Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Decentralized Output Feedback Control of Large-Scale Nonlinear Systems Interconnected by Unmeasurable States

Michael T. Frye, Yuanlin Lu, and Chunjiang Qian

***Abstract*— This paper studies the problem of global decentralized control by output feedback for large-scale uncertain systems whose subsystems are interconnected not only by their outputs but also by their *unmeasurable states*. We show that under a linear growth condition, there is a decentralized output feedback controller rendering the closed-loop systems globally exponentially stable. This is accomplished by extending the output feedback domination design [14] that only needs little information of the nonlinear systems. A time-varying output feedback controller is also constructed for the large-scale systems with unknown parameters.**

Index Terms: Decentralized control, interconnected systems, large-scale systems, global stabilization, output feedback

I. INTRODUCTION

Decentralized control of interconnected systems has been an area of considerable research due to its obvious practical application to current problems in the field of controls. Large-scale systems have very complex dynamic models due to the uncertain environment, the varying system parameters, and the interconnected structure of the system. Also it is inevitable that nonlinearities are prevalent throughout the dynamics of the interconnected systems. All these make the stabilization of such large-scale systems a difficult control problem. Though quite challenging, the research of large-scale systems are relevant to such areas as communication networks, a system of satellites, and formation flying of autonomous vehicles, and hence are important in control practice.

The research of large-scale nonlinear systems began in the late 1960's and early 1970's. One of the earliest investigations into the nonlinear issues of large-scale systems centered around time-varying stabilization [2]. The early research in [7] demonstrated a method of using high-gain state feedback to stabilize the nonlinearities of the large-scale systems. The research in the early 1980's focused on the use of state feedback to globally stabilize large-scale nonlinear systems. Adaptive control was applied in [3] to stabilize a class of large-scale nonlinear systems with success. Output feedback had also be applied to linear large-scale system in such papers as [1], [16], and [12] during the same time. The use of output feedback has certain apparent advantages because of the fact that not all of the state variables of a large-scale system can be measured. An interesting paper [18], which applied static output feedback to nonlinear large-scale systems, used a linear quadratic method to develop a sufficient condition for stability of the large-scale system on a closed connected set. The use of adaptive control and output feedback was applied in [5] and [4]. A series of papers [6], [8] discussed output feedback and disturbance rejection for large-scale systems with disturbance. Robustness issue and output feedback was studied in [11].

Note that most of the existing decentralized output feedback results are developed for large-scale systems interconnected only by the outputs. There are very few results dealing with large-scale systems interconnected by the unmeasurable states. One existing result is the work [8], in which each subsystem within the large-scale system is dominated by a system of its own states and the outputs of other subsystems. All the unmeasurable states of other subsystems disappear in the bounding functions. The work [18] dealt with nonlinear functions that can depend on unbounded unmeasurable states, however the result is not a global one. In fact, due to the use of a quadratic method, the static output controller proposed in [18] could only stabilize the system on a closed connected set. Currently, the problem of *global decentralized control* of large-scale systems interconnected by unbounded unmeasurable states is quite open. The major difficulty in implementing an output feedback controller for those highly interconnected large-scale systems is that for each subsystem, the presence of unmeasurable states of other subsystems makes the design of the decentralized output feedback controller very complicated. In other words, it is very challenging to design a global stabilizer for one subsystem only using its output, while this subsystem is also driven by the unmeasurable states of the other subsystems.

On the other hand, recently there are a number of interesting output feedback stabilization results outside the area of decentralized control. For example, in the work [9], a necessary and sufficient condition was given for a nonlinear system to be equivalent to an observable linear system perturbed by a vector field that depends only on the output and input of the system. As a consequence, global stabilization by output feedback is achievable for a class of nonlinear systems that are diffeomorphic to a system in the nonlinear observer form [9], [10]. Other work are based on the common assumption that the system is *linear or Lipschitz* in the unmeasurable states (see [13] and its

This work was supported in part by the U.S. NSF under grant ECS-0239105 and UTSA Faculty Research Award.

The authors are with the Department of Electrical Engineering, The University of Texas at San Antonio, San Antonio, Texas 78249 mtfrye@lonestar.utsa.edu and cqian@utsa.edu

references). When the nonlinear functions are not Lipschitz in unmeasurable states or have uncertainties associated with unmeasurable states, the paper [14] was able to provide a method of constructing a linear output feedback stabilizer using a feedback domination design method under a linear growth condition.

In this paper, we will show that the result [14] can be applied to m subsystems that are highly interconnected through all the *unmeasurable* states. Under a linear growth condition imposed on the uncertain nonlinear vector fields, we will design a linear controller for each subsystem only using its own output. As shown in [14], this output feedback controller needs no information of the uncertain nonlinearities. The new structure of the observer and controller will enable us to overcome the difficulty in dealing with the output feedback control problem in the presence of unmeasurable states in each subsystem. A combination of the observers and controllers constructed for subsystems will globally stabilize the whole large-scale system.

This paper is organized as follows, Section 2 presents the *Problem Statement*, where we will present our assumption which utilizes a growth condition for bounding the nonlinearities of m subsystems. In Section 3, we will present our main results and demonstrate that a linear observer coupled with its output feedback controller can globally stabilize a large-scale systems comprised of m subsystems. Section 4, *An Example*, implements a high gain L output feedback controller on a simple 2-system case. In Section 5, we extend the results to the case when the growth rate is unknown by using a time varying gain. We summary our results in Section 6.

II. PROBLEM STATEMENT

In this paper, we consider a class of large-scale uncertain nonlinear systems comprised of m subsystems.

$$\text{Subsystem 1:}\quad \begin{cases} \dot{x}_{11} = x_{12} + f_{11}(x, d(t)) \\ \dot{x}_{12} = x_{13} + f_{12}(x, d(t)) \\ \quad\vdots \\ \dot{x}_{1n} = u_1 + f_{1n}(x, d(t)) \\ \quad y_1 = x_{11} \end{cases}$$
$$\vdots$$
$$\text{Subsystem i:}\quad \begin{cases} \dot{x}_{i1} = x_{i2} + f_{i1}(x, d(t)) \\ \dot{x}_{i2} = x_{i3} + f_{i2}(x, d(t)) \\ \quad\vdots \\ \dot{x}_{in} = u_i + f_{in}(x, d(t)) \\ \quad y_i = x_{i1} \end{cases} \qquad (2.1)$$
$$\vdots$$
$$\text{Subsystem m:}\quad \begin{cases} \dot{x}_{m1} = x_{m2} + f_{m1}(x, d(t)) \\ \dot{x}_{m2} = x_{m3} + f_{m2}(x, d(t)) \\ \quad\vdots \\ \dot{x}_{mn} = u_m + f_{mn}(x, d(t)) \\ \quad y_m = x_{m1} \end{cases}$$

where $x_i = (x_{i1}, \cdots, x_{in})$, $i = 1, \cdots, m$, $x = (x_1, \cdots, x_m)$, is the state, y_i is the output, u_i is the control input, $d(t)$ is a bounded unknown disturbance, and f_{ij} is a function satisfying the following condition.

Assumption 2.1: For $i = 1, \cdots, m$ and $j = 1, \cdots, n$, there is a constant $c \geq 0$ such that

$$|f_{ij}(x, d(t))| \leq c(|x_{11}| + \cdots + |x_{1j}| + |x_{21}| + \cdots + |x_{2j}| + \cdots + |x_{m1}| + \cdots + |x_{mj}|) \qquad (2.2)$$

The objective of this paper is to show that Assumption 2.1 guarantees the existence of dynamic compensators and controllers of the form

$$\begin{aligned} \dot{\xi}_i &= M\xi_i + Ny_i, \quad M \in \mathbb{R}^{n\times n}, \quad N \in \mathbb{R}^n \\ u_i &= K\xi_i, \qquad K \in \mathbb{R}^{1\times n}, i = 1, \cdots, m \end{aligned} \qquad (2.3)$$

such that the closed-loop system (2.1) and (2.3) is *globally exponentially stable* (GES) at the equilibrium $(x, \xi) = (0, 0)$.

Remark 2.2: Apparently, system (2.1) under Assumption 2.1 represents a class of large-scale systems whose subsystems are interconnected through not only the outputs but also the *unmeasurable states*. Moreover, those unmeasurable states in (2.1) will not disappear in the bounding functions in (2.2). For example, in the following system

$$\text{Subsystem 1}\quad \begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = u_1 + d(t)x_{12} + d(t)x_{22}, \\ y_1 = x_{11}, \quad d(t) \in [1, 2]. \end{cases}$$
$$\text{Subsystem 2}\quad \begin{cases} \dot{x}_{21} = x_{22} \\ \dot{x}_{22} = u_2 + \ln(1 + x_{12}^2) + x_{22}\sin(x_{22}) \\ y_2 = x_{21}, \end{cases}$$

Subsystem 1 and Subsystem 2 are interconnected through the unmeasurable state x_{12} and x_{22} that cannot be eliminated even in the bounding functions. Due to this reason, the problem of global decentralized control of Subsystem 1 and Subsystem 2 by output feedback is challenging and interesting.

This paper will show how a *linear output* feedback controller of the form (2.3) can be recursively constructed to globally stabilize system (2.1) under Assumption 2.1. An advantage of our design method is that the precise knowledge of the nonlinearities or uncertainties of the systems needs not to be known. What is really needed is the growth rate of the bounding function of the uncertain nonlinearities as shown in Assumption 2.1. This feature makes it possible to stabilize m uncertain interconnected subsystems using very limited information even all the subsystems are interconnected through *unmeasurable states*.

III. GLOBAL DECENTRALIZED CONTROL BY OUTPUT FEEDBACK

In this section, we prove that under Assumption 2.1 there exists a globally stabilizing output feedback controller for system (2.1). This is done by using a new output feedback domination design which explicitly construct a *linear output* feedback control law without requiring the knowledge of the nonlinearities in system (2.1).

Theorem 3.1: Under Assumption 2.1, there exists a linear output feedback controller (2.3) that renders the large-scale interconnected system (2.1) globally exponentially stable.

Proof. In order to prove Theorem 3.1, we utilize the output feedback domination design proposed in [14] to design a linear observer and controller for each individual subsystem. With these m observers and controllers, it can be shown that the closed-loop system is globally exponentially stable after a large enough gain has been carefully chosen.

A. *LINEAR OBSERVER DESIGN:*

We begin with by designing the following linear observer for subsystem i

$$\begin{aligned} \dot{\hat{x}}_{i1} &= \hat{x}_{i2} + La_1(x_{i1} - \hat{x}_{i1}) \\ &\vdots \\ \dot{\hat{x}}_{i(n-1)} &= \hat{x}_{in} + L^{(n-1)}a_{n-1}(x_{i1} - \hat{x}_{i1}) \\ \dot{\hat{x}}_n &= u_i + L^n a_n(x_{i1} - \hat{x}_{i1}) \end{aligned} \tag{3.1}$$

where $L \geq 1$ is a gain parameter to be determined later, and $a_j > 0,\ j = 1, \cdots, n$, are coefficients of the Hurwitz polynomial $p(s) = s^n + a_1 s^{(n-1)} + \cdots + a_{n-1}s + a_n$. Define the following term $e_{ij} = x_{ij} - \hat{x}_{ij}, \quad j = 1, \cdots, n$. A simple calculation yields the following error dynamics:

$$\begin{aligned} \dot{e}_{i1} &= e_{i2} - La_1 e_{i1} + f_{i1}(x, d(t)) \\ &\vdots \\ \dot{e}_{i(n-1)} &= e_{in} - L^{n-1}a_{n-1}e_{i1} + f_{i(n-1)}(x, d(t)) \\ \dot{e}_{in} &= -L^n a_n e_{i1} + f_{in}(x, d(t)) \end{aligned}$$

Next, we introduce the change of coordinates $\varepsilon_{ij} = \frac{e_{ij}}{L^{j-1}}, \quad j = 1, \cdots, n$ to obtain a new error dynamic

$$\dot{\varepsilon}_i = LA\varepsilon_i + \begin{bmatrix} f_{i1}(x, d(t)) \\ \frac{f_{i2}(x,d(t))}{L} \\ \vdots \\ \frac{1}{L^{n-1}} f_{in}(x, d(t)) \end{bmatrix} \tag{3.2}$$

where

$$\varepsilon_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{in} \end{bmatrix}, \qquad A = \begin{bmatrix} -a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & \cdots & 1 \\ -a_n & 0 & \cdots & 0 \end{bmatrix}.$$

Clearly, A is a Hurwitz matrix. Therefore, there is a positive definite matrix $P = P^T > 0$ such that

$$A^T P + PA = -I.$$

Consider the Lyapunov function $V_{\varepsilon_i} = \varepsilon_i^T P \varepsilon_i$. The derivative of V_{ε_i} along (3.2) is,

$$\begin{aligned} \dot{V}_{\varepsilon_i} &= -L\|\varepsilon_i\|^2 + 2\varepsilon_i^T P \begin{bmatrix} f_{i1}(x, d(t)) \\ \frac{f_{i2}(x,d(t))}{L} \\ \vdots \\ \frac{f_{in}(x,d(t))}{L^{n-1}} \end{bmatrix} \\ &\leq -L\|\varepsilon\|^2 + 2\|P\|\|\varepsilon\| \\ &\quad \times \left(|f_{i1}| + \frac{1}{L}|f_{i2}| + \cdots + \frac{1}{L^{n-1}}|f_{in}| \right). \end{aligned}$$

By Assumption 2.1,

$$\begin{aligned} |f_{i1}(x, d(t))| &\leq c(|x_{11}| + \cdots + |x_{m1}|) \\ \frac{|f_{i2}(x, d(t))|}{L} &\leq \frac{c}{L}(|x_{11}| + \cdots + |x_{m1}| + |x_{12}| \\ &\quad + \cdots + |x_{m2}|) \\ &\vdots \\ \frac{|f_{in}(x, d(t))|}{L^{n-1}} &\leq \frac{c}{L^{n-1}}(|x_{11}| + \cdots + |x_{m1}| + |x_{12}| \\ &\quad + \cdots + |x_{m2}| + \cdots + |x_{1n}| \\ &\quad + \cdots + |x_{mn}|) \end{aligned}$$

Therefore,

$$\begin{aligned} \dot{V}_{\varepsilon_i} \leq\ & -L\|\varepsilon\|^2 + c_1\|\varepsilon\|[(1 + \frac{1}{L} + \cdots + \frac{1}{L^{n-1}})|x_{11}| \\ & +(\frac{1}{L} + \cdots + \frac{1}{L^{n-1}})|x_{12}| + \cdots + \frac{1}{L_{n-1}}|x_{1n}| \\ & +(1 + \frac{1}{L} + \cdots + \frac{1}{L^{n-1}})|x_{21}| \\ & +(\frac{1}{L} + \cdots + \frac{1}{L^{n-1}})|x_{22}| \\ & + \cdots + \frac{1}{L^{n-1}}|x_{2n}| \\ & + \cdots + (1 + \frac{1}{L} + \cdots + \frac{1}{L^{n-1}})|x_{m1}| \\ & + \cdots + \frac{1}{L^{n-1}}|x_{mn}|] \\ \leq\ & -L\|\varepsilon\|^2 + c_1\|\varepsilon\|[n|x_{11}| + \frac{1}{L}(n-1)|x_{12}| \\ & + \cdots + \frac{1}{L_{n-1}}|x_{1n}| + n|x_{21}| + \frac{n-1}{L}|x_{22}| \\ & + \cdots + \frac{1}{L^{n-1}}|x_{2n}| + \cdots + n|x_{m1}| \\ & + \cdots + \frac{1}{L^{n-1}}|x_{mn}|] \\ \leq\ & -L\|\varepsilon\|^2 + c_2\|\varepsilon\|[(|x_{11}| + \cdots + |x_{m1}|) \\ & +\frac{1}{L}(|x_{12}| + \cdots + |x_{m2}|) + \cdots + \\ & \frac{1}{L^{n-1}}(|x_{1n}| + \cdots + |x_{mn}|)], \end{aligned}$$

for a constant $c_2 > 0$.

Define $z_{ij} = \frac{\hat{x}_{ij}}{L^{j-1}}, \quad j = 1, \cdots, n$. This, together with the fact that $x_{ij} = \hat{x}_{ij} + L^{n-1}\varepsilon_{ij}$, implies

$$\left|\frac{1}{L^{j-1}}x_{ij}\right| \leq \left|\frac{1}{L^{j-1}}\hat{x}_{ij}\right| + |\varepsilon_{ij}| = |z_{ij}| + |\varepsilon_{ij}|, \quad j = 1, \cdots, n.$$

With this in mind, it is not difficult to deduce that

$$\begin{aligned} \dot{V}_{\varepsilon_i} &\leq -L\|\varepsilon_i\|^2 + \sqrt{n}c_2\|\varepsilon_i\| \\ &\quad \times (\|z_1\| + \cdots + \|z_m\| + \|\varepsilon_1\| + \cdots + \|\varepsilon_m\|) \\ &\leq -L\|\varepsilon_i\|^2 + c_3\|z_1\|^2 + c_3\|z_2\|^2 + \cdots + c_3\|z_m\|^2 \\ &\quad + c_3\|\varepsilon_1\|^2 + c_3\|\varepsilon_2\|^2 + \cdots + c_3\|\varepsilon_m\|^2 \qquad (3.3) \end{aligned}$$

for a constant $c_3 > 0$.

B. ***CONTROLLER DESIGN:***

Under the new coordinates $z_{ij} = \frac{\hat{x}_{ij}}{L^{j-1}}, \quad j = 1, \cdots, n$, system (3.1) becomes

$$\begin{aligned} \dot{z}_{i1} &= Lz_{i2} + La_1\varepsilon_{i1} \\ &\vdots \\ \dot{z}_{i(n-1)} &= Lz_{in} + La_{n-1}\varepsilon_{i1} \\ \dot{z}_{in} &= \frac{1}{L^{n-1}}u_i + La_n\varepsilon_{i1} \qquad (3.4) \end{aligned}$$

Construct $u_i = -L^n[k_1z_{i1} + k_2z_{i2} + \cdots + k_nz_{in}]$, where $k_1, \cdots, k_n$ are the coefficients of the Hurwitz polynomial $s^n + k_ns^{n-1} + \cdots + k_2s + k_1 = 0$. Under this controller, system (3.4) can be written as the following compact form.

$$\dot{z}_i = LBz_i + LD\varepsilon_{i1} \qquad (3.5)$$

where

$$z_i = \begin{bmatrix} z_{i1} \\ z_{i2} \\ \vdots \\ z_{in} \end{bmatrix}, D = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ k_1 & k_2 & \cdots & k_n \end{bmatrix}.$$

For Hurwitz matrix B, there is a positive definite matrix $Q = Q^T > 0$ such that

$$B^TQ + QB = -I.$$

Consider the following Lyapunov function $V_{z_i} = z_i^TQz_i$. By the necessary substitution we are arrive at the following equations,

$$\begin{aligned} \dot{V}_{z_i} &= -L\|z_i\|^2 + 2z_i^TLQD\varepsilon_{i1} \\ &\leq -L\|z_i\|^2 + Lc_4\|z_i\|\|\varepsilon_i\| \\ &\leq -\frac{1}{2}L\|z_i\|^2 + Lc_5\|\varepsilon_i\|^2, \qquad c_5 = \frac{c_4^2}{2} \quad (3.6) \end{aligned}$$

Choice of gain L: Construct the following Lyapunov function

$$W_i = (1/2 + c_5)V_{\varepsilon_i} + V_{z_i}.$$

Using equations (3.6) and (3.3), one has

$$\begin{aligned} \dot{W}_i &= (1/2 + c_5)\dot{V}_{\varepsilon_i} + \dot{V}_{z_i} \\ &\leq -L(1/2 + c_5)\|\varepsilon_i\|^2 + c_6\|z_1\|^2 + c_6\|z_2\|^2 \\ &\quad + \cdots + c_6\|z_m\|^2 + c_6\|\varepsilon_1\|^2 + c_6\|\varepsilon_2\|^2 \\ &\quad + \cdots + c_6\|\varepsilon_m\|^2 - \frac{1}{2}L\|z_i\|^2 + Lc_5\|\varepsilon_i\|^2 \\ &= -\frac{1}{2}L\|\varepsilon_i\|^2 - \frac{1}{2}L\|z_i\|^2 + c_6\|z_1\|^2 \\ &\quad + \cdots + c_6\|z_m\|^2 + c_6\|\varepsilon_1\|^2 \\ &\quad + \cdots + c_6\|\varepsilon_m\|^2, \\ &\quad c_6 = (1/2 + c_5)c_3. \qquad (3.7) \end{aligned}$$

Consequently, for m subsystems we have

$$\begin{aligned} \sum_{i=1}^{m}\dot{W}_i &\leq -\left(\frac{1}{2}L - mc_6\right)\sum_{i=1}^{m}\|\varepsilon_i\|^2 \\ &\quad -\left(\frac{1}{2}L - mc_6\right)\sum_{i=1}^{m}\|z_i\|^2. \qquad (3.8) \end{aligned}$$

If the gain L is made large enough, the right hand side (3.8) will be negative definite. Hence, the closed-loop system will be globally exponentially stable (GES). ∎

Remark 3.2: In contrast to the common observer design that typically uses a copy of the nonlinear system, we design only a *linear* observer for each subsystem in the large-scale system (2.1). Such a construction, has enabled us to deal with difficult issues caused by the uncertainties or nonlinearities of the systems in the simple system case [14]. In this paper, this new construction of the observer and controller also lets us avoid dealing with the nonlinear functions of the interconnected unmeasurable states. Consequently, this feedback domination design leads to a solution to the problem of decentralized output feedback control of system (2.1).

Remark 3.3: It is worthwhile pointing out that the observer and controller for each system have the same structure. Hence, after we construct one output feedback controller for one subsystem, we can duplicate the controller for the other $m-1$ subsystems. This property will reduce the design time and implementation cost for the control design of system (2.1).

Remark 3.4: Note that in system (2.1) all the subsystems have the same dimension (i.e. n). However, if the dimensions of m subsystems are different, we are still able to achieve similar stabilization result under Assumption 2.1 with different dimensional variables. The only difference is that the dimension of the observer will be consistent with the dimension of the corresponding subsystem.

IV. AN EXAMPLE

In this section, a two-system model will be simulated based on the design procedures in Section 3.

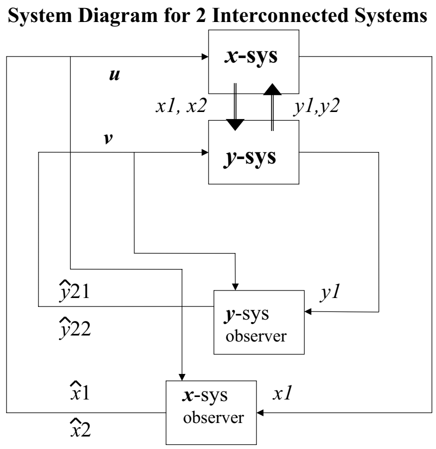

Fig. 1. System Diagram Overview

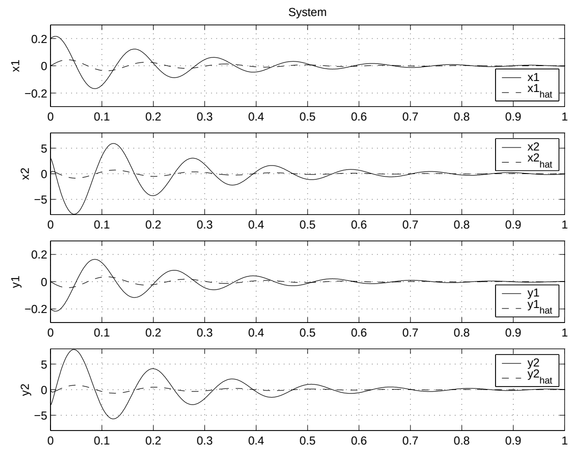

Fig. 2. $(x_1(0), x_2(0)) = (.2, 3)$ $(\hat{x}_1(0), \hat{x}_2(0)) = (0, 0)$ $(y_1(0), y_2(0)) = (-.2, -3)$ $(\hat{y}_1(0), \hat{y}_2(0)) = (0, 0)$

Example 4.1: Consider the following interconnected nonlinear system.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + y_2 \sin x_2 \\ x_{output} &= x_1 \\ \dot{y}_1 &= y_2 \\ \dot{y}_2 &= v + d(t)\ln(1+y_2^2) + d(t)x_2 \\ y_{output} &= y_1 \end{aligned} \tag{4.1}$$

where $d(t)$ is a disturbance bounded by a known constant. As Equation (4.1) shows, the x-system and y-system are coupled through the unmeasurable states ($x_2,\ y_2$). Moreover, the unmeasurable states are associated with unknown disturbances. Therefore, most of the existing output feedback control design procedures will fail to be applicable to the system (4.1). On the other hand, it is easy to verify that Assumption 2.1 holds for system (4.1). By Theorem 3.1, we are able to design an output feedback controller for (4.1). Figure 1 illustrates in block diagram form the control strategy that will be implemented for the 2-system example.

Specifically, we construct the observer as follows,

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + 0.42L(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= u + 4.2L^2(x_1 - \hat{x}_1) \\ \dot{\hat{y}}_1 &= \hat{y}_2 + 0.42L(y_1 - \hat{y}_1) \\ \dot{\hat{y}}_2 &= v + 4.2L^2(y_1 - \hat{y}_1) \end{aligned} \tag{4.2}$$

The control laws to be implemented are

$$\begin{aligned} u &= -28.6L^2\hat{x}_1 - 25.7L\hat{x}_2 \\ v &= -28.6L^2\hat{y}_1 - 25.7L\hat{y}_2, \end{aligned} \tag{4.3}$$

where the gain L was calculated to be **20**. Figure 2 below illustrates the response of the closed-loop system (4.1)-(4.2)-(4.3).

Remark 4.2: As shown in [14], the output feedback domination design has the universal property that enables us to use a single output feedback controller to stabilize a family of nonlinear systems satisfying same growth condition. This nice property is also valid in the decentralized case. For example, the exactly same output feedback controller (4.2)-(4.3) for (4.1) will also stabilize the following system.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + d(t)\sqrt{|x_2 y_2|} \\ x_{output} &= x_1 \\ \dot{y}_1 &= y_2 \\ \dot{y}_2 &= v + d(t)(1 - e^{-|y_2|}) + x_2 \sin x_2 \\ y_{output} &= y_1 \end{aligned}$$

V. EXTENSION TO LARGE-SCALE SYSTEMS WITH UNKNOWN GROWTH RATE

For the interconnected system (2.1), there might be circumstances when the growth rate c in Assumption 2.1 is unknown. In such situations there arises the probelm if it is still possible to achieve global regulation for system (2.1). In this section, we show that using the time-varying observer developed in [15], a decentralized output feedback controller with time-varying gain *L(t)* can be designed to globally regulate system (2.1) whose nonlinear function $f_{i,j}(x, d(t))$ linearly grows at an unknown rate.

Theorem 5.1: Suppose system (2.1) satisfy Assumption 2.1 with *unknown* growth rate c. Then, there exists an output feedback controller of the form,

$$\begin{aligned} \dot{\xi}_i &= M(t)\xi_i + N(t)y_i, \quad M(t) \in \mathbb{R}^{n\times n}, \quad N(t) \in \mathbb{R}^n \\ u_i &= K(t)\xi_i, \qquad\quad K(t) \in \mathbb{R}^{1\times n}, i = 1, \cdots, m, \end{aligned} \tag{5.1}$$

such that all the states of (2.1)-(5.1) are ultimately bounded. Moreover,

$$\lim_{t\to+\infty} (x(t), \xi(t)) = 0.$$

Proof. Theorem 5.1 can be easily proved by combining the time-varying observer and controller proposed in [15] with the design procedure for Theorem 3.1. The linear structure of the observer will avoid the difficulty in dealing with the uncertain nonlinear functions while the time-varying gain will suppress the effect of the unknown growth rate. The proof is very parallel to the one of Theorem 3.1 and hence is omitted here. ∎

Next, we see how system (4.1) can be globally regulated by output feedback when $d(t)$ is bounded by an *unknown* constant c. As a matter of fact, by Theorem 5.1, the following observer is developed using the varying gain L,

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + L(t)(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= u + L^2(t)(x_1 - \hat{x}_1) \\ \dot{\hat{y}}_1 &= \hat{y}_2 + L(t)(y_1 - \hat{y}_1) \\ \dot{\hat{y}}_2 &= v + L^2(t)(y_1 - \hat{y}_1). \end{aligned} \quad (5.2)$$

The control laws to be implemented will be

$$\begin{aligned} u &= -L^2(t)\hat{x}_1 - L(t)\hat{x}_2 \\ v &= -L^2(t)\hat{y}_1 - L(t)\hat{y}_2. \end{aligned} \quad (5.3)$$

Figure 3 illustrates the result due to a implementation of the varying gain $L(t) = t + 1$.

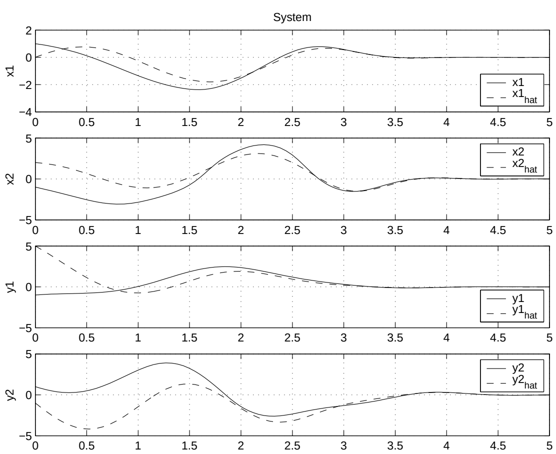

Fig. 3. $(x_1(0), x_2(0)) = (1, -1)$ $(\hat{x}_1(0), \hat{x}_2(0)) = (0, 2)$ $(y_1(0), y_2(0)) = (-1, 0)$ $(\hat{y}_1(0), \hat{y}_2(0)) = (5, -1)$

Remark 5.2: Using a similar argument proposed in [15], we can prove that $(x, y, \hat{x}, \hat{y})$ tend to zero exponentially. As a consequence, the observers and controllers are ultimately bounded even though L is not bounded. In fact, as shown in figure 3 the states of the system (4.1) and observers (5.2) tend to zero very quick. In real control practice, to avoid the use of unbounded L, we can saturate the gain after sufficiently long time.

VI. CONCLUSION

We have presented in this paper, a method of using output feedback to globally stabilize a large-scale nonlinear systems whose m subsystems are highly interconnected by unmeasurable states. Under the linear growth condition, we explicitly construct m sets of linear observers and controllers only using the output feedback information of each subsystem. It is shown that global output feedback stabilization is achieved for the closed-loop system. Also, observers and controllers using time-varying gain are developed to control the large-scale systems with unknown parameters. The universal feature of our feedback domination design enables us to only design one output feedback controller and apply it to all the different systems satisfying the same growth condition.

REFERENCES

[1] W. Bachmann and D. Konik, On Stabilization of Interconnected Systems using Output Feedback, *System Control Letter,* Vol. 5, pp. 89-95 (1984).
[2] E. J. Davison, The Decentralized Stablization and Control of a Class of Unknown Nonlinear Time-Varying Systems, *Automatica,* Vol. 10, No. 3, pp. 309-316 (1974).
[3] P. Ioannou, Decentralized Adaptive Control of Interconnected Systems, *IEEE Trans. Aut. Contr.,* Vol. 31, pp. 291-298 (1986).
[4] S. Jain and F. Khorrami, Decentralized Adaptive Output-Feedback Design for Large-Scale Nonlinear Systems, *IEEE Trans. Automat. Contr.,* Vol. 42, pp. 729-735 (2001).
[5] Z. P. Jiang, Decentralized and Adaptive Nonlinear Tracking of Large-Scale Systems via Output Feedback, *IEEE Trans. Automat. Contr.,* Vol. 45, pp. 2122-2128 (2000).
[6] Z. P. Jiang, F. Khorrami, and D. Hill, Decentralized Nonlinear Output-Feedback Stabilization with Disturbance Attenuation, *IEEE Trans. Automat. Contr.,* Vol. 46, pp. 1623-1629 (2001).
[7] H. Khalil and A Saberi, Decentralized stabilization of nonlinear interconnected systems using high-gain feedback, *IEEE Trans. Automat. Contr.,* Vol. 27, pp. 265-268 (1982).
[8] P. Krishnamurthy, F. Khorrami, Decentralized Control and Disturbance Attenuation for Large-Scale Nonlinear Systems in Generalized Output-Feedback Canonical Form, *Automatica,* (2003).
[9] A. J. Krener and A. Isidori, Linearization by output injection and nonlinear observer, *Systems & Control Letters,* Vol. 3, pp. 47-52, 1983.
[10] A.J. Krener and W. Respondek, Nonlinear observers with linearizable error dynamics, *SIAM J. Contr. Optimiz.* 23 (1985), 197–216.
[11] B. Labibi, B. Lohmann, A. K. Sedigh, and P. J. Maralani, Output Feedback Decentralized Control of Large-Scale Systems Using Weighted Sensitivity Functions Minimization , *Systems & Control Letters,* Vol. 47, pp. 191-198, (2002).
[12] A. Linnemann, Decentralized Control of Dynamically Interconnected Systems, *IEEE Trans. Aut. Contr.,* Vol. 29, 1052-1054 (1984).
[13] L. Praly, Asymptotic stabilization via output feedback for lower triangular systems with output dependent incremental rate, *IEEE Trans. Aut. Contr.,* Vol.48, 1103 -1108 (2003).
[14] C. Qian and W. Lin, Output Feedback Control of a Class of Nonlinear Systems: A Nonseparation Principle Paradigm, *IEEE Trans. Aut. Contr.,* Vol. 47, 1710-1715 (2002).
[15] C. Qian, C. B. Schrader, and W. Lin. Global Output Feedback Control of a Class of Nonlinear Systems with Unknown Parameters, *Proceedings of 2003 American Control Conference,* (2003).
[16] A. Saberi and H. K. Khalil, Decentralized Stabilization of Interconnected Systems using Output Feedback, *Int. J. Contr.,* Vol. 41, pp. 1461-1475 (1985).
[17] X. H. Xia and W.B. Gao, Nonlinear observer design by observer error linearization, SIAM J. Contr. Optimiz. 27 (1989), pp. 199-216.
[18] D. Z. Zheng, Decentralized output feedback stabilization of a class of nonlinear interconnected systems, *IEEE Trans. Aut. Contr.,* Vol. 34, 1297-1300 (1989).