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Fault Diagnosis in a Class of Differential-Algebraic Systems

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***Abstract*— An Iterative Learning Observer (ILO) is proposed for fault diagnosis in a class of differential-algebraic nonlinear systems described by so-called semi-explicit form with index 1. The main feature of this ILO-based fault diagnosis strategy is that the ILO can estimate both system states and algebraic variable. This is important since both fault detection and estimation can be achieved. As a result the ILO can still track the post-fault system. Moreover, the ILO input can be used to isolate faults. The simulation study presented shows the effectiveness of this ILO-based fault detection and estimation strategy for differential-algebraic systems.**

I. INTRODUCTION

A large class of engineering systems are described by a mix of differential and algebraic equations. Example of such systems are power systems, robotic manipulators, and electric circuits, and chemical processes [5], [13], [14]. For example, in a chemical process, the differential equations may arise from dynamic conservation equations, while the algebraic equations commonly arise from thermodynamic equilibrium relations, empirical correlations, pseudo-steady-state assumptions, and so on.

Research on the control of differential-algebraic systems has attracted a great deal of attention during the past few years. The problem of feedback controller synthesis has been addressed only for restricted classes of differential-algebraic systems that mainly arise from mechanical systems [7], [10]. A framework for study of Lyapunov stability of equilibria in differential-algebraic systems is presented in [5]. [9] addresses the output feedback control problem for a class of nonlinear multivaraible high-index differential-algebraic equation systems in semi-explicit form.

At the same time over the last two decades, fault diagnosis has attracted a lot of attention. The majority work is to design, analyze fault detection and isolation issue for both linear and nonlinear systems [1], [2], [8], [11], [15], [16].

In this work, fault diagnosis problem will be discussed in a class of differential-algebraic systems based on an Iterative Learning Observer. A nonlinear sliding mode observer is proposed in [13] for a linear differential-algebraic system whose model is first realized by converting it into an equivalent control problem via the singularly perturbed sliding manifold (SPSM) approach. A robust sliding observer is then designed, ensuring asymptotic stability in the presence of disturbances.

Over past decade, few works have been done for fault diagnosis in differential-algebraic systems. The main contribution of [14] is to design and analyze a numerically feasible learning scheme for robust and stable fault diagnosis of differential-algebraic systems. The proposed fault diagnosis architecture monitors the physical system for any off-nominal behavior using online modelling techniques and learning algorithms. Online approximators, in the form of neural networks, are utilized in the detection of faults and in the derivation of models for the fault function, which can be used for fault isolation, fault identification, and fault accommodation.

In this paper, fault detection and estimation issues, based on an ILO, will be discussed in a class of differential-algebraic systems that are in the so-called semi-explicit form with index 1.

II. PROBLEM STATEMENT AND SYSTEM FORMULATION

Consider a class of differential-algebraic systems described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \Phi(x(t), z(t), u(t)) + f_a(t) \\ 0 &= k(x) + g(x)z(t) \\ y(t) &= Cx(t) + Dz(t) \end{aligned} \tag{1}$$

with compatible initial conditions, where $x(t) \in \Re^n$ is unmeasurable system state vector; $z(t) \in \Re^l$ is algebraic variable; $y(t) \in \Re^p$ is measurable output; $u(t) \in \Re^m$ is system control input; $\Phi(x, z, u) : \Re^n \times \Re^l \times \Re^m \rightarrow \Re^n$ is a Lipschitz nonlinearity; $f_a(t)$ represents system faults, say aged components or actuator faults; $A \in \Re^{n\times n}, C \in \Re^{p\times n}$ and $D \in \Re^{p\times l}$ are constant matrices; $k(x) : \Re^n \rightarrow \Re^l$; $g(x) : \Re^n \rightarrow \Re^{l\times l}$.

Remark 1: Incompatible initial conditions will typically lead to jumps in the constraint [5].

Remark 2: The above description of differential-algebraic systems is in the so-called semi-explicit form [4] with the algebraic variables $z(t)$ appearing linearly. The semi-explicit differential-algebraic system model is motivated by some practical applications, such as chemical processes. Moreover, the linear form of the algebraic variables $z(t)$ is also typical in chemical processes.

The problem in question is to construct an ILO in a class of differential-algebraic systems described in (1) for the purpose of fault diagnosis.

Following nomenclature is adopted in this work:

A^T: Transpose of matrix A; $\|A\| = [\lambda_{max}(A^TA)]^{\frac{1}{2}}$: Matrix norm, where λ_{max} is the maximum eigenvalue; $\|x\| = (x^Tx)^{\frac{1}{2}}$: Euclidean norm of vector x; $\lambda_{min}(A)$: minimum eigenvalue of matrix A.

Additionally, throughout this paper, following assumptions are required

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Assumption 1: Fault $f_a(t)$ is bounded with $\|f_a(t)\| \leq b_f, \quad \forall t \geq 0.$

Assumption 2: $g(x) \in \Re^{l\times l}$ has full rank.

Assumption 3: $\Phi(x,z,u)$ is bounded and satisfies Lipschitz condition with Lipschitz constants b_1 and b_2 i.e.

$$\|\Phi(x,z,u)-\Phi(\hat{x},\hat{z},u)\| \leq b_1\|x(t)-\hat{x}(t)\|+b_2\|z(t)-\hat{z}(t)\|. \tag{2}$$

Assumption 4: $g^{-1}(x)k(x)$ satisfies Lipschitz condition with Lipschitz constants b_3 i.e.

$$\|g^{-1}(x)k(x) - g^{-1}(\hat{x})k(\hat{x})\| \leq b_3\|x(t) - \hat{x}(t)\|. \tag{3}$$

Assumption 5: Matrix A is Hurwitz and system (1) is bounded input-bounded state stable.

Remark 3: Because of Assumption 2, the algebraic variable $z(t)$ can be directly solved as

$$z(t) = -g^{-1}(x)k(x). \tag{4}$$

In addition, with the help of Assumptions 3 and 4, following result can be further derived

$$\begin{array}{ll} & \|\Phi(x,z,u) - \Phi(\hat{x},\hat{z},u)\| \leq b_1\|x(t) - \hat{x}(t)\| \\ & +b_2\|g^{-1}(x)k(x) - g^{-1}(\hat{x})k(\hat{x})\| \\ \leq & b_1\|x(t) - \hat{x}(t)\| + b_2b_3\|x(t) - \hat{x}(t)\| \\ = & b_4\|x(t) - \hat{x}(t)\| \end{array} \tag{5}$$

where $b_4 = b_1 + b_2b_3$.

III. Main Result

In this section, we present an ILO for the considered differential-algebraic systems, then its stability will be proved.

A. An ILO for Differential-Algebraic Systems

The ILO was first suggested in [3]. The main characteristic of it is that its states are updated or driven successively by the previous system output errors and the previous ILO input. Here, $Iterative$ indicates that ILO repeats the same operation, i.e. the operation of the ILO input being always updated by the previous information.

An ILO for the differential-algebraic systems (1) is proposed in following form

$$\begin{array}{rcl} \dot{\hat{x}}(t) & = & A\hat{x}(t) + \Phi(\hat{x}(t),\hat{z}(t),u) + L(y(t) - \hat{y}(t)) - v(t) \\ \hat{z}(t) & = & -g^{-1}(\hat{x})k(\hat{x}) \\ v(t) & = & K_1v(t-\tau) + K_2[y(t-\tau) - \hat{y}(t-\tau)] \\ \hat{y}(t) & = & C\hat{x}(t) + D\hat{z}(t) \end{array} \tag{6}$$

where $\hat{x}(t) \in \Re^n$ is the estimated system state; $\hat{z}(t) \in \Re^l$ is the estimated algebraic variable; $\hat{y}(t) \in \Re^p$ is the estimated system output at time t; τ is sampling time interval; $y(t-\tau) \in \Re^p$ is the immediate past measurable output, i.e. the output at time $t-\tau$; $v(t)$ is called *ILO input*; L and $K_i's$ are gain matrices with appropriate dimensions to be determined, where $i = 1, 2$. It is noted that the algebraic variable $z(t)$ is directly estimated from the estimated $x(t)$ because of Assumption 2.

Subtracting observer (6) from (1) leads following estimation error equation:

$$\begin{array}{rcl} \dot{\tilde{x}}(t) & = & (A - LC)\tilde{x}(t) + [\Phi(x,z,u) - \Phi(\hat{x},\hat{z},u)] \\ & & -LD\tilde{z}(t) + f_a(t) + v(t) \\ \tilde{z}(t) & = & -g^{-1}(x)k(x) + g^{-1}(\hat{x})k(\hat{x}) \\ \tilde{y}(t) & = & C\tilde{x}(t) + D\tilde{z}(t) \end{array} \tag{7}$$

where $\tilde{x}(t) = x(t) - \hat{x}(t)$ is system state estimation error, $\tilde{z}$ is algebraic variable estimation error, matrix $(A-LC)$ can be a stable matrix by selecting an appropriate gain matrix L.

The ILO input $v(t)$ is updated by the immediate past values such that $v(t)$ can estimate fault $f_a(t)$ that will be seen in the simulation. Our next task is to prove the stability of the proposed ILO (6).

B. Stability Analysis

To prove theorem 1 that states the stability conditions of the proposed ILO, lemma 1 is required.

Lemma 1: If ILO input $v(t)$ is defined by (6), then following inequality holds

$$\begin{array}{rcl} v^T(t)v(t) & \leq & 3v^T(t-\tau)K_1^TK_1v(t-\tau) \\ & & +3\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) \\ & & +3\tilde{z}^T(t-\tau)(K_2D)^T(K_2D)\tilde{z}(t-\tau). \end{array} \tag{8}$$

Proof:

Substituting expression of ILO input $v(t)$ in (6) into $2v^T(t)v(t)$, we have:

$$\begin{array}{l} 2v^T(t)v(t) = 2v^T(t-\tau)K_1^TK_1v(t-\tau) \\ +2v^T(t-\tau)K_1^TK_2C\tilde{x}(t-\tau) \\ +2v^T(t-\tau)K_1^TK_2D\tilde{z}(t-\tau) \\ +2\tilde{x}^T(t-\tau)(K_2C)^TK_1v(t-\tau) \\ +2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) \\ +2\tilde{x}^T(t-\tau)(K_2C)^T(K_2D)\tilde{z}(t-\tau) \\ +2\tilde{z}^T(t-\tau)(K_2D)^TK_1v(t-\tau) \\ +2\tilde{z}^T(t-\tau)(K_2D)^T(K_2C)\tilde{x}(t-\tau) \\ +2\tilde{z}^T(t-\tau)(K_2D)^T(K_2D)\tilde{z}(t-\tau). \end{array} \tag{9}$$

By applying the following inequality

$$2a^Tb \leq a^Ta + b^Tb \qquad \forall \;\; a,b \in \Re^n, \tag{10}$$

we have:

$$\begin{array}{l} 2v^T(t-\tau)K_1^TK_2C\tilde{x}(t-\tau) \\ \leq v^T(t-\tau)K_1^TK_1v(t-\tau) \\ +\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) \end{array} \tag{11}$$

$$\begin{array}{l} 2v^T(t-\tau)K_1^TK_2D\tilde{z}(t-\tau) \\ \leq v^T(t-\tau)K_1^TK_1v(t-\tau) \\ +\tilde{z}^T(t-\tau)(K_2D)^T(K_2D)\tilde{z}(t-\tau) \end{array} \tag{12}$$

$$\begin{array}{l} 2\tilde{x}^T(t-\tau)(K_2C)^TK_1v(t-\tau) \\ \leq v^T(t-\tau)K_1^TK_1v(t-\tau) \\ +\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) \end{array} \tag{13}$$

$$\begin{array}{l} 2\tilde{x}^T(t-\tau)(K_2C)^T(K_2D)\tilde{z}(t-\tau) \\ \leq \tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) \\ +\tilde{z}^T(t-\tau)(K_2D)^T(K_2D)\tilde{z}(t-\tau) \end{array} \tag{14}$$

$$\begin{array}{l}2\tilde{z}^T(t-\tau)(K_2D)^TK_1v(t-\tau)\\ \le \tilde{z}^T(t-\tau)(K_2D)^T(K_2D)\tilde{z}(t-\tau)\\ +v^T(t-\tau)K_1^TK_1v(t-\tau)\end{array} \tag{15}$$

$$\begin{array}{l}2\tilde{z}^T(t-\tau)(K_2D)^T(K_2C)\tilde{x}(t-\tau)\\ \le \tilde{z}^T(t-\tau)(K_2D)^T(K_2D)\tilde{z}(t-\tau)\\ +\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau).\end{array} \tag{16}$$

Substituting (11) through (16) into (9), we have

$$\begin{aligned}v^T(t)v(t) \quad \le \quad & 3v^T(t-\tau)K_1^TK_1v(t-\tau)\\ & +3\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau)\\ & +3\tilde{z}^T(t-\tau)(K_2D)^T(K_2D)\tilde{z}(t-\tau).\end{aligned} \tag{17}$$

This completes the proof. ■

Theorem 1: Consider differential-algebraic systems (1) satisfying Assumptions 1-5, and the ILO is given in (6). If (23) and (24) hold, then system state estimation error is bounded.

Proof:

Consider following Lyapunov function candidate:

$$\begin{aligned}V(t) \quad = \quad & \tilde{x}^TP\tilde{x}+\int_{t-\tau}^{t}\tilde{x}^T(\theta)R\tilde{x}(\theta)d\theta\\ & +\int_{t-\tau}^{t}v^T(\alpha)v(\alpha)d\alpha\end{aligned} \tag{18}$$

where $P=P^T>0$ and $R=R^T>0$.

Substituting (7) into the derivative of Lyapunov function candidate V leads

$$\begin{aligned}\dot{V} \quad = \quad & \tilde{x}^T((A-LC)^TP+P(A-LC))\tilde{x}-2\tilde{x}^TPLD\tilde{z}\\ & +2\tilde{x}^TP(\Phi(x,z,u)-\Phi(\hat{x},\hat{z},u))+2\tilde{x}^TPf_a\\ & +2\tilde{x}^TPv(t)+v^T(t)v(t)-v^T(t-\tau)v(t-\tau)\\ & +\tilde{x}^T(t)R\tilde{x}(t)-\tilde{x}^T(t-\tau)R\tilde{x}(t-\tau).\end{aligned} \tag{19}$$

By applying inequality (10), following inequality holds

$$2\tilde{x}^TPv(t)\le\tilde{x}^TPP\tilde{x}+v^T(t)v(t). \tag{20}$$

Substituting (5) and (20) into (19), we get

$$\begin{aligned}\dot{V} \quad \le \quad & \tilde{x}^T((A-LC)^TP+P(A-LC)+PP+R)\tilde{x}\\ & +2\lambda_{max}(P)b_3\|LD\|\|\tilde{x}\|^2+2\lambda_{max}(P)b_4\|\tilde{x}\|^2\\ & +(2+\gamma)v^T(t)v(t)-v^T(t-\tau)v(t-\tau)\\ & +2\tilde{x}^TPf_a-\gamma v^T(t)v(t)-\tilde{x}^T(t-\tau)R\tilde{x}(t-\tau)\end{aligned} \tag{21}$$

where $I_l\in\Re^{l\times l}$ is an identity matrix.

Using Lemma 1, the above equation can be further extended as

$$\begin{aligned}\dot{V} \quad \le \quad & \tilde{x}^T((A-LC)^TP+P(A-LC)+PP+R)\tilde{x}\\ & +2\lambda_{max}(P)b_3\|LD\|\|\tilde{x}\|^2+2\lambda_{max}(P)b_4\|\tilde{x}\|^2\\ & +v^T(t-\tau)[(6+3\gamma)K_1^TK_1-I]v(t-\tau)\\ & +(6+3\gamma)\lambda_{max}[(K_2D)^T(K_2D)]b_3^2\|\tilde{x}(t-\tau)\|^2\\ & +(6+3\gamma)\lambda_{max}[(K_2C)^T(K_2C)]\|\tilde{x}(t-\tau)\|^2\\ & -\lambda_{min}(R)\|\tilde{x}(t-\tau)\|^2\\ & +2\lambda_{max}(P)b_f\|\tilde{x}\|-\gamma v^T(t)v(t)\end{aligned} \tag{22}$$

where $I\in\Re^{n\times n}$ is an identity matrix.

For any $Q=Q^T>0$, there exists a $P=P^T>0$ satisfying following equation

$$(A-LC)^TP+P(A-LC)+PP+R=-Q, \tag{23}$$

and let

$$\begin{array}{l}(6+3\gamma)K_1^TK_1\le I,\\ \lambda_{min}(R)\ge(6+3\gamma)\lambda_{max}[(K_2D)^T(K_2D)]b_3^2\\ +(6+3\gamma)\lambda_{max}[(K_2C)^T(K_2C)],\end{array} \tag{24}$$

(22) can be simplified as

$$\begin{aligned}\dot{V} \quad \le \quad & -\lambda_{min}(Q)\|\tilde{x}\|^2-\gamma v^T(t)v(t)\\ & +2\lambda_{max}(P)b_3\|LD\|\|\tilde{x}\|^2+2\lambda_{max}(P)b_4\|\tilde{x}\|^2\\ & +2\lambda_{max}(P)b_f\|\tilde{x}\|\end{aligned} \tag{25}$$

According to [6], [12], the above inequality has following form by some operation

$$\begin{aligned}\dot{V} \quad & \le \quad -\alpha\|\tilde{x}\|^2+c\|\tilde{x}\|-\gamma v^T(t)v(t)\\ & \le \quad -\alpha/2\|\tilde{x}\|^2-\gamma v^T(t)v(t)+k,\end{aligned} \tag{26}$$

where $c=2\lambda_{max}(P)b_f$, $\alpha=\lambda_{min}(Q)-2\lambda_{max}(P)b_3\|LD\|-2\lambda_{max}(P)b_4$, then the ILO for the differential-algebraic systems is stable. Furthermore, Assumption 4 can guarantee the boundedness of $\tilde{z}(t)$. The proof is complete. ■

Remark 4: In fact, $\dot{\tilde{x}}$ can be proved bounded, to this end, let $w(t):=\dot{\tilde{x}}(t)$, and differentiate state estimation error (7) to obtain

$$\dot{w}=(A-LC)w+s-LD\dot{\tilde{z}}+\dot{f}_a(t)+\dot{v}(t) \tag{27}$$

where $\dot{v}(t)=K_1\dot{v}(t-\tau)+K_2Cw(t-\tau)+K_2D\dot{\tilde{z}}(t-\tau)$ and

$$\begin{aligned}s \quad := \quad & \frac{d}{dt}(\Phi(x,z,u)-\Phi(\hat{x},\hat{z},u))\\ & =\left[\frac{\partial\Phi}{\partial x}(x,z,u)\dot{x}+\frac{\partial\Phi}{\partial z}(x,z,u)\dot{z}\right]\\ & -\left[\frac{\partial\Phi}{\partial x}(\hat{x},\hat{z},u)\dot{\hat{x}}+\frac{\partial\Phi}{\partial z}(\hat{x},\hat{z},u)\dot{\hat{z}}\right]\\ & +\left[\frac{\partial\Phi}{\partial u}(x,z,u)\dot{u}-\frac{\partial\Phi}{\partial u}(\hat{x},\hat{z},u)\dot{u}\right].\end{aligned} \tag{28}$$

Letting $h(x):=g^{-1}(x)k(x)$ and differentiating $\tilde{z}(t)$ leads following results

$$\begin{aligned}\dot{\tilde{z}}(t) \quad = \quad & -\frac{\partial h}{\partial x}(x)\dot{x}+\frac{\partial h}{\partial x}(\hat{x})\dot{\hat{x}}+\frac{\partial h}{\partial x}(\hat{x})\dot{x}-\frac{\partial h}{\partial x}(\hat{x})\dot{x}\\ = \quad & \left[\frac{\partial h}{\partial x}(\hat{x})-\frac{\partial h}{\partial x}(x)\right]\dot{x}+\frac{\partial h}{\partial x}(\hat{x})\left[\dot{\hat{x}}-\dot{x}\right]\end{aligned} \tag{29}$$

Assume that $\frac{\partial h}{\partial x}(\hat{x})-\frac{\partial h}{\partial x}(x)$ satisfies Lipschitz condition with Lipschitz constant b_h, and that $\frac{\partial h}{\partial x}(\hat{x})$ is bounded. $\dot{x}$ is also bounded because of Assumptions 1, 3, and 5, then take norms on both sides of equation (29), we obtain

$$\|\dot{\tilde{z}}\|\le b_h\|\tilde{x}\|\|\dot{x}\|+\|\frac{\partial h}{\partial x}(\hat{x})\|\|w\| \tag{30}$$

Assume that $\left[\frac{\partial \Phi}{\partial a}(x,z,u)-\frac{\partial \Phi}{\partial a}(\hat{x},\hat{z},u)\right]$, where $a = \{x,z,u\}$, satisfies Lipschitz condition as that in Assumption 3, and that $\frac{\partial \Phi}{\partial b}(\hat{x},\hat{z},u))$, $\dot{u}$, $-\frac{\partial h(x)}{\partial x}$ are bounded, where $b=\{x,z\}$, therefore $\dot{z}$ is bounded, moreover

$$
\begin{aligned}
\|s\| \leq\ & \left\|\frac{\partial \Phi}{\partial x}(x,z,u)\dot{x}-\frac{\partial \Phi}{\partial x}(\hat{x},\hat{z},u)\dot{x}\right\| \\
& +\left\|\frac{\partial \Phi}{\partial x}(\hat{x},\hat{z},u)\dot{x}-\frac{\partial \Phi}{\partial x}(\hat{x},\hat{z},u)\dot{\hat{x}}\right\| \\
& +\left\|\frac{\partial \Phi}{\partial z}(x,z,u)\dot{z}-\frac{\partial \Phi}{\partial z}(\hat{x},\hat{z},u)\dot{z}\right\| \\
& +\left\|\frac{\partial \Phi}{\partial z}(\hat{x},\hat{z},u)\dot{z}-\frac{\partial \Phi}{\partial z}(\hat{x},\hat{z},u)\dot{\hat{z}}\right\| \\
& +\left\|\frac{\partial \Phi}{\partial u}(x,z,u)\dot{u}-\frac{\partial \Phi}{\partial u}(\hat{x},\hat{z},u)\dot{u}\right\| \\
\leq\ & \left\|\frac{\partial \Phi}{\partial x}(x,z,u)-\frac{\partial \Phi}{\partial x}(\hat{x},\hat{z},u)\right\|\|\dot{x}\| \\
& +\left\|\frac{\partial \Phi}{\partial x}(\hat{x},\hat{z},u)\right\|\|\dot{\tilde{x}}\|+\left\|\frac{\partial \Phi}{\partial z}(\hat{x},\hat{z},u)\right\|\|\dot{\tilde{z}}\| \\
& +\left\|\frac{\partial \Phi}{\partial z}(x,z,u)-\frac{\partial \Phi}{\partial z}(\hat{x},\hat{z},u)\right\|\|\dot{z}\| \\
& +\left\|\frac{\partial \Phi}{\partial u}(x,z,u)-\frac{\partial \Phi}{\partial u}(\hat{x},\hat{z},u)\right\|\|\dot{u}\| \\
\leq\ & r_1+r_2\|w\|
\end{aligned}
\tag{31}
$$

where r_1 and r_2 are two positive constants.

Using an analysis similar to that used in the analysis of the estimation error dynamics, one can show that $\|w\|$ is bounded with the assumption that the derivative of faults is bounded.

Remark 5: In fact, in the course of the proof of Theorem 1, a systematic method has been derived for selecting each parameter of the ILO, as shown in (23) and (24).

Remark 6: From the proof of Theorem 1, and remark 4, it is known that both estimation error $\tilde{x}(t)$ and its derivative $\dot{\tilde{x}}(t)$ are bounded, accordingly, $v(t)+f_a(t)$ is also bounded, thereby, the ILO input $v(t)$ can detect, estimate or reconstruct fault $f_a(t)$. This will be seen in the simulation. Because of its capability of reconstructing actuator faults, ILO input can be utilized to isolate faults. In addition, the boundedness of $v(t)+f_a(t)$ also explains that the robustness of ILO results from ILO input $v(t)$, it is $v(t)$ that compensates the effect of $f_a(t)$ on estimate error dynamics, such that the ILO can still track the post-faulty system.

IV. An Illustrative Example

To illustrate the effectiveness of the ILO-based fault detection and estimation in a class of differential-algebraic systems, we consider following system given by

$$
\begin{aligned}
\dot{x}_1(t) &= -x_1-0.2zx_1+f_a(t) \\
\dot{x}_2(t) &= x_1-3x_2+2u(t) \\
z(t) &= 0.4x_1 \\
y(t) &= x_1+0.02z
\end{aligned}
\tag{32}
$$

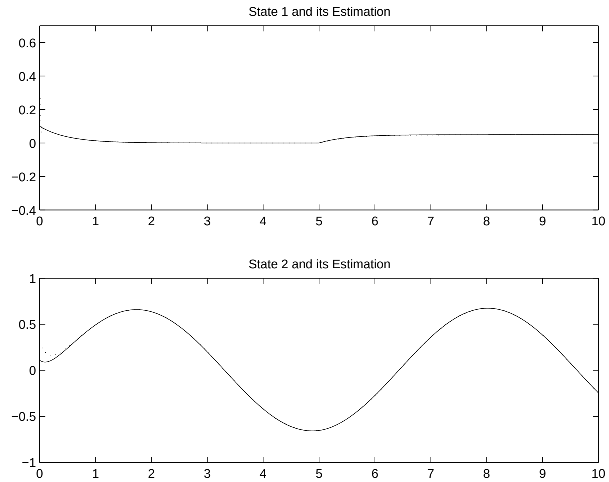

Fig. 1. State estimation by ILO.

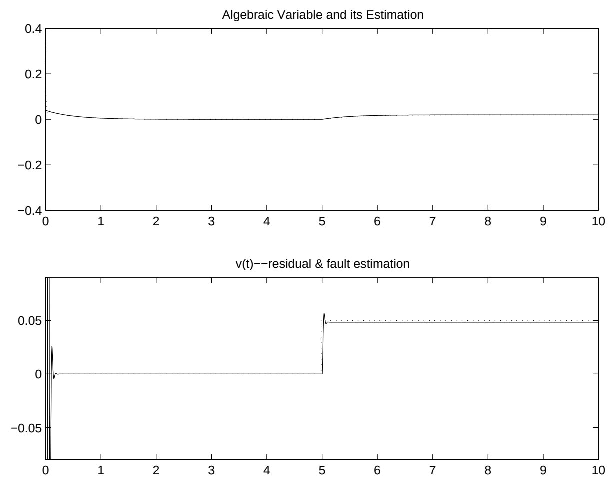

Fig. 2. Algebraic Variable and ILO input V(t).

Based on equation (6), an ILO is constructed as follows

$$
\begin{aligned}
\dot{\hat{x}}_1(t) &= -\hat{x}_1-0.2\hat{z}\hat{x}_1+0.1\tilde{y}(t)+v(t) \\
\dot{\hat{x}}_2(t) &= \hat{x}_1-3\hat{x}_2+2u(t) \\
v(t) &= 0.4v(t-\tau)+20\tilde{y}(t-\tau) \\
\hat{z}(t) &= 0.4\hat{x}_1 \\
\hat{y}(t) &= \hat{x}_1+0.02\hat{z}
\end{aligned}
\tag{33}
$$

The state and algebraic variables are initialized to consistent values. The sampling time interval is taken as 0.01.

Figures 1 and 2 show that both the estimated algebraic variable and system states can quickly converge to their real values. It can been seen that after an abrupt fault occurs at time t=5, all ILO states can still follow the system states after a transition because the ILO input compensates the effect of the fault. Therefore, ILO input $v(t)$ can be selected as a residual and can be used to estimate the fault (the dotted line) as shown in Figure 2.

V. Conclusions

An ILO-based fault diagnosis strategy has been proposed in a class of differential-algebraic systems with semi-explicit form. The ILO is updated at each sampling instance, therefore, its input that can reconstruct actuator faults can be used as a residual because it can keep alert on system any variation. Once fault reconstruction is achieved, fault isolation can be done by inspecting each component of ILO input. The illustrative example demonstrates that this kind of ILO-based fault diagnosis approach is successful.

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