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A Semiempirical Identification Method by Using a Multiestimation Technique via Reduced-Order Nominal Models

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Abstract— **A multiestimation scheme is presented to identify a partially unknown plant. Several reduced-order linear nominal models of the plant are considered to compose the multiestimation scheme. Each reduced-order nominal model is built as a parallel connection of first-order filters and contains some, but not all, natural modes, which are supposed known, of the true plant to be identified. The assumption that the elementary filters are of first-order is identical to assume that all poles are real and distinct, which is feasible in many practical situations. The assumption that the modes are known may work in an acceptable way when nominal values and a small range of uncertainty are known. A supervisor with a switching law selects the most appropriate estimation model of the plant at certain time instants according to a index related with the identification error of each estimator. In this way, a system identification scheme which incorporates model order reduction issues can be designed.**

I. Introduction

DYNAMICS of almost all real systems is non-linear and then their behaviour change abruptly according to the operation conditions, [1]. These operation conditions depend on the magnitude and type of the input signal applied to the system, particularly, on the frequency rank at which the input signal belongs to. As a consequence, in many industrial applications it is not reasonable to assume that the same plant model remains adequate as time progresses. Models in which the environment and related parameters undergo abrupt change at certain time instants are found to be relevant in a much wider class of practical situations, [2]. In this sense, it is crucial to elucidate either the number of dominant modes to be considered for identification purposes or the method to be used in the identification process as for instance in [3] where the accuracies of three different identification methods are compared for the same example. Two of them are the so-called empirical transfer function estimate (EFTE) and the experimental transfer function estimate which is a raw EFTE requiring smoothing. Another estimated is based on the implementation of a long Fourier transform and then using the spectral Daniell window. The obtained identification performance was found very different from one method to another one depending on the input frequency. This proves the importance of the selection of the identification method and, as a result, it is also foreseen the high dependence of the identification performance on the model and its order and relative degree. One of the fields where it is more relevant the order of the model used with respect to the dominant frequencies of the signal input applied is the design of controlled systems such as robots and space structures with structural flexibility, [4]. In such a type of problems the selection and placement of sensors and actuators is an important step. That selection and placement must be optimized according, for instance, to the key points in the flexible structure shape to measure the relevant modes. The modes in the response are very important, with respect to the associated gain, to non zero initial conditions but they all are not equally important at any input frequency and under zero initial conditions.

In the present paper, the use of simplified models based on the input dominant frequencies is proposed. A method to integrate the on-line model order choice with the reference input spectrum is presented. The mechanism is to implement a switching rule between several estimators of different orders. The overall process is stated as an automatic task that does not require any on-line designer operation. The time intervals between consecutive switches are subject to a minimum residence time that guarantees

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acceptable transient behaviour. The main idea behind the proposed scheme is that the model order reduction techniques may be addressed as linked with multiestimation techniques while taking into account the transient response generated from each particular reference input used.

Therefore, the objective of this paper is to identify a reduced-order estimated model for a real plant under different operation points. The procedure to achieve this objective consists of a multiestimation scheme. Several estimation algorithms run in parallel and each of them estimates the parameters of each proposed nominal plant model. Each of these nominal models represents one of the possible operation points of the real plant. They can be of different orders and all of them are under-parameterized. In this way, a reduced-order estimated model can be obtained. The different nominal models are built as a parallel connection of first-order filters with poles belonging to the set of modes of the plant, which are supposed known. The numerators of those filters are unknown and then they have to be estimated by means of some estimation algorithms, each of them for each of the nominal models considered. In summary, all of the nominal models considered contain some of the modes, but potentially not all, of the real plant. A supervisor with a suitable switching law selects the estimation model that optimizes an appropriate cost function which depends on the error signal between the real output signal and the issued output from each estimated model. This process runs automatically as an identification scheme which issues an estimated and reduced-order model of a real plant under different signal inputs. This can be relevant for designing reduced-order controllers for certain mechanical systems as in [4]. There exists a considerable number of papers where the multiestimation technique have already used satisfactorily. For instance, in the area of adaptive control of, partially or fully, unknown linear plants, as in [5-8], the use of the multiestimation technique has been used to improve the output performance with respect to that obtained with a single estimation algorithm.

II. Problem Statement

The behaviour of a non-linear plant at any operation point can be described by means of the following time-varying difference equation,

$$A^{(i)}(q^{-1})y_k = B^{(i)}(q^{-1})u_k \tag{1}$$

where u_k and y_k are the input and output sequences, respectively, q^{-1} is the one-step delay operator and i denotes a generic operating point of the plant, $i \in \{1,\ 2,\ \ldots,\ n_p\}$ with n_p being the number of possible operation conditions of the plant. The orders of the time-invariant polynomials $A^{(i)}$ and $B^{(i)}$ depend on the operating point of the plant. The possible transition from one operation point to another one can be produced by a change in the input signal applied to the plant.

It is assumed that the plant modes are simple, stable and known. Then, the roots of the polynomials $A^{(i)}$, and then its coefficients, associated with each operation point are known. However, the parameters of the polynomials $B^{(i)}$ will be unknown and then an estimation algorithm will be need to identify them. Moreover, each operation point can be nominally modeled as the parallel connection of strictly proper first-order filters, with poles belonging to the set of the modes of the plant, of unknown numerator factors. Each transfer function associated with each operation point does not include all the modes of the plant in order to obtain a reduced-order nominal model of the plant. In summary, the behaviour of the plant at the *i*-th operation point would be described by means of the transfer function,

$$H^{(i)}(z) = \frac{B^{(i)}(z)}{A^{(i)}(z)} = \frac{b_{n(i)-1}^{(i)} z^{n-1} + b_{n(i)-2}^{(i)} z^{n-2} + \ldots + b_0^{(i)}}{(z-a_1^{(i)})(z-a_2^{(i)})\ldots(z-a_{n(i)}^{(i)})} = \sum_{j=1}^{n(i)} \frac{\alpha_j^{(i)}}{z-a_j^{(i)}} \tag{2}$$

where $a_j^{(i)}$ and $\alpha_j^{(i)}$ for $j \in \{1,\ 2,\ \ldots,\ n(i)\}$ and $i \in \{1,\ 2,\ \ldots,\ n_p\}$ are supposed known and unknown, respectively, with $n(i)$ denoting the order of the nominal model at each plant operating condition and subject to the condition that $1 \le n(i) < n_m$ where n_m is the number of plant modes. The following figure shows a scheme representation of the plant at the *i*-th operation point.

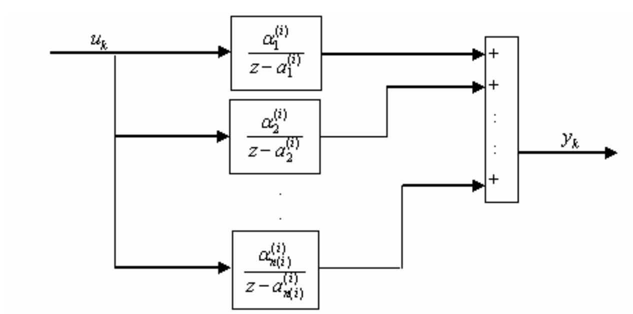

Fig. 1. Plant nominal model at the *i*-th operation point.

The values of $\alpha_j^{(i)}$ are considered constant or smoothly time-varying functions into a small rank in order to be available the use of estimation methods to identify time invariant plants.

The main idea is to eliminate some of the modes of the plant to obtain a reduced-order model of the plant in the current operation point. The motivation is the fact that a full description of all modes of the plant can not be necessary for some input frequencies. i.e, the gain of some of the plant modes can be very small with respect to the gain of other ones for certain input frequencies and then such modes could be non considered in (2) under inputs belonging to this frequency rank. Then, n_p nominal models of the plant will be considered, one for each possible plant operating condition. Each of them does not contain at least one of the plant modes. Then, a multiestimation scheme identify on-line the numerators of the terms of the n_p possible eqns. (2) and a supervisor chooses the model which better approach to the real plant. The obtained model will be which suppresses the mode or modes with minor gain for the applied input. The result is the automatically issue of a reduced-order estimated model of the plant for the operation condition associated with the applied input.

III. MULTIESTIMATION SCHEME

The motivation for the use of a multiestimation technique is to have several models which represent the plant behaviour. Each estimator works separately estimating the parameters of its associated model. The main idea is to know which of the estimated models is the best approach to the behaviour of the plant at each instant. Obviously, the model which better represent the plant behaviour will not be always the same since it will depend on the operation condition of the plant. i.e., if the plant is submitted to changes in its operation conditions, which can be produced by abrupt changes in the applied input, then the model which better approaches the plant behaviour will be different at different time intervals. This means that a different model can represent the plant behaviour at different time intervals. Then, it is necessary to make a comparison among all estimated models issued by the multiestimation scheme at certain time instants to know which is the most approximated to the plant in the current instant. For that, a index which measures the approximation of each estimated model to the plant is defined. Each of these indexes compares the true plant output y_k with the estimated one by the corresponding estimated model $H^{(i)}(z)$ in the following way:

$$J_k^{(i)} = \sum_{l=0}^{k} \lambda^{k-l} (y_l - \hat{y}_l^{(i)})^2 \tag{3}$$

where $y_k^{(i)}$ is the output of the estimated model $H^{(i)}(z)$ and $\lambda < 1$ is a forgetting factor introduced for giving more importance to the last samples. Obviously, the estimated model with the smallest value for the index $J_k^{(i)}$ is the best approach to the true plant behaviour.

The algorithms for the adaptation of the unknown parameters $\alpha_j^{(i)}$ of the different estimation models can be of diverse types. All of them are based in the following scheme,

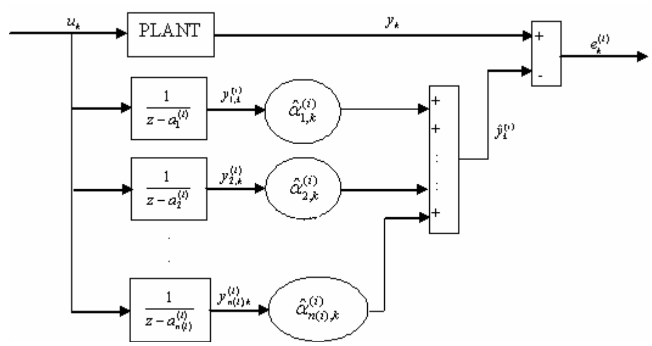

Fig. 2. Estimation scheme for a single algorithm.

Note that one scheme of this kind is proposed for each of the possible $i \in \{1,\ 2,\ \dots,\ n_p\}$ estimated models of the plant. All of them compose the parallel multiestimation scheme.

A. Estimation Algorithms

Two different types of estimation algorithms are proposed.

1) Algorithm 1

A least-squares type estimation algorithm is considered for adapting the parameters $\hat{\alpha}_j^{(i)}$ for all $j \in \{1,\ 2,\ \dots,\ n(i)\}$. This algorithm is defined by,

$$\hat{\theta}_{k+1}^{(i)} = \hat{\theta}_k^{(i)} + \frac{P_k^{(i)} \varphi_k^{(i)} e_k^{(i)}}{1 + \varphi_k^{(i)T} P_k^{(i)} \varphi_k^{(i)}} \quad ; \quad P_{k+1}^{(i)} = P_k^{(i)} - \frac{P_k^{(i)} \varphi_k^{(i)} \varphi_k^{(i)T} P_k^{(i)}}{1 + \varphi_k^{(i)T} P_k^{(i)} \varphi_k^{(i)}} \tag{4}$$

where $P_0^{(i)} = P_0^{(i)T} > 0$, $\varphi_k^{(i)} = \left[y_{1,k}^{(i)},\ y_{2,k}^{(i)},\ \dots,\ y_{n(i),k}^{(i)} \right]^T$, $\hat{\theta}_k^{(i)} = \left[\hat{\alpha}_{1,k}^{(i)},\ \hat{\alpha}_{2,k}^{(i)},\ \dots,\ \hat{\alpha}_{n(i),k}^{(i)} \right]^T$ and $e_k^{(i)} = y_k - \hat{y}_k^{(i)} = y_k - \hat{\theta}_k^{(i)T} \varphi_k^{(i)}$ is the identification error corresponding to the i-th estimation model at the k sample time instant.

2) Algorithm 2

This algorithm is of least-squares type too, but the parameters estimation is uncoupled with respect to the previous algorithm. This is obtained if the covariance matrix $P_k^{(i)}$ is diagonal. The estimates are given by,

$$\hat{\alpha}_{j,k+1}^{(i)} = \hat{\alpha}_{j,k}^{(i)} + \frac{p_{j,k}^{(i)} \hat{y}_{j,k}^{(i)} e_k^{(i)}}{1 + \sum_{l=1}^{n(i)} p_{l,k}^{(i)} \hat{y}_{l,k}^{(i)2}} \quad ; \quad p_{j,k+1}^{(i)} = p_{j,k}^{(i)} - \frac{p_{j,k}^{(i)2}\, \hat{y}_{j,k}^{(i)2}}{1 + \sum_{l=1}^{n(i)} p_{l,k}^{(i)} \hat{y}_{l,k}^{(i)2}} \tag{5}$$

for all $j \in \{1,\ 2,\ \dots,\ n(i)\}$ with $p_{j,0}^{(i)} > 0$ and where the scalars $p_{j,k}^{(i)}$ are the elements of the diagonal of $P_k^{(i)}$ and $e_k^{(i)} = y_k - \hat{y}_k^{(i)} = y_k - \sum_{l=1}^{n(i)} \hat{\alpha}_{l,k}^{(i)}\ \hat{y}_{l,k}^{(i)}$ is the identification error corresponding to the i-th estimation model at the k sample time instant.

IV. SIMULATIONS

A discrete stable non-linear plant with three known modes $a_1 = 0.5$, $a_2 = 0.00001$ and $a_3 = -0.9$ is considered. Therefore, the behaviour of the plant at a certain operating point can be described by,

$$H(z) = \frac{\alpha_1}{z-0.5} + \frac{\alpha_2}{z-0.00001} + \frac{\alpha_3}{z+0.9} \tag{6}$$

where α_1, α_2 and α_3 are unknown and they have to be estimated. $T = 0.1\,s$ is the sampling time used. At least one of the terms of (6) is suppressed in order to obtain a reduced-order estimated model of the plant for each possible operating point. In this way, six possible reduced-order nominal models can be obtained. However, a subset composed by only the three estimation models of second order is considered in the multiestimation scheme. Each one misses information about one of the simple plant modes. Namely, the three estimation models of two poles are:

$$H^{(4)}(z) = \frac{\hat{\alpha}_{1k}^{(4)}}{z-0.5} + \frac{\hat{\alpha}_{2k}^{(4)}}{z-0.00001} \; ; \; H^{(5)}(z) = \frac{\hat{\alpha}_{1k}^{(5)}}{z-0.5} + \frac{\hat{\alpha}_{3k}^{(5)}}{z+0.9}$$

$$H^{(6)}(z) = \frac{\hat{\alpha}_{2k}^{(6)}}{z-0.00001} + \frac{\hat{\alpha}_{3k}^{(6)}}{z+0.9} \tag{7}$$

The gains of each simple fraction of each reduced-order model of (7) are estimated by using any of the estimation algorithms 1 or 2 of *Section III*. Then, a supervisor selects the estimated model with the best approach to the plant behaviour from the multiestimation scheme at each certain time interval. Under other point of view, the simple fraction with the smallest influence in the plant behaviour is detected and suppressed by means of this strategy.

All the estimation algorithms are initialized with $\hat{\alpha}_{j0}^{(i)} = 0.8$, with $i \in \{4, 5, 6\}$ and $j \in \{1, 2, 3\}$, and $P_0^{(i)} = 10^{16} I_2$, where I_2 denotes the second order identity matrix. The estimates are updated at intervals of five sampling instants and the selection of the best estimated model by the supervisor is performed after each ten samples. The forgetting factor which appears in the quality index $J_k^{(i)}$, for each estimated model, is 0.95.

A. Simulation 1

The input applied to the plant is the sum of two sinusoidal signals of frequencies $0.0628\ \text{rad}/s$ and $0.0698\ \text{rad}/s$, namely, $u_k = \sin\left(\frac{\pi k}{50} + \frac{\pi}{4}\right) + 0.1\,\sin\left(\frac{\pi k}{45}\right)$. The behaviour of the plant subject to this input is given by (6) with the unknown values $\alpha_i = 0.45$ for $i \in \{1, 2, 3\}$. The three estimation models of (7) are included in the multiestimation scheme in order to obtain a reduced-order estimation model for the plant at this operation point. Therefore, two parameters are estimated in each estimation model. Algorithm 1 is used to estimate these parameters. Fig. 3 displays the estimated output issued by the multiestimation scheme together with the true plant output. Fig. 4 shows the estimation model chosen by the supervisor during the simulation. The estimated output follows the true plant output with a good approach after a transient interval with a small tracking-error. The supervisor selects the three estimation models at different time intervals during the simulation and finally chooses the estimation model $H^{(4)}(z)$ which results the best reduced-order estimation model for the true plant at this operation point.

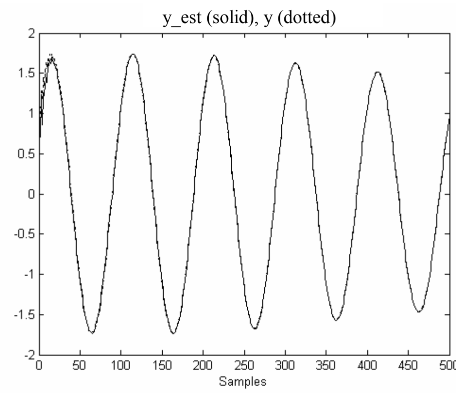

Fig. 3. Evolution of the estimated and true plant outputs.

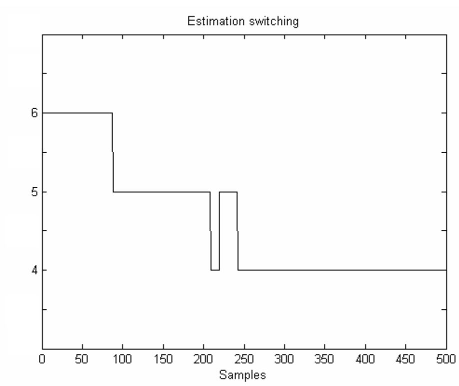

Fig. 4. Reduced-order estimation model chosen by the supervisor.

The reason of the final choice of the estimation model $H^{(4)}(z)$ by the supervisor is based in the frequency spectrum of the applied input. For that, the representation of the amplitude Bode diagram for each of the reduced-order estimation models is displayed in Fig. 5 supposed that $\hat{\alpha}_{jk}^{(i)} = 1$ for all integer $k \geq 0$, $i \in \{4, 5, 6\}$ and $j \in \{1, 2, 3\}$. This figure shows as the model $H^{(4)}(z)$ posses the highest gain for the frequencies which characterizes the applied input. Consequently, it is finally chosen by the supervisor. Finally, Fig. 6 displays the evolution of the estimated parameters of the model

$H^{(4)}(z)$. They converge to constant values which do not coincide with the corresponding true values of (6) because the estimation model is of a reduced-order.

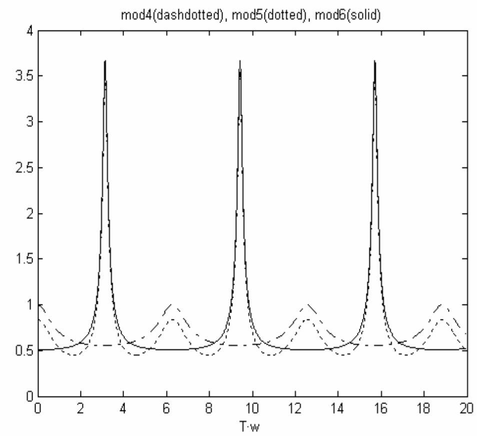

Fig. 5. Amplitude Bode diagram of the three estimation models for $\omega \in [0 \text{ rad/s}, 200 \text{ rad/s}]$.

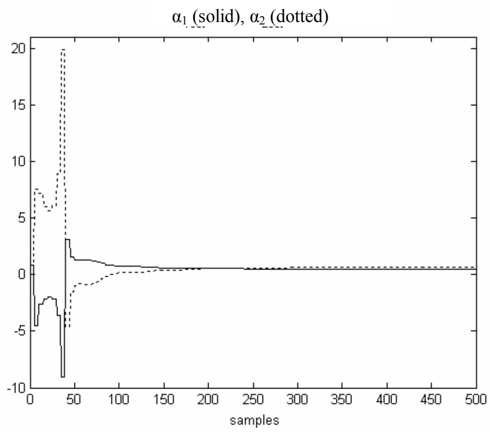

Fig. 6. Evolution of the estimates $\hat{\alpha}_1^{(4)}$ and $\hat{\alpha}_2^{(4)}$ of the reduced-order model $H^{(4)}(z)$.

The convergence of the estimates to the true values would be possible if the gain of the suppressed plant mode were sufficiently small compared with the gains of the modes included in the reduced-order estimation model. This fact has been empirically corroborated with numerical simulations which have been omitted by space reasons.

B. Simulation 2

In this case Algorithm 2 is used in each reduced-order estimation model. The unknown values of (6) are $\alpha_1 = \alpha_2 = 0.45$ and $\alpha_3 = 0.01$. The initialization for the estimation algorithms and the applied input are equal to those used for Simulation 1. The simulation results are displayed in the following figures.

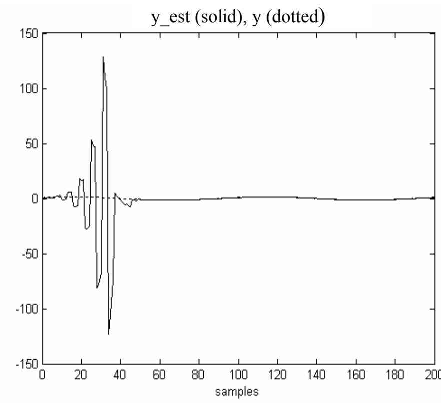

Fig. 7. Evolution of the estimated and true plant outputs between the samples 0 and 200.

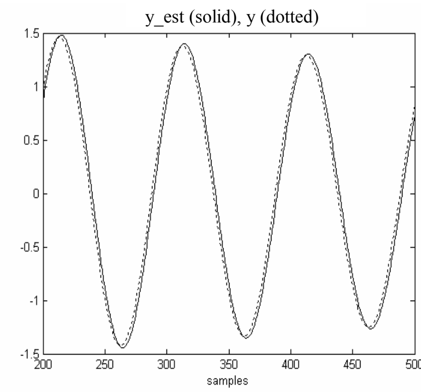

Fig. 8. Evolution of the estimated and true plant outputs between the samples 200 and 500.

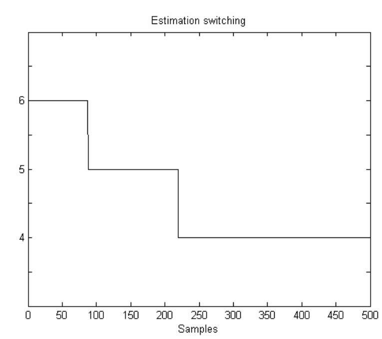

Fig. 9. Reduced-order estimation model chosen by the supervisor.

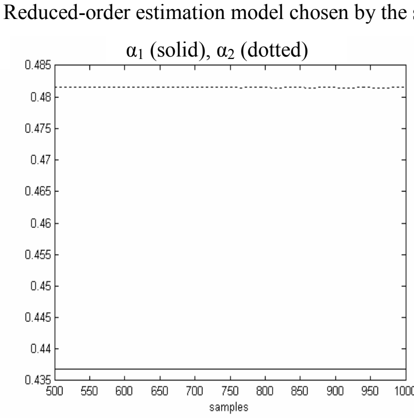

Fig. 10. Evolution of the estimates $\hat{\alpha}_1^{(4)}$ and $\hat{\alpha}_2^{(4)}$ of the reduced-order model $H^{(4)}(z)$ between the samples 500 and 1000.

C. Simulation 3

In this case, the values of α_i, for $i \in \{1,\ 2,\ 3\}$, in (6) are slowly time-varying. The variation of these values are showed in the following figures,

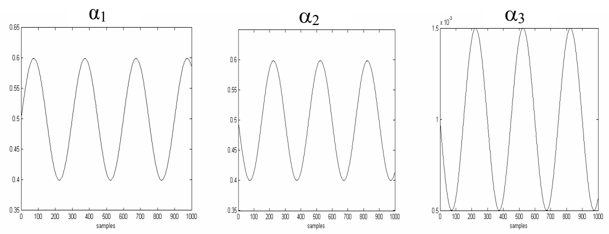

Fig. 11: Evolution of the values of α_i corresponding to the operating point of the true plant.

Algorithm 1 is utilized to estimate the parameters of each of the three reduced-order estimation models of (7). The same initialization for the estimation algorithms that in the previous simulations is considered. Fig. 12 displays the estimated output and the true plant output and Fig. 13 shows the switching map between the reduced-order estimation models of the multiestimation scheme. This mapping is carried out by means of the supervisor action.

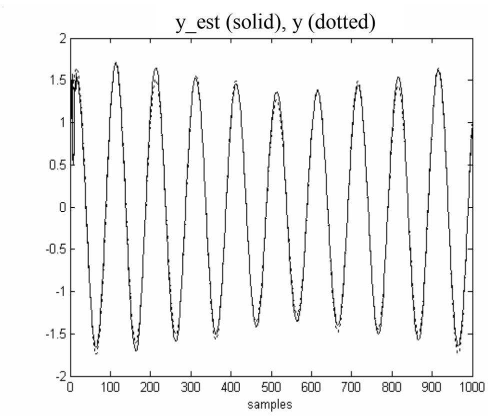

Fig. 12. Evolution of the estimated and true plant outputs during the simulation.

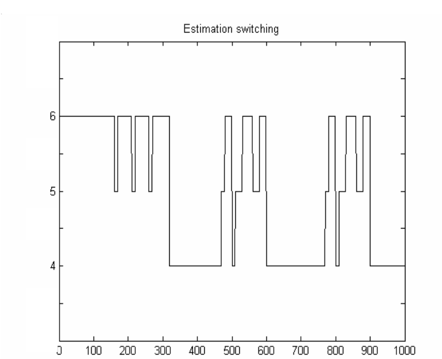

Fig. 13. Reduced-order estimation model chosen by the supervisor.

Note that the selection of the estimation model by the supervisor does not converge to a particular model since the variation of the parameters α_i of (6) makes the operating point of the plant to change with the time.

V. Conclusions

A multiestimation scheme for obtaining a reduced-order estimated model for a stable real plant, possibly non-linear, at each different operating points has been presented. The modes of the plant to be identified are distinct and known. Each of the nominal estimation models suppressed at least one of the plant modes. The estimation algorithms which compose the multiestimation scheme run in parallel and estimate the gain associated with each mode included in its corresponding nominal model. The simulation results corroborate the achievable performance of obtaining distinct reduced-order models of the true plant at different operation points. However, the exact identification of the true plant parameters is only possible if the gain of the suppressed mode in the reduced-order estimation model chosen by the supervisor is sufficiently small with respect to the gains of the modes of the estimation model. This last feature can not be ensured since an a-priori knowledge of the gains of the plant modes is not available. However, the presented strategy lets to obtain a reduced-order estimation model of the true plant at certain operating point, which can be later used, for instance, to implement adaptive controllers to achieve certain control objectives.

Acknowledgment

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