High gain observer : Attenuation of the peak phenomena

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***Abstract*— In many problems of estimation, the high gain technic is used to compensate for nonlinear terms in order to guarantee the convergence of the estimator. However, the use of the high gain generally generates the so-called peak phenomena (at the beginning, the estimated trajectory deviates from the desired one). In this paper we extend the observer stated in [5] to a class of MIMO nonlinear systems. The proposed observer takes into account the peak phenomena. An application to a heat exchanger is illustrated in simulation.**

I. INTRODUCTION

Consider the nonlinear system:

$$\left\{ \begin{array}{lcl} \dot{x} & = & f(x,u) \\ y & = & h(x) \end{array} \right. \tag{I.1}$$

where the state $x(t) \in \mathbb{R}^n$, the input $u(t) \in \mathbb{R}^m$ and the output $y(t) \in \mathbb{R}^p$.
An observer is generally a dynamical system of the form:

$$\left\{ \begin{array}{lcl} \dot{\hat{x}} & = & f(\hat{x},u) - k(t)(h(\hat{x}) - y(t)) \\ \dot{g} & = & G(\hat{x},g,y,u) \\ k(t) & = & K(g(t)) \end{array} \right. \tag{I.2}$$

where $k(t)$ is the gain of the observer, $\hat{x}(t) \in \mathbb{R}^n$, $g(t)$ belongs to some $\mathbb{R}^N$, such that:
$g(t)$ is bounded, $\lim_{t \longrightarrow \infty} \|\hat{x}(t) - x(t)\| = 0$ and if $\hat{x}(0) = x(0)$ then $\hat{x}(t) = x(t)$, $\forall t \geq 0$.
In many observation problems the second equation of (I.2) is an algebraic one ($\dot{g} = 0$), see for instance [1], [2], [3], [4], [5], [6],[10], [11], [12].
To design an observer, many approaches can be considered: One of them consists in applying the E.K.F. (Extended Kalman Filter). The gain of this observer derives from a Ricatti differential equation.
The second approach is a structural one. It consists in characterizing a special class of nonlinear systems for which an observer can be designed: linear systems up to output injection [10], [11], [12], state affine systems up to output injection [7], [8], [9].
For many systems, the use of a high gain is necessary to guarantee the convergence of the observer [2], [3], [4], [5], [6]. However, the use of such gain generates the so-called peak phenomena: $\|\hat{x}(t) - x(t)\|$ becomes large at the starting of the observer. This phenomena is generally undesirable in the control problems.
This paper is organized as follows:
In section 2, we extend the high gain observer stated in [5] to a multi-output case. This observer takes into account the peak phenomena. In section 3, an application to a heat exchanger is illustrated in simulation.

II. HIGH GAIN OBSERVER : PEAK PHENOMENA

First, let us recall the high gain observer stated in [5]. Consider the class of single output systems of the form:

$$\left\{ \begin{array}{lcl} \dot{x} & = & Ax + \psi(x,u) \\ y & = & Cx \end{array} \right. \tag{II.1}$$

$$A = \begin{pmatrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ \vdots & & & 1 \\ 0 & \cdots & \cdots & 0 \end{pmatrix}, \quad C = \left[\begin{array}{cccc} 1 & 0 & \dots & 0 \end{array} \right],$$

$$\psi_i(x,u) = \psi_i(x_1, \dots, x_i, u)$$

Under the assumption that the nonlinear terms ψ are global Lipschitz functions w.r.t. x, the authors in [5], show that the following system:

$$\dot{\hat{x}} = A\hat{x} + \psi(\hat{x},u) - S_\theta^{-1} C^T (C\hat{x} - y) \tag{II.2}$$

forms an exponential observer, where $\theta > 0$ is a constant which must be chosen sufficiently large and S_θ is the solution of:

$$\theta S_\theta + A^T S_\theta + S_\theta A = C^T C$$

More precisely, it is shown that $e^T(t) S_\theta e(t) \leq e^{-(\theta-\lambda)t} e^T(0) S_\theta e(0)$, where $e(t) = \hat{x}(t) - x(t)$ is the error equation and λ is a positive constant which only depends on the Lipschitz constant of ψ. This last inequality implies that:

$$\|e(t)\|^2 \leq \theta^n \alpha e^{-(\theta-\lambda)t} \|e(0)\|^2 \tag{II.3}$$

for some positive constant α.
Hence $\|e(t)\|$ may become close to $\theta^n \|e(0)\|$, for t close to 0. This implies that at the beginning, the estimate state may deviate from the unknown state of the system (the peak phenomena).

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In this section, we will give an extension of the high gain observer (II.2) which takes into account this peak phenomena. The considered class of systems is given by:

$$\left\{\begin{array}{rcl}\dot{x}^1 & = & A_1(s,y)x^1+\varphi^1(x,u)\\ \vdots & & \\ \dot{x}^p & = & A_p(s,y)x^p+\varphi^p(x,u)\\ y & = & \left[C_1x^1,\ldots,C_px^p\right]^T\end{array}\right. \tag{II.4}$$

u is a known input which takes its values in a compact subset U of R^m, s(t) is a known signal and $x^i = \left[\begin{array}{ccc} x_1^i, & \ldots, & x_{n_i}^i\end{array}\right]^T \in \mathbb{R}^{n_i}$; $x = \left[\begin{array}{c} x^{1^T},\ldots,x^{p^T}\end{array}\right]^T \in \mathbb{R}^n$.

$$A_i(s,y)=\left(\begin{array}{cccc} 0 & a_{i1}(s,y) & & 0\\ \vdots & & \ddots & \\ \vdots & & & a_{in_i-1}(s,y)\\ 0 & \cdots & \cdots & 0\end{array}\right),$$

$C_i = \left[\begin{array}{cccc} 1 & 0 & \ldots & 0\end{array}\right]$ and $\varphi^i = \left[\begin{array}{ccc}\varphi_1^i & \ldots & \varphi_{n_i}^i\end{array}\right]^T$,
The a_{ij}'s and the φ^i's are of class C^1 w.r.t. their arguments and are satisfying the following triangular structure:

$$\left\{\begin{array}{rcl}\varphi_1^i & = & \varphi_1^i(y,u),\\ \varphi_j^i & = & \varphi_j^i(\pi_j^i(x^i),y,u),\ 2\leq j\leq n_i-1\\ \varphi_{n_i}^i & = & \varphi_{n_i}^i(x,u)\end{array}\right. \tag{II.5}$$

where π_j^i is the projection from $\mathbb{R}^{n_i}$ into $\mathbb{R}^{j-1}$ defined by: $\pi_j^i(x^i) = (x_2^i,\ldots,x_j^i)$.
In the sequel, we will assume the following hypotheses:
H1) The φ_j^i's are global Lipschitz w.r.t. $\pi_j^i(x^i)$:
$\exists c\geq 0; \forall x; \forall\bar{x}, |\varphi_j^i(\pi_j^i(x^i),y,u)-\varphi_j^i(\pi_j^i(\bar{x}^i),y,u)|\leq c\|x-\bar{x}\|$
H2) $y(t)$, $s(t)$ and their derivatives $\dot{y}(t)$, $\dot{s}(t)$ are bounded.
H3) The a_{ij}'s never vanish on the compact subset $\overline{Y\times S}$ where Y and S are the bounded sets containing the signals $y(t)$ and $s(t)$.
Our candidate observer takes the following form:

$$\left\{\begin{array}{rcl}\dot{\hat{x}}^1 & = & A_1(s,y)\hat{x}^1+\hat{\varphi}^1(\hat{x},u)+\Lambda_1^{-1}K_1(C_1\hat{x}^1-y_1)\\ \vdots & & \\ \dot{\hat{x}}^p & = & A_p(s,y)\hat{x}^p+\hat{\varphi}^p(\hat{x},u)+\Lambda_p^{-1}K_p(C_p\hat{x}^p-y_p)\\ \dot{\theta} & = & -k(\theta-\theta_0)\end{array}\right. \tag{II.6}$$

where k is a positive constant. The K_i's are such that the real parts of eigenvalues of $(\tilde{A}_i+K_iC_i)$ are negative, where $\tilde{A}_i = \left(\begin{array}{cccc} 0 & 1 & & 0\\ \vdots & & \ddots & \\ \vdots & & & 1\\ 0 & \cdots & \cdots & 0\end{array}\right)$ is the $n_i\times n_i$ matrix and Λ_i is the $n_i\times n_i$ diagonal matrix: $diag\left(\theta^{-\delta_i},\ \theta^{-2\delta_i}a_{i1}(s,y),\ldots,\ \theta^{-n_i\delta_i}\Pi_{j=1}^{n_i-1}a_{ij}(s,y)\right)$.
Noticing that from **H3**), $\Lambda_i^{-1}(t)$ is well defined for every $t\geq 0$. Finally, $\delta_1>0,\cdots,\delta_p>0$ are constants satisfying the following linear program:

$$-n_i\delta_i+n_j\delta_j<\tfrac{\delta_i}{2};\quad 1\leq i,\ j\leq p \tag{II.7}$$

Noticing that one solution of (II.7) consists in taking $\delta_i=\frac{1}{n_i}$. Now, we can state our main result:

Theorem 1: Let $\delta_1>0,\cdots,\delta_p\geq 0$ be any parameters satisfying (II.7). Then there exists $\underline{\theta}_0>0$ such that: $\forall\theta_0\geq\underline{\theta}_0; \exists t(\theta_0)\geq 0; \forall t\geq t(\theta_0); \forall x(0); \forall\hat{x}(0); \forall\theta(0)>0$, we have:

$$\|\hat{x}(t)-x(t)\|\leq\lambda e^{-\mu t}\|\hat{x}(0)-x(0)\|$$

where $\lambda>0,\ \mu>0$ are constants.

Remark 1: if $\theta(0)=\theta_0$, then $\forall t\geq 0,\ \theta(t)=\theta_0$, and hence observer (II.2) becomes a particular case of (II.6).

Remark 2: The choice of small value $\theta(0)$ permits to obtain a small gain at the starting of the observer. This allows to reduce the peak phenomena (see simulations below).

Proof of theorem 1
Set $e^i(t)=\hat{x}^i(t)-x^i(t)$ and $e(t)=[e^{1^T}(t),\cdots,e^{p^T}(t)]^T$, we get:

$$\dot{e}^i=(A_i(s,y)+\Lambda_i^{-1}K_iC_i)e_i+\delta\varphi^i\qquad 1\leq i\leq p$$

where $\delta\varphi^i=\hat{\varphi}^i-\varphi^i$.
Now consider the following change of coordinates:

$$\bar{e}^i=\Lambda_ie^i \tag{II.8}$$

A simple calculation gives:

$$\dot{\bar{e}}^i=\theta^{\delta_i}[\tilde{A}_i+K_iC_i]\bar{e}^i+\Lambda_i\delta\varphi^i+\tilde{\Lambda}_i(t)\bar{e}^i\qquad 1\leq i\leq p \tag{II.9}$$

where

$$\tilde{\Lambda}_i(t)=diag\left(-\frac{\delta_i\dot{\theta}}{\theta},\ \frac{-2\delta_i\dot{\theta}}{\theta}+\frac{\dot{a}_{i1}}{a_{i1}},\ldots,\ \frac{-n_i\delta_i\dot{\theta}}{\theta}+\frac{\dot{\overbrace{\Pi_{l=1}^{n_i-1}a_{il}}}}{\Pi_{l=1}^{n_i-1}a_{il}}\right) \tag{II.10}$$

Now, set $V=V_1+\cdots+V_p$, where $V_i=\bar{e}^{i^T}P_i\bar{e}^i$ and P_i is a S.P.D. matrix such that:

$$(\tilde{A}_i+K_iC_i)^TP_i+P_i(\tilde{A}_i+K_iC_i)=-I_i \tag{II.11}$$

where I_i is the $n_i\times n_i$ identity matrix.
To prove theorem 1, it suffices to show that $V(t)$ exponentially asymptotically decreases to zero. Indeed, from hypotheses **H1**),**H2**), $\|\Lambda_i(t)\|$, $\|\Lambda_i^{-1}(t)\|$ become bounded. Hence, there exist two constants $\lambda_1>0$, $\lambda_2>0$ which may depend on θ_0 such that $\lambda_1\|e\|^2\leq V=\sum_{i=1}^{p}V_i\leq\lambda_2\|e\|^2$. Thus $V(t)\to 0$ is equivalent to $\|e(t)\|\xrightarrow{exp}0$.
Now, let us prove that $V(t)$ exponentially converges to zero. Combining (II.9), (II.11) we deduce:

$$\begin{array}{rcl}\dot{V}_i & = & -\theta^{\delta_i}\|\bar{e}^i\|^2+2\bar{e}^{i^T}P_i\Lambda_i\delta\varphi^i+2\bar{e}^{i^T}\tilde{\Lambda}(t)P_i\bar{e}^i\\ & \leq & -\eta_i^{-1}\theta^{\delta_i}V_i+2\|P_i\|\|\bar{e}^i\|\|\Lambda_i\delta\varphi^i\|\\ & & +2\|\tilde{\Lambda}_i(t)\|\|P_i\|\|\bar{e}^i\|^2\end{array} \tag{II.12}$$

Where η_i is the largest eigenvalue of P_i.
In what follows, we will give some upper bounds of $\|\Lambda_i\delta\varphi^i\|$ and $\|\tilde{\Lambda}_i(t)\|$.
Using the triangular structure (II.5) together with the expression of $\hat{\varphi}^i$, we get:

- for $j=1$

$$(\Lambda_i \delta\varphi^i)_1 = \theta^{-\delta_i}(\hat{\varphi}^i_1 - \varphi^i_1) = 0 \quad \text{(II.13)}$$

- for $2 \le j \le n_i - 1$;

$$\begin{array}{rcl} |(\Lambda_i \delta\varphi^i)_j| & = & \theta^{-j\delta_i}|\Pi_{l=1}^{j-1} a_{il}||\hat{\varphi}^i_j - \varphi^i_j| \\ & \le & c\theta^{-j\delta_i} a_i^{j-1}\sqrt{(e^i_2)^2 + \cdots + (e^i_j)^2} \end{array} \quad \text{(II.14)}$$

- for $j = n_i$;

$$\begin{array}{rcl} |(\Lambda_i \delta\varphi^i)_{n_i}| & = & \theta^{-n_i\delta_i}|\Pi_{l=1}^{n_i-1} a_{il}||\hat{\varphi}^i_{n_i} - \varphi^i_{n_i}| \\ & \le & c\theta^{-n_i\delta_i} a_i^{n_i-1}\|e\| \end{array} \quad \text{(II.15)}$$

  where c is the Lipschitz constant of the φ^i's given in **H1)** and a_i is the upper bound of the $|a_{ij}(s,y)|$'s, when (s,y) describes $\overline{S \times Y}$.

Now, chose $\bar{\theta}_0 > 1$ and $\theta_0 \ge \bar{\theta}_0$, and taking into account the fact that $\dot{\theta} = -k(\theta - \theta_0)$ with $\theta(0) > 0$, it follows that there exists $t_1(\theta_0) > 0$ such that $\forall t \ge t_1(\theta_0)$, we have $\theta(t) \ge 1$.
Hence, from (II.13), (II.14) and (II.15), the following inequalities hold for every $t \ge t_1(\theta_0)$:

- for $1 \le j \le n_i - 1$

$$|(\Lambda_i \delta\varphi^i)_j| \le \; c\|\vec{e^i}\| \quad \text{(II.16)}$$

- for $j = n_i$

$$|(\Lambda_i \delta\varphi^i)_{n_i}| \;\; \le \;\; c\sum_{l=1}^{p}\theta^{n_l\delta_l - n_i\delta_i}\|\vec{e^l}\| \quad \text{(II.17)}$$

Now set,

$$\alpha_i = \max_{1\le l\le p}\{0, n_l\delta_l - n_i\delta_i\} \quad \text{(II.18)}$$

and combining (II.16), (II.17) and (II.18) , then we have:

$$\forall t \ge t_1(\theta_0), \quad \|\Lambda_i \delta\varphi^i\| \le \beta_i\theta^{\alpha_i}\|\vec{e}\| \quad \text{(II.19)}$$

where β_i is a positive constant which doesn't depend on θ_0.
The following holds for $\tilde{\Lambda}_i$:
There exists $t_2(\theta_0) > 0$; there exists a constant $\tilde{\beta}_i > 0$ which doesn't depend on θ_0 such that for every $t \ge t_2(\theta_0)$, we have:

$$\|\tilde{\Lambda}_i(t)\| \le \tilde{\beta}_i \quad \text{(II.20)}$$

Indeed, since $\dot{\theta}(t) \to 0$ as $t \to \infty$ ($\dot{\theta} = -k(\theta - \theta_0)$) and $\tilde{\Lambda}_i$ only depends continuously on the bounded signals $y(t), \dot{y}(t), s(t), \dot{s}(t)$, we can deduce (II.20).
Now combining (II.12), (II.19) and (II.20) and taking $t \ge max(t_1(\theta_0), t_2(\theta_0))$, and set $\tilde{\eta}_i$ to be the smallest eigenvalue of P_i, we obtain:

$$\begin{array}{rcl} \dot{V}_i & \le & -\eta_i^{-1}\theta^{\delta_i}V_i + 2\beta_i\theta^{\alpha_i}\|P_i\|\|\vec{e^i}\|\|\vec{e}\| + 2\tilde{\beta}_i\tilde{\Lambda}_i\|P_i\|\|\vec{e^i}\|^2 \\ & \le & -(\eta_i^{-1}\theta^{\delta_i} - 2\tilde{\beta}_i\|P_i\|\tilde{\eta}_i^{-1})V_i + 2\beta_i\theta^{\alpha_i}\|P_i\|\|\vec{e^i}\|\|\vec{e}\| \end{array} \quad \text{(II.21)}$$

Using the inequality: $2\theta^{\alpha_i}\|\|\vec{e^i}\|\|\vec{e}\| \le \theta^{2\alpha_i}\|\|\vec{e^i}\|^2 + \|\vec{e}\|^2$ and set $\tilde{\eta} = min\{\tilde{\eta}_1, \cdots, \tilde{\eta}_p\}$, we get:

$$\begin{array}{rcl} \dot{V}_i & \le & -(\eta_i^{-1}\theta^{\delta_i} - 2\tilde{\beta}_i\|P_i\|\tilde{\eta}_i^{-1})V_i \\ & & +\beta_i\|P_i\|(\theta^{2\alpha_i}\|\|\vec{e^i}\|^2 + \|\vec{e}\|^2) \\ & \le & -(\eta_i^{-1}\theta^{\delta_i} - 2\tilde{\beta}_i\|P_i\|\tilde{\eta}_i^{-1} - \beta_i\theta^{2\alpha_i}\|P_i\|\tilde{\eta}_i^{-1})V_i \\ & & +\tilde{\eta}^{-1}\beta_i\|P_i\|V \end{array} \quad \text{(II.22)}$$

Hence:

$$\begin{array}{rcl} \dot{V} & \le & \sum_{i=1}^{p}\{-(\eta_i^{-1}\theta^{\delta_i} - 2\tilde{\beta}_i\|P_i\|\tilde{\eta}_i^{-1} - \beta_i\theta^{2\alpha_i}\|P_i\|\tilde{\eta}_i^{-1})V_i \\ & & +\tilde{\eta}^{-1}\beta_i\|P_i\|V\} \\ & \le & -(\alpha(\theta) - \nu)V \end{array} \quad \text{(II.23)}$$

where $\alpha(\theta) = min\{\eta_i^{-1}\theta^{\delta_i} - \beta_i\theta^{2\alpha_i}\|P_i\|\tilde{\eta}_i^{-1}; \quad 1 \le i \le p\}$ and
$\nu = max\{2\tilde{\beta}_i\|P_i\|\tilde{\eta}_i^{-1}; \quad 1 \le i \le p\} + \tilde{\eta}^{-1}\sum_{i=1}^{p}\beta_i\|P_i\|$ is a positive constant which doesn't depend on θ. From inequality (II.7) and expression (II.18), we deduce that $\alpha(\theta_0) \to \infty$ as $\theta_0 \to \infty$. Now choose $\underline{\theta}_0$, such that $\mu(\underline{\theta}_0) - \nu > 0$ and consider $\theta_0 > \underline{\theta}_0$. Since $\theta(t) \to \theta_0$ as $t \to \infty$, it follows that there exists $t_3(\theta_0)$ such that $\forall t \ge t_3(\theta_0)$, we have $\alpha(\theta(t)) - \nu \ge \rho$ for some constant $\rho > 0$. Finally, set $t(\theta_0) = max(t_1(\theta_0), t_2(\theta_0), t_3(\theta_0))$, it follows that:
$\forall t \ge t(\theta_0), \quad \dot{V}(t) \le -(\alpha(\theta(t)) - \nu)V \le -\rho V(t)$.
This ends the proof of the theorem.

III. Application

The aim of this section consists in applying the above observer synthesis to a heat exchange process described by figure 1.

A. Modelling of the heat exchange system

The process is mainly built around a counter-flow tubular heat exchanger. The warm water flows in a closed circuit, the temperature in the hot water tank is fixed by an independently controlled electric heater. The cold water flows in an open circuit. The flows of both warm and cold water are controlled by two electro-pneumatic valves. T_1, T_3 are respectively the inlet temperatures of the warm and the cold water and T_2, T_4 are the corresponding outlet temperatures. The dynamics of actuators (electro-pneumatic valves) cannot be neglected. Indeed their time constants are equivalent to the residence time constants of the heat exchanger (0.5s - 1s). The corresponding state variables are the displacements and the velocities of the electro-pneumatic valves. The temperatures are assumed to be homogeneous in the tubular heat exchanger. Under the hypothesis that the circuit of the thermal exchange is a

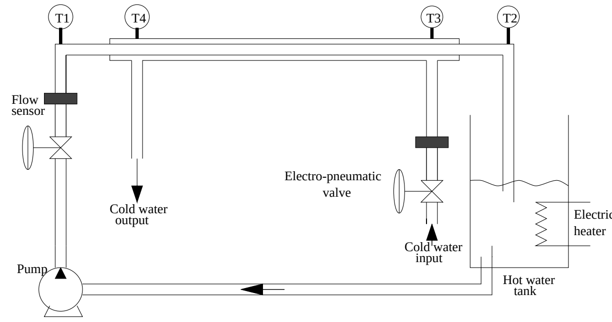

Fig. 1. heat exchanger plant

closed system which contains a constant mass of water, the inertia of the fluid is negligible and the flow is turbulent, a model of the process takes the form:

$$\left\{ \begin{array}{lcl} \dot{x} & = & f(x) + u_1 g_1(x) + u_2 g_2(x) \\ y & = & (y_1, y_2) = (x_1, x_4) \end{array} \right. \qquad \text{(III.1)}$$

where: $$f(x) = \begin{pmatrix} (e_1 - a_1 x_1)x_2 - b_1(x_1 - x_4) \\ x_3 \\ -\omega_0^2 x_2 - 2\xi\omega_0 x_3 \\ (e_2 - a_2 x_4)x_5 + b_2(x_1 - x_4) \\ x_6 \\ -\omega_0^2 x_5 - 2\xi\omega_0 x_6 \end{pmatrix}$$

$$g_1(x) = \begin{pmatrix} 0 \\ 0 \\ k_0\omega_0^2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ and } g_2(x) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ k_0\omega_0^2 \end{pmatrix}.$$

a_1, a_2, b_1, b_2, e_1, e_2, are physical constants which derive from the energy balance transfer. k_0 is the static gain of the valve, ω_0 is the undamped natural frequency and finally, ξ is the damping factor.
$x = [x_1, \cdots, x_6]^T$, $u = (u_1, u_2)$ is the control vector; $(x_1, x_4) = (T_2, T_4)$ is the output measurements; x_2, x_5 are respectively the displacement of the warm water valve and the cold water valve and finally, x_3, x_6 are respectively the velocity of the warm water valve and the cold water valve.
Noticing that the above system takes the form (II.4), with $p = 2$, $x^1 = [x_1, x_2, x_3]^T$, $x^2 = [x_4, x_5, x_6]^T$, $a_{11} = (e_1 - a_1 y_1)$, $a_{12} = 1$, $a_{21} = (e_2 - a_2 y_2)$, $a_{22} = 1$. Moreover, taking into account the physical data, the terms: $e_1 - a_1 y_1(t)$, $e_2 - a_2 y_2(t)$ never vanish on the corresponding physical domain. Clearly, the hypotheses **H1), H2), H3)** are satisfied. Hence the observer (II.6) can be applied.
To show the relationship between the gain of the observer and the peak phenomena, we compare two simulations: the state estimation obtained from an observer with constant gain $(\theta(t) = \theta_0)$ and that derived from a similar observer with variable gain $\theta(t)$ (with a small initial condition $\theta(0)$), see figure 2 below.
All simulations use the following physical data:
$a_1 = 222.55$, $a_2 = 137.95$, $b_1 = 0.163$, $b_2 = 0.108$,
$e_1 = 76.353\ 10^3$, $e_2 = 41.103\ 10^3$, $k_0 = 0.93$, $\omega_0 = 6.28$,
$\xi = 0.7$, $u_1 = 5mA$, $u_2 = 5mA$ and initial conditions:
$x(0) = [338,\ 0,\ 0,\ 301,\ 0,\ 0]^T$,
$\hat{x}(0) = [343,\ 4.\ 10^{-3},\ 4.\ 10^{-3},\ 296,\ 4.\ 10^{-3},\ 4.\ 10^{-3}]^T$,
$\theta(0) = 1,\ \theta_0 = 60,\ \delta_1 = \delta_2 = 1,\ k = 1.5$,
$K_1 = K_2 = [1.O5,\ 0.37,\ 0.04]^T$
The output measurements are disturbed by adding a Gaussian noise with zero mean and amplitude equivalent to 5%.

IV. CONCLUSION

In this paper, we have extended the high gain observer synthesis stated in [5] to a multi-output class of uniform observable systems. The proposed gain $\theta(t)$ derives from a first order dynamic linear equation. This method permits to initialize the gain of the observer at a small value which allows to reduce the peak phenomena. Noticing that the use of the first order dynamic equation is not necessary and one can choose an other adequate stable reference model $\dot{\theta}(t) = \Theta(\theta(t), u(t), y(t))$ to calculate the gain $\theta(t)$.

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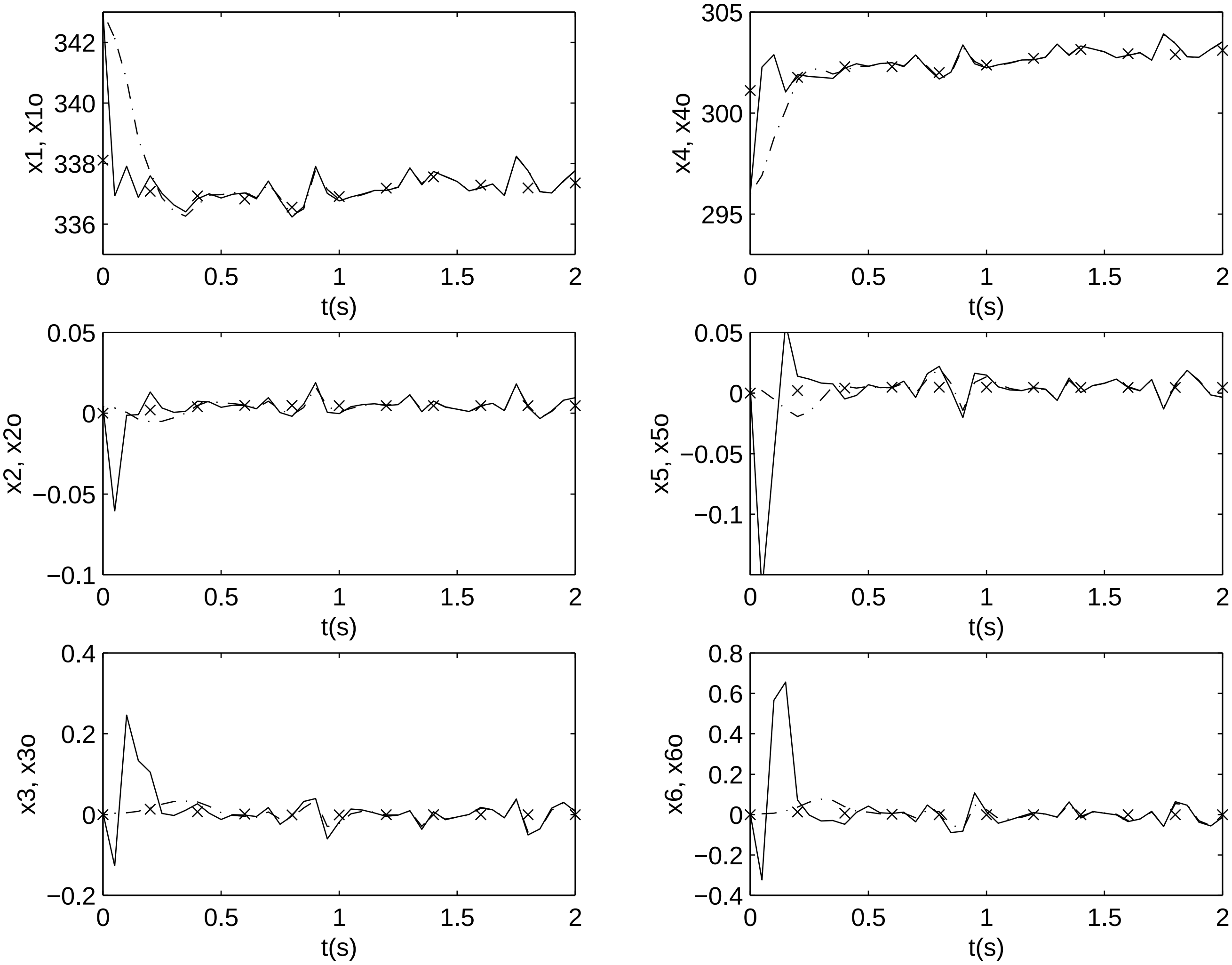

Fig. 2. — Observer with constant gain, – - Observer with variable gain, $\times\ \times$ System