Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Supervisor Synthesis for Bounded Petri Nets Based on a Transformation Function

Yu Ru, Weimin Wu, Hongye Su and Jian Chu
National Laboratory of Industrial Control Technology
& Institute of Advanced Process Control
Zhejiang University, Hangzhou, 310027, P.R.China
robinyru@yahoo.com.cn

***Abstract*— This paper addresses the supervisor synthesis on the forbidden state problem, in which the forbidden markings can not be easily expressed as linear inequality constraints using reported methods. The system to be controlled is modelled by bounded Petri nets with uncontrollable transitions. Through the analysis of the reverse net, we obtain the weakly forbidden markings in order to deal with uncontrollable transitions. By introducing a transformation function, which facilitates not only tracking the system state but also determining the control pattern, we propose a synthesis method to obtain the maximally permissive supervisor. The method need not analyze the reachability graph and the online computation has the complexity of polynomial times. In addition, for a special class of generalized Petri nets called the Input Dominant Petri nets, the synthesis method can be applied conveniently, as illustrated by an example in the reported literature.**

I. INTRODUCTION

Petri nets have been used as a model for supervisor synthesis of discrete events systems (DES) due to their advantages, which include a higher language complexity, a compact and graphical description of the state space and the ability of synthesis in a modular way compared with finite state machines [1]. Therefore we study supervisor synthesis of DES modelled by Petri nets on the forbidden state problem.

The forbidden state problem was first introduced by Ramadge and Wonham in [2]. In the case that the forbidden state problem is represented by linear inequality constraints, the various methods proposed in the literature [3]–[7] can be applied. Though there exists a set of generalized mutual exclusion constraints (GMEC) equivalent to any forbidden marking constraints for a safe and conservative Petri net [8], in other condition several individual states may constitute the forbidden states that can not be easily expressed as linear inequality constraints using reported methods.

The earlier work of this aspect was completed by Holloway and Krogh [9], in which the forbidden state problem in cyclic controlled marked graphs was solved. The maximally permissive state feedback control logic is obtained based on the path algorithm and predicates. Though the method is computationally efficient, it is only applicable to safe cyclic marked graphs. Another work [10] done by Ghaffari addresses the general case where forbidden states may not be expressed as linear inequality constraints and provides a synthesis approach that is maximally permissive for general Petri nets and that prevents deadlocks. However it does not avoid the state explosion problem in its design phase.

In this paper, we consider the forbidden state problem in which the forbidden states can not be easily expressed as linear inequality constraints using reported methods. The system is modelled by controlled Petri nets (CtlPNs) that are bounded and may have uncontrollable transitions. By introducing a transformation function, which facilitates not only tracking the system state but also determining the control pattern, we propose a synthesis method to obtain the maximally permissive supervisor. The method can deal with generalized Petri nets only if they are bounded, so it can be applied to more general Petri nets than that in the literature [9]. At the same time the method need not analyze the reachability graph (RG) that inevitably results in the state explosion problem, so it is more efficient than that in the literature [10].

The remainder of this paper is arranged as follows. Next Section introduces some preliminaries about generalized Petri nets and controlled Petri nets. In Section III, the details of the supervisor synthesis method are given. An example in the reported literature is used to show the synthesis procedure in Section IV. Finally, Section V gives the conclusions.

II. PRELIMINARIES

In this section, we first introduce some relevant concepts, notations and properties of generalized Petri nets, then define a special class of generalized Petri nets, finally review controlled Petri nets.

A. Basic concepts and notations of generalized Petri nets

A generalized Petri net structure is a directed graph represented by a quadruple $N = (P, T, F, W)$, where P is a finite set of places $\{p_1, p_2, ...p_m\}$; T is a finite set of transitions $\{t_1, t_2, ...t_n\}$; $F \subseteq (P \times T) \cup (T \times P)$ is the incidence relation, representing the set of directed arcs connecting places to transitions and vice versa; $W : F \rightarrow N$ is the weight function, where N is the positive integer set. It is assumed that $P \cap T = \Phi$ and $P \cup T \neq \Phi$. Graphically, places are represented by circles, transitions are represented by bars and the weight bigger than one is labeled near the corresponding arc, as illustrated in Fig.1.

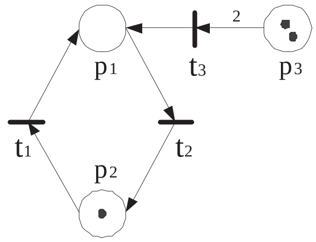

Fig. 1. A generalized Petri net

The sets of all input and output places of a transition $t \in T$ are defined as ${}^\bullet t = \{p \in P | (p,t) \in F\}$ and $t^\bullet = \{p \in P | (t,p) \in F\}$ respectively. Similarly, the sets of all input and output transitions of a place $p \in P$, namely ${}^\bullet p$ and $p^\bullet$, can be defined. Under the assumption that the Petri net is pure (i.e., it has no self-loops) or is made pure by adding a dummy pair of a transition and a place [11], we define the incidence matrix $D \in Z^{m \times n}$,

$$D_{ij} = \begin{cases} W(t_j, p_i) & if\ p_i \in t_j^\bullet \\ -W(p_i, t_j) & if\ p_i \in {}^\bullet t_j \\ 0 & otherwise \end{cases}$$

where Z is the integer set, $i = 1, ...m$ and $j = 1, ...n$.

A marking is a function $\mu : P \rightarrow N_0 = \{0, 1, ...\}$ that assigns to each place a non-negative integer number of tokens, represented by black dots as shown in Fig.1, where $\mu(p_i)$ denotes the number of tokens in p_i. A generalized Petri net $G = \langle N, \mu_0 \rangle$ is a net structure N with an initial marking μ_0. A transition t is state-enabled by a marking μ if for each $p \in {}^\bullet t$ such that $\mu(p) \geq W(p,t)$. In this paper, the no concurrency (NC) assumption holds, i.e., only a single transition can fire at any instant. For a given marking μ, a state-enabled transition t may be fired and will result in a marking μ' defined by the following equation:

$$\mu'(p) = \begin{cases} \mu(p) + W(t,p) & if\ p \in t^\bullet \\ \mu(p) - W(p,t) & if\ p \in {}^\bullet t \\ \mu(p) & otherwise \end{cases} \quad (1)$$

The evolution of a Petri net from μ to μ' after firing a transition t is denoted by $\mu[t\rangle\mu'$, where t is state-enabled by μ and μ' is defined by (1). A (possibly empty) firing sequence from a marking μ_0 is a sequence of transitions $\sigma = (t_1, t_2, ..., t_k)$ such that $\mu_0[t_1\rangle\mu_1[t_2\rangle\mu_2 \cdots [t_k\rangle\mu_k$, which is denoted by $\mu_0[\sigma\rangle\mu_k$. The marking μ_k is given by the state equation below:

$$\mu_k = \mu_0 + D\bar{\sigma} \quad (2)$$

where $\bar{\sigma}$ is the non-negative characteristic vector of the firing sequence σ. The set of transitions occurred in σ is denoted by T_σ.

A marking μ' is reachable from μ in G if there exists a firing sequence σ such that $\mu[\sigma\rangle\mu'$. Given a net G, the set of all the reachable markings (also called the reachability set) from μ_0 is denoted by $R(G)$.

B. Properties of generalized Petri nets

The property of boundedness plays an important role in the supervisor synthesis method based on a transformation function, so we first introduce the concept of boundedness.

Definition 1: A Petri net G is said to be *k-bounded* or simply *bounded* if for every $p \in P$ and every $\mu \in R(G)$, $\mu(p) \leq k$, where k is a finite positive integer.

Definition 2: A Petri net G is said to be *structurally bounded* if it is bounded for any finite initial marking μ_0.

The definition of bounded Petri nets implies that to determine if a Petri net is bounded, RG analysis is required. To avoid this difficulty, the following lemma is proposed in [11].

Lemma 1: A Petri net G is structurally bounded if and only if there exists an *m-vector* y of positive integers such that $y^T D \leq \mathbf{0}$.

Unluckily, the lemma tells nothing about how to find the positive integer vector. However for a special class of generalized Petri nets introduced in this paper, which is a superset of state machines (SM, in which $W = 1$ and each transition has exactly one input and exactly one output place), its boundedness can be proven.

Definition 3: A Petri net G is called the *Input Dominant Petri net* if for each transition $t \in T$, the sum of input arcs' weight is bigger than or equal to that of output arcs', i.e.,

$$\sum_{p_i \in {}^\bullet t} W(p_i, t) \geq \sum_{p_j \in t^\bullet} W(t, p_j)$$

Using Lemma 1, we prove that Input Dominant Petri nets are structurally bounded, which is demonstrated in the following theorem.

Theorem 1: An Input Dominant Petri net G is structurally bounded and one upper bound is $\sum_{i=1}^{m} \mu_0(p_i)$ for every place $p \in P$.

Proof: Given an Input Dominant Petri net $G = \langle N, \mu_0 \rangle$, the incidence matrix is $D \in Z^{m \times n}$. Considering the *m-vector* $y = (1 \cdots 1)^T$, we get

$$y^T D(:,j) = \sum_{i=1}^{m} D(i,j) = \sum_{p \in t_j^\bullet} W(t_j, p) - \sum_{p \in {}^\bullet t_j} W(p, t_j) \leq 0$$

where $D(:,j)$ represents the jth column of D. As it is true for each $j = 1, 2, ...n$, $y^T D \leq \mathbf{0}$. Therefore according to Lemma 1, an Input Dominant Petri net is structurally bounded.

Multiplying (2) by y^T, we obtain

$$y^T \mu_k = y^T \mu_0 + y^T D \bar{\sigma}$$

Since $y^T D \leq \mathbf{0}$ and $\bar{\sigma}$ is non-negative, it can be deduced that

$$y^T \mu_k \leq y^T \mu_0$$

Taking into account $y = (1 \cdots 1)^T$, then

$$\sum_{i=1}^{m} \mu_k(p_i) \leq \sum_{i=1}^{m} \mu_0(p_i)$$

therefore for each $p \in P$

$$\mu_k(p) \leq \sum_{i=1}^{m} \mu_0(p_i)$$

which proves the latter part of Theorem 1. $\diamondsuit$

Remark 1: The upper bound given in Theorem 1 is not necessarily the supremum, which depends on the net structure and the initial marking. Considering the Petri net in Fig.1, obviously it is an Input Dominant Petri net. According to Theorem 1, we obtain the upper bound 3 for every place, but the supremum is 2.

To deal with uncontrollable transitions, we introduce the definition of reverse net [12].

Definition 4: A Petri net structure $N' = (P, T, F', W')$ is said to be the *reverse net* of $N = (P, T, F, W)$, if $F' = \{(x, y)|(y, x) \in F\}$ and $W'(x, y) = W(y, x), \forall(x, y) \in F'$.

In words, the reverse net changes nothing of the original net but reversing the arcs' direction. One property of the reverse net is given in the following lemma.

Lemma 2: If there is a firing sequence $\sigma = (t_1, t_2, ..., t_k)$ such that $\mu_0[\sigma\rangle\mu_k$ in the net N, then there exists a firing sequence σ' such that $\mu_k[\sigma'\rangle\mu_0$ in the reverse net N' and vice versa.

Proof: We first prove that if there is a state-enabled transition t such that $\mu[t\rangle\mu'$ in N then $\mu'[t\rangle\mu$ in N' holds. In N, μ' is defined by (1), which implies that $\mu'(p) \geq W(t, p)$ for each $p \in t^\bullet$. Therefore for each $p \in {}^\bullet t$ in N', $\mu'(p) \geq W'(p, t)$, i.e., t is state-enabled at μ' in N'. It is easy to verify that μ is obtained after firing t in N'.

Applying the result to $\mu_0[t_1\rangle\mu_1[t_2\rangle\mu_2 \cdots [t_k\rangle\mu_k$ in N, we obtain $\mu_k[t_k\rangle\mu_{k-1}[t_{k-1}\rangle\mu_{k-2} \cdots [t_1\rangle\mu_0$ in N', i.e., $\sigma' = (t_k, t_{k-1}, ..., t_1)$. Similarly the latter part can be proven. $\diamondsuit$

C. Controlled Petri nets

Controlled Petri nets are a class of Petri nets with external enabling conditions. Formally a CtlPN is a tuple $G_c = (G, T_c)$, where G is a generalized Petri net and T_c ($T_{uc} = T \backslash T_c$) is the controllable transition subset. The state of G_c is determined by the marking of G.

A control pattern u for the controlled Petri net G_c is a subset of transitions such that $T_{uc} \subseteq u \subseteq T$. We write $u_1 \geq u_2$ ($u_1 > u_2$) if $u_1 \supseteq u_2$ ($u_1 \supset u_2$). A control pattern u_1 is said to be more permissive than a control pattern u_2 if $u_1 > u_2$. The most permissive control pattern is $u_{one} = T$ and the least is $u_{zero} = T_{uc}$.

A transition $t \in T$ is said to be control-enabled under u if $t \in u$. If a transition t is both state-enabled by μ and control-enabled by u, it is enabled and can be fired. After firing t, the obtained state can be decided by (1). Given a controlled Petri net G_c, the set of all the reachable markings from μ under u is denoted by $R_\infty(\mu, u)$. Similarly $R_\infty(M, u) = \bigcup_{\mu \in M} R_\infty(\mu, u)$ denotes the set of all the reachable markings from a marking set M under u.

III. Supervisor synthesis

In this section, we demonstrate the supervisor synthesis method for bounded Petri nets, the idea of which is inspired from a simple example in [5]. In Section IV we will use the example to illustrate the synthesis method.

A. The forbidden state problem

Given a bounded Petri net G, the control goal considered in this paper is to prevent the controlled Petri net from reaching any marking within a designated set of forbidden markings M_F, which can not be easily expressed as linear inequality constraints using reported methods.

In general, there is a larger set of markings that must be avoided, due to the possibility of uncontrollable firing sequences. We call this larger set the weakly forbidden markings (WFM) defined as follows.

Definition 5: Given a set of forbidden markings M_F, $W(M_F)$ is the set of *weakly forbidden markings* with respect to M_F defined by $W(M_F) = \{\mu | R_\infty(\mu, u_{zero}) \bigcap M_F \neq \Phi\}$ [9].

In words, $W(M_F)$ is the set of markings from which a marking in M_F can be reached uncontrollably. There exists a unique maximally permissive state feedback control policy if and only if $\mu_0 \notin W(M_F)$ [1].

To compute $W(M_F)$, we give the following theorem.

Theorem 2: Given a net N, $W(M_F) = R'_\infty(M_F, u_{zero})$, where $R'_\infty(M_F, u_{zero})$ denotes all the reachable markings from M_F under u_{zero} in the reverse net N'.

Proof: First we prove $W(M_F) \subseteq R'_\infty(M_F, u_{zero})$. $\forall\mu' \in W(M_F)$, $\exists\mu \in M_F$ and σ' which satisfies $T_{\sigma'} \subseteq u_{zero}$, such that $\mu'[\sigma'\rangle\mu$ in N. According to Lemma 2, there exists a firing sequence σ which obviously satisfies $T_\sigma \subseteq u_{zero}$, such that $\mu[\sigma\rangle\mu'$ in N'. Therefore $\mu' \in R'_\infty(M_F, u_{zero})$, which implies $W(M_F) \subseteq R'_\infty(M_F, u_{zero})$.

Similarly we can prove $R'_\infty(M_F, u_{zero}) \subseteq W(M_F)$. Therefore $W(M_F) = R'_\infty(M_F, u_{zero})$. $\diamondsuit$

According to Theorem 2, we can obtain $W(M_F)$ by finding all the reachable markings from M_F under u_{zero} in the reverse net N'. Through the analysis of N', we obtain the weakly forbidden markings without constructing influence paths [9]. Obviously if all the transitions are controllable, i.e., $u_{zero} = \Phi$, then $W(M_F) = M_F$.

B. The transformation function

Given a *k-bounded* Petri net G, the transformation function $\Gamma : R(G) \rightarrow N_0$ is defined below,

$$\Gamma(\mu) = \mu(p_1) + \mu(p_2)B + \mu(p_3)B^2 + \cdots + \mu(p_m)B^{m-1}$$

where $B = k + 1$, $\mu \in R(G)$.

Proposition 1: The transformation function defined above is injective.

[1] It is equivalent to Theorem 2 of [13]

Proof: By contradiction. Suppose $\mu_1, \mu_2 \in R(G)$ and $\mu_1 \neq \mu_2$, but $\Gamma(\mu_1) = \Gamma(\mu_2)$. Then we obtain

$$\mu_1(p_1) + \mu_1(p_2)B + \cdots + \mu_1(p_m)B^{m-1} = \mu_2(p_1) + \mu_2(p_2)B + \cdots + \mu_2(p_m)B^{m-1}$$

which can be written in another form

$$(\mu_1(p_1) - \mu_2(p_1)) + B\{(\mu_1(p_2) - \mu_2(p_2)) + \cdots + (\mu_1(p_m) - \mu_2(p_m))B^{m-2}\} = 0 \quad (3)$$

Let $L = (\mu_1(p_2)-\mu_2(p_2))+\cdots+(\mu_1(p_m)-\mu_2(p_m))B^{m-2}$, (3) can be expressed as below

$$(\mu_1(p_1) - \mu_2(p_1)) + BL = 0$$

Thus

$$L = \frac{\mu_2(p_1) - \mu_1(p_1)}{B}$$

Since the Petri net G is *k-bounded* and $B = k+1$, it is true that $\mu(p_i) < B$ for each $\mu \in R(G)$ and each $p \in P$, which implies

$$-B < \mu_2(p_1) - \mu_1(p_1) < B$$

Therefore $-1 < L < 1$. Because L is an integer, it must be that $L = 0$ and $\mu_1(p_1) = \mu_2(p_1)$. Similarly, we can prove $\mu_1(p_i) = \mu_2(p_i)$ for $i = 2, 3, ..m$ from $L = 0$. Then $\mu_1 = \mu_2$, a contradiction. The conclusion holds. $\diamondsuit$

Using Γ, $R(G)$ is mapped to a discrete non-negative integer set with equal number of elements. Especially a weakly forbidden marking $\mu_f \in W(M_F)$ is mapped to $\Gamma(\mu_f)$. Therefore it can be represented by the following equation

$$\mu(p_1) + \mu(p_2)B + \cdots + \mu(p_m)B^{m-1} = \Gamma(\mu_f) \quad (4)$$

Similarly $W(M_F) = \{\mu_{f1}, \mu_{f2}, ...\mu_{fl}\}$ with l elements is mapped to a set $I_F = \{\ \Gamma(\mu_f) \mid \mu_f \in W(M_F)\ \}$, which can be sorted into a strictly ascending sequence $S_F : (f_1\ f_2\ ...\ f_l)$ since $\Gamma(\mu)$ is injective. Then the forbidden state problem can be interpreted as finding a control scheme to prevent $\Gamma(\mu)$ from reaching I_F.

According to (4), the admissible markings can be represented by a disjunction of two inequalities

$$\mu(p_1) + \mu(p_2)B + \cdots + \mu(p_m)B^{m-1} \leq \Gamma(\mu_f) - 1 \bigvee$$
$$\mu(p_1) + \mu(p_2)B + \cdots + \mu(p_m)B^{m-1} \geq \Gamma(\mu_f) + 1 \quad (5)$$

To enforce the above OR constraint, the place invariant method introduces the negative token number within the control places [5]. In the worst situation, a disjunction of $l+1$ inequalities in the form of (5) is needed to express the admissible markings when considering $W(M_F)$. To enforce the constraints using the place invariant method, $2l$ control places have to be added and extended transition enabling rules for the controller subnet are required. Obviously it is too complicated though most of the connection relation of those controllers with transitions is the same. In the next part we propose a supervisor synthesis method based on the transformation function to enforce these constraints efficiently.

C. The synthesis method

The synthesis method is composed of three steps. First we judge whether the net is bounded or not and compute the weakly forbidden markings. Then to track the value of $\Gamma(\mu)$ (i.e., the state of G), we construct a monitor place that is similar to the one in [14], but with simpler construction procedures. Finally we determine the control pattern u according to the monitor place's token number.

First step: Boundedness judgement and WFM' computation

The following algorithm gives details of the step.

```
Algorithm 1 (Boundedness judgement and
WFM' computation)
Input: A Petri net G and a set of
forbidden markings M_F
```

1) Judge G is bounded or not
  IF G is unbounded, the synthesis procedure exits
  ELSE, find one upper bound k
  END IF
2) Compute the weakly forbidden markings $W(M_F)$ according to Theorem 2
  IF $\mu_0 \in W(M_F)$, the supervisor does not exist and the synthesis procedure exits
3) Construct the transformation function $\Gamma(\mu)$ with $B = k + 1$
4) Compute S_F based on $W(M_F)$ and $\Gamma(\mu)$

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Output: A petri net G, a strictly
ascending sequence S_F and B
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Remark 2: If G is an Input Dominant Petri net, then it is bounded and $k = \sum_{i=1}^{m} \mu_0(p_i)$. Therefore the method can be applied to Input Dominant Petri nets conveniently. We can also judge the boundedness of G from Lemma 1 if y can be easily obtained.

Remark 3: If possible, we should assign the supremum to k so as to decrease the value of $\Gamma(\mu)$, which will make the monitor place realized easily in real applications.

Second step: Construction of the monitor place

For each transition $t_i \in T$, where $i = 1, 2, ...n$, we compute the value of $\triangle\Gamma(i)$ using the following formula

$$\triangle\Gamma(i) = \sum_{p_j \in t_i^{\bullet}} W(t_i, p_j)B^{j-1} - \sum_{p_k \in {}^{\bullet}t_i} W(p_k, t_i)B^{k-1} \quad (6)$$

Then we can construct the monitor place p_{mon} using $\triangle\Gamma$ conveniently, as shown below.

```
Algorithm 2 (Construction of the monitor
place)
Input: A Petri net G and B
```

1) Compute $\triangle\Gamma$ with B
2) FOR $i = 1, 2, ...n$
  IF $\triangle\Gamma(i) > 0$, draw an arc from t_i to p_{mon} with the weight $\triangle\Gamma(i)$
  ELSE IF $\triangle\Gamma(i) < 0$, draw an arc from p_{mon} to t_i with

the weight $-\triangle\Gamma(i)$
END IF
END FOR
3) The initial token number of p_{mon} is $\Gamma(\mu_0)$.
Output: A monitored Petri net G and $\triangle\Gamma$

Remark 4: Since the Petri net G is pure, $\triangle\Gamma(i)=0$ will not happen.

Next, we will show $\Gamma(\mu)$ is realized by the monitor place in the following theorem.

Theorem 3: For each $\mu \in R(G)$, $\mu_{p_{mon}}=\Gamma(\mu)$.

Proof: By induction on μ. Obviously it is true for μ_0. Now we assume the marking is μ_k that satisfies $\mu^k_{p_{mon}}=\Gamma(\mu_k)$, where $\mu^k_{p_{mon}}$ represents the token number in p_{mon} when the marking is μ_k. Once a transition t_i enabled by μ_k is fired, the token number of $p_j \in t_i^\bullet$ increases by $W(t_i,p_j)$ and that of $p_k \in {}^\bullet t_i$ decreases by $W(p_k,t_i)$, which leads the marking from μ_k to μ_{k+1}. According to the definition of $\Gamma(\mu)$ and (6), we obtain

$$\Gamma(\mu_{k+1})=\Gamma(\mu_k)+\sum_{p_j\in t_i^\bullet} W(t_i,p_j)B^{j-1}-\sum_{p_k\in {}^\bullet t_i} W(p_k,t_i)B^{k-1}=\mu^k_{p_{mon}}+\triangle\Gamma(i)$$

According to Algorithm 2, $\mu^k_{p_{mon}}+\triangle\Gamma(i)$ is equal to $\mu^{k+1}_{p_{mon}}$, i.e., $\mu^{k+1}_{p_{mon}}=\Gamma(\mu_{k+1})$. Therefore the conclusion holds for each $\mu \in R(G)$. $\diamondsuit$

Since the monitor place's token number tracks the value of $\Gamma(\mu)$ according to the evolution of Petri nets, it tracks the system state efficiently.

Third step: Determination of the control pattern

Considering the current marking μ, each transition t in the net can be determined to be control-enabled or not by $\mu_{p_{mon}}$, $\triangle\Gamma$ and S_F in the following algorithm.

Algorithm 3 (Determination of the control pattern)
Input: A monitored Petri net G, $\triangle\Gamma$ and S_F
1) $T_{ua}=\Phi$
2) FOR $i=1,2,...n$
Let $\Gamma_P=\mu_{p_{mon}}+\triangle\Gamma(i)$
FOR $j=1,2,...l$
IF $\Gamma_P=f_j$, then t_i is added to the set T_{ua} and jump out of the inner for loop
END FOR
END FOR
3) $u=T\backslash T_{ua}$
Output: The control pattern u

Remark 5: The control pattern is obtained by judging whether a transition's firing will lead to weakly forbidden markings or not. If it does not, the transition is control-enabled, otherwise control-disabled. The method can realize the forbidden state specification in that the token number of the monitor place tracks the system state, $\triangle\Gamma$ reflects the effect of transitions' firings on the monitor place and S_F represents the weakly forbidden markings. Therefore the method is correct by construction.

Remark 6: The supervisor obtained is maximally permissive in the sense that it only acts to inhibit these transitions whose firings will lead to the forbidden states and permits as many transitions enabled to fire as possible.

Remark 7: Algorithm 1 and 2 can be computed offline. The main online computation comes from Algorithm 3 that has the complexity of polynomial times $O(nl)$, where n and l represents the number of transitions and weakly forbidden markings respectively. Note that here l refers to the number of weakly forbidden markings, not that of forbidden markings. As the computation of WFM is reachability analysis in essence, l may be large for some problems, which causes computational difficulties. In this sense, it is impossible that Algorithm 1 is efficient for all problems.

Remark 8: The controlled system may be not live, even deadlock (either exists originally or caused by supervisor) as we have not considered these issues in the synthesis method.

IV. EXAMPLE

In this section the example in [5] is used to show the synthesis procedure presented in Section III.

Consider the Petri net G in Fig.2, where the initial marking μ_0 is $(1\ 2\ 1)^T$. We want to prevent three individual forbidden states $\mu_{f1}=(1\ 3\ 0)^T$, $\mu_{f2}=(0\ 3\ 1)^T$ and $\mu_{f3}=(1\ 1\ 2)^T$, which make up of M_F and can not be easily expressed as linear inequality constraints using reported methods. Note that in [5] only μ_{f3} is considered. In addition, we suppose t_3 is uncontrollable.

From Theorem 2, it is easy to verify that $W(M_F)=\{\ \mu_{f1}\ \mu_{f2}\ \mu_{f3}\ \mu_{f4}\ \}$ where $\mu_{f4}=(0\ 1\ 3)^T$. Thus $\mu_0 \notin W(M_F)$, which implies the maximally permissive supervisor exists. Since the net is an Input Dominant Petri net, $B=\sum_{i=1}^{3}\mu_0(p_i)+1=5$. The transformation function is $\Gamma(\mu)=\mu(p_1)+5\mu(p_2)+25\mu(p_3)$, which maps $W(M_F)$ to an ascending sequence S_F : (16 40 56 80).

For each transition $t_i \in T$, we compute $\triangle\Gamma(i)$ and the result is $\triangle\Gamma=(4\ 20\ -24)^T$. According to Algorithm 2, we construct p_{mon} with the initial token number $\Gamma(\mu_0)=36$, as illustrated in Fig.3.

For the Petri net G with μ_0, the control pattern $u=\{t_3\}$ is obtained using Algorithm 3. After firing the enabled transition t_3, $\mu_1=(2\ 2\ 0)^T$ is reached and $\mu_{p_{mon}}=12$ accordingly, which is shown in Fig.4. Applying Algorithm 3 to G with μ_1, we get the control pattern $u=\{t_2,t_3\}$. Since t_3 is not state-enabled, only t_2 is enabled to fire. In such a way, the system progresses under the control of the supervisor so as to meet the forbidden state specifications.

V. CONCLUSIONS

This paper has addressed the supervisor synthesis problem for bounded Petri nets with control specifications described as a set of forbidden markings, which can not be easily expressed as linear inequality constraints using reported methods. Through the analysis of the reverse net, we obtain the weakly forbidden markings in order to deal with uncontrollable transitions without constructing influence paths. By introducing the transformation function that facilitates not only tracking the system state but also determining the control pattern, we propose a method to synthesize the maximally permissive supervisor. The method need not analyze the RG and the online computation has the complexity of polynomial times. In addition for Input Dominant Petri nets that are structurally bounded and easy to construct the transformation function, the method can be applied conveniently, as illustrated by the example in Section IV.

The boundedness assumption seems somewhat restrictive, however unboundedness does not arise very often in the supervisory control context. One drawback is that when the number of places or the upper bound increases, so does the weight of arcs between the monitor place and transitions and the monitor place's token number. As a result it may be hard to realize. In the future we will improve the method to deal with concurrent transitions.

VI. ACKNOWLEDGMENTS

The authors would like to thank Dr Dong L., Profs. Z. Retchkiman, T. Murata and K.M. Passino sincerely for their help and gratefully acknowledge anonymous reviewers' comments.

This work was supported in part by the National Outstanding Youth Science Foundation of China (60025308), Teaching and Research Award Program for Outstanding Young Teachers in Higher Education Institutions of MOE, P.R. China., the key project of CIMS under the National High Technology Research and Development Program of China (863 Program) (2001AA413020) and Doctor Degree Program Foundation of China (20020335103).

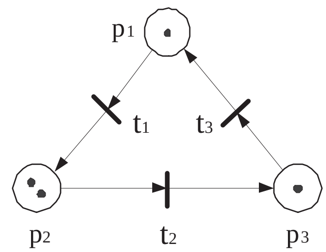

Fig. 2. The uncontrolled Petri net

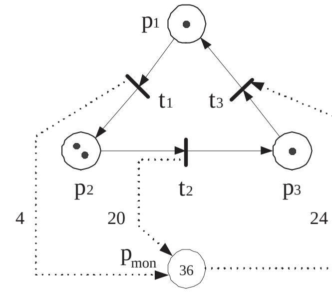

Fig. 3. The monitored Petri net with μ_0

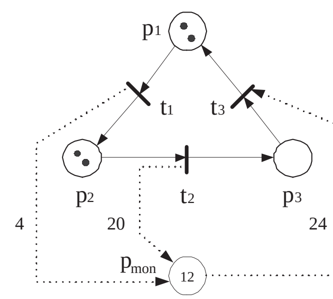

Fig. 4. The monitored Petri net with μ_1

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