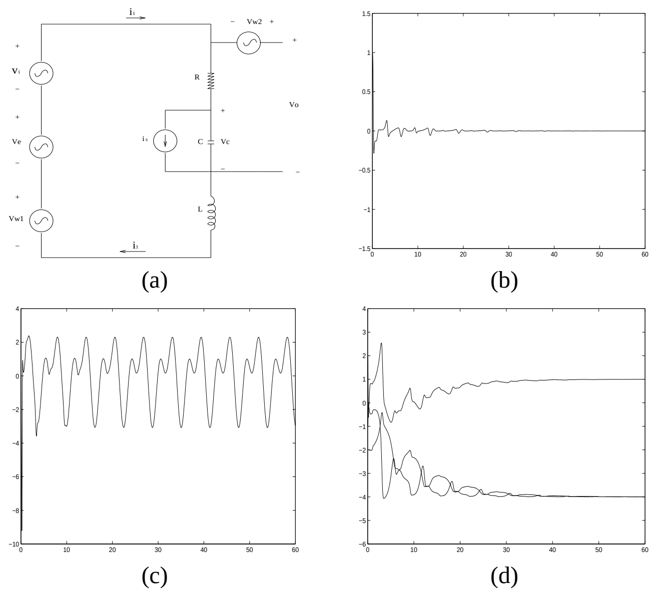

Fig. 1. System response under command input $d(t) = 0$, $\check{w}(t) = 2.5\sin(t)$, $\grave{w}_1(t) = 0$, and $\grave{w}_2(t) = 0$.
(a) Diagram of Circuit (b) Tracking error; (c) Control input; (d) Parameter estimate.

1) the controller μ achieves disturbance attenuation level γ with respect to w and disturbance attenuation level 0 with respect to $\check{w}$.
2) $\forall (x_0, \theta, \grave{w}_{[0,\infty)}, \check{w}_{[0,\infty)}) \in \grave{\mathcal{W}}$ with $\grave{w}_{[0,\infty)} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\check{w}_{[0,\infty)} \in \mathcal{L}_\infty$, then $\lim_{t\to\infty}(x_1(t) - y_d(t)) = 0$.

Remark 1: The adaptive control design can be extended to treat linear systems with partially known control vector £elds. Consider the following design model

$$\begin{aligned} \dot{x} &= Ax + (y\bar{A}_{211} + u\bar{A}_{212} + \sum_{j=1}^{\check{q}} \check{w}_j \bar{A}_{213j})\theta + Bu + Dw + \check{D}\check{w} \\ y &= Cx + Ew \end{aligned}$$

Compared to the model (2), there is an additional term Bu, where B is known and has the following structure $B = \begin{bmatrix} \mathbf{0}_{1\times(r-1)} & b_{p0} & b_{p1} & \cdots & b_{p,n-r} \end{bmatrix}'$. If the high frequency gain b_0 is unknown, b_{p0} will be absorbed into θ. In this case, the identi£er will involve equations (9), (10), and (12) except there will be an additional term Bu in (12e). We can follow the same steps in the control design. The same results as Theorem 1 and Corollary 1 still hold.

V. Example

In this section, we present one example to illustrate the main results of this paper. The designs were carried out using MATLAB symbolic computation tools, and the closed-loop systems were simulated using SIMULINK.

Consider the following circuit problem in Figure 1(a), where v_i is the input voltage source; v_o is the measured output; v_e is an unknown sinusoidal voltage source; v_{w1} is an unmeasured exogenous voltage source; v_{w2} is an unmeasured exogenous voltage disturbance in the output channel; i_s is a measured exogenous current source. The objective is to achieve asymptotic tracking of $v_o - v_{w2}$ to the reference trajectory y_d.

The equations that describe the circuit are obtained as

$$\begin{aligned}\dot{\grave{x}}_1 &= \frac{1}{L}\left(u + v_e + \grave{w}_1 - R\grave{x}_1 - \grave{x}_2\right); & \grave{x}_1(0) = 1\\ \dot{\grave{x}}_2 &= \frac{1}{C}\left(\grave{x}_1 - \breve{w}\right); & \grave{x}_2(0) = 1\\ y &= R\grave{x}_1 + \grave{x}_2 + \grave{w}_2\end{aligned}$$

where $\grave{x}_1 = i_1$, $\grave{x}_2 = v_c$, $u = v_i$, $y = v_o$, $\breve{w} = i_s$, $\grave{w}_1 = v_{w1}$, and $\grave{w}_2 = v_{w2}$. v_e can be modeled as the output of a second-order linear system as following,

$$\begin{aligned}\dot{\grave{x}}_3 &= \grave{x}_4; & \grave{x}_3(0) = 1;\\ \dot{\grave{x}}_4 &= \theta_2\grave{x}_3; & \grave{x}_4(0) = 2;\end{aligned}$$

The initial conditions are set for illustration purposes. Note that the true system satis£es the Assumptions 1, 2, and 9. The adaptive controller will achieve zero disturbance attenuation level with respect to $\breve{w}$. We assume that $R = 1\Omega$ and $L = 1$H. We de£ne $\theta = [\theta_1; \theta_2; \theta_3]'$, where $\theta_1 = \frac{1}{C}$ and $\theta_3 = \frac{\theta_2}{C}$. The true value for the parameter θ is $[1, -4, -4]$. Introduce the appropriate state and disturbance transformations, we transform the ture system into the design model. For the adaptive control design, the ultimate lower bound for the achievable disturbance attenuation level is 2.5 with respect to the transformed disturbance w. We set the desired disturbance attenuation level $\gamma = 12.5$.

The reference trajectory, y_d, is generated by the following linear system, $\dot{x}_d = -x_d + d$; $y_d = x_d$, with initial condition $x_d(0) = 1$, where d is the command input signal.

We select $\check{x}_0 = [\ 1 \quad 0 \quad 0 \quad 0\ ]'$; $\check{\theta}_0 = [\ 2 \quad -1 \quad -2\ ]'$; $Q_0 = 0.00008I_3$; $K_c = 0.038$; $\bar{\beta}_1 = 0.1$; $Y = 1.7988I_3$.

Select the input signals as those of Figure 1, the simulation results are shown in the Figure 1(b)-(d). We observe that the parameter estimates converge to the true values, and the output tracking error converge to zero. The transient of the system is well-behaved, and the control magnitude is again upper bounded by 9.

VI. Conclusions

In this paper, we studied the adaptive control design for tracking and disturbance attenuation for SISO linear systems with noisy output measurements and partly measured disturbances. We consider the same linear system as [1] except that part of the disturbance inputs are measured. We make the same assumptions as those of [1], which includes a known upper bound of the dynamic order, a known relative degree, a strictly minimum phase transfer function from the control input to the output, known sign of the high-frequency gain with nonvanishing bound away from zero. The same design paradigm is used as that of [1] to obtain the adaptive controllers, where cost-to-come function analysis is £rst applied to obtain the £nite-dimensional estimator and the integrator backstepping methodology is then applied to obtain the controllers. The controllers then achieve the desired disturbance attenuation level, with the ultimate lower bound of the attenuation level being the noise intensity in the measurement channel. They guarantee the total stability of the closed-loop system and achieves asymptotic tracking of the refence trajectory when the disturbance is of £nite energy and uniformly bounded. These results are the same as those of [1]. By proper scaling, one may achieve arbitrary positive disturbance attenuation level with respect to the measured disturbances. The advantage of the measurements of some disturbance inputs is that, when the relative degrees from the measured disturbances to the output are greater than or equal to that from the control input, the controllers guarantee *zero* disturbance attenuation level with respect to measured disturbance inputs. Furthermore, asymptotic tracking is achieved even though the measured disturbances are uniformly bounded without being of £nite energy. These theoretical results are illustrated by numerical examples in the paper.

Future research directions that are of interest are described as follows. One fruitful direction lies in the extension of the results to more general multiple-input multiple-output systems. Another direction lies in the generalization of the results to nonlinear systems.

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