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A Wavelet Packet Based Sifting Process and Its Application for Structural Health Monitoring

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Abstract

The paper presents an innovative wavelet packet based sifting process to decompose a signal into its components with different frequency content. The method is illustrated for simulation data of a linear three degree-of-freedom system and the results are compared with those using the empirical mode decomposition (EMD) method. Both methods provide good approximations for the modal responses from the modal analysis. Incorporated with the classical Hilbert transform, the proposed sifting process may be effectively used for structural health monitoring, including both detecting abrupt structural stiffness loss and monitoring development of progressive stiffness degradation, as demonstrated by two case studies. Results from a preliminary study for experimental data are also presented.

1. Introduction

Wavelets have been widely used in damage detection [1] and image processing [2]. Using its capability in multi-resolution and time-frequency analysis, wavelet analysis has become a promising technique for structural health monitoring of large-scale structures [3], [4].

The Empirical Mode Decomposition (EMD) method was recently developed as a systematic and robust tool for signal processing for non-linear and non-stationary data [5]. Using this technique a signal can be decomposed into its mono-components, called as Intrinsic Mode Functions (IMF) by an empirical sifting process. By incorporating Hilbert transform with the sifting process, the instantaneous frequency and instantaneous amplitude variation of intrinsic mode functions can be found and used in applications of damage detection and system identification [6],[7]. The empirical nature of the approach may be partially attributed to a subjective definition of the envelope and the intrinsic mode function involved in its sifting process. For example, an impulse response of a simple linear damped oscillator, which is physically mono-component with a single frequency, may not be necessarily fit the definition of IMF and envelope function.

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This paper intends to present a sifting process based on wavelet packet decomposition of a signal. Grouping its wavelet packet components of a signal based on the minimum entropy, the original signal can be decomposed into its dominant components with nearly distinct frequency contents. This wavelet theory based sifting process may produce comparable decomposition of a signal as compared with the EMD approach. An attempt has been made to apply the proposed sifting process for structural health monitoring, as illustrated in two case studies.

2. Background

This section presents brief background of the wavelet packet analysis and the empirical mode decomposition method. The reader is referred to the relevant references for details.

2.1 Wavelet Packet Analysis

In contrast to Short Time Fourier Transform, which uses a single analysis window, the wavelet uses short windows at high frequencies and long windows at low frequencies providing ‘zoom in-zoom out’ effect. This property makes wavelet transform as a potential tool to analyze non-stationary signal. The signal is mapped on Time-Scale plane where the scale is introduced as an alternative to frequency. This section summarizes the information about continuous and discrete wavelets. For detailed information, readers are referred to [8], [9], and [10].

Continuous Wavelet Transform (CWT) of a signal *f(t)* is defined as

$$W_f(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) * \overline{\Psi}(\frac{t-b}{a}) dt \tag{1}$$

Where $\overline{\Psi}$ is a conjugate of mother wavelet Ψ, ‘*a*’ and ‘*b*’ are dilation and translational parameters, respectively. Both are real and ‘*a*’ must be positive.

The continuous wavelet transform implies that local data in the time domain are examined by a shifted wavelet window with a variable window size and the wavelet coefficient measures correlation between the local data and the shifted and scaled wavelet windows. As a result, the transient frequency content of the signal can be revealed. The original signal can be reconstructed by an inverse wavelet transform. The mother wavelet needs to satisfy certain admissibility condition to ensure existence of the inverse wavelet transform.

Discrete Wavelet Transform (DWT) is often used for more efficient implementation in a practical application. In DWT the dilation parameter '*a*' and the translational parameter '*b*' is discretized by using the dyadic scale i.e.

$$a = 2^j \quad b = k.2^j \quad j, k \in z \tag{2}$$

where z is the set of positive integers.

In the discrete wavelet transform, the wavelet plays a role as filters. Using shifted and scaled wavelet filters the signal is examined locally in the time domain at different levels of scale. As a result, the signal can be decomposed into a tree structure with wavelet details and wavelet approximations at various levels as follows

$$f(t) = \sum_{i=1}^{i=j} D_i(t) + A_j(t) \tag{3}$$

where $D_i(t)$ denotes the wavelet detail and $A_j(t)$ stands for the wavelet approximation at the j^{th} level, respectively. Note that at each level, the DWT decomposition results in halving the time resolution and doubling the frequency resolution. Reconstruction of the signal can be easily implemented as the dyadic wavelet filter family forms an orthonormal basis [9].

In certain applications, where the important information is located in higher frequency components, the frequency resolution of discrete wavelet decomposition may not be fine enough to meet certain requirements. The necessary frequency resolution may be achieved by using wavelet packet transform [10], an extension of regular wavelet analysis. In the wavelet packet analysis the wavelet details at each level is, in addition to decomposition of only the wavelet approximation in the regular wavelet analysis, further decomposed to its own approximation and details. By this process, some lower frequency contents leaked in the wavelet details at the previous level can be further sifted out at the current level and also the frequency resolution for signal analysis increases. As a result, the wavelet packet analysis may provide better accuracy in both higher and lower frequency components of the signal.

2.2 Empirical Mode Decomposition (EMD) Method

The empirical mode decomposition (EMD) method was recently proposed to decompose a signal into its monocomponents, referred as the Intrinsic Mode Functions (IMF), by an innovative sifting process [5]. The IMF is defined as (i) a function in which the number of extrema and the number of zero crossings must either equal or differ at most by one; and (ii) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by local minima is zero. The sifting process separates the components from the signal called as Intrinsic Mode Functions (IMFs) by extracting the highest frequency component first. The signal is decomposed into IMFs and residue as

$$f(t) = \sum_{i=1}^{n} c_i(t) + r_n \tag{4}$$

where C_i(t) is the i^{th} Intrinsic Mode Function and r_n is the residue.

This technique along with Hilbert Transform can be used in variety of applications including damage detection and coronal oscillations analysis.

2.3 Instantaneous Frequency and Hilbert Transform

As a general definition, the instantaneous frequency of a signal at time t can be expressed as the rate of change of phase angle function of the analytic function obtained by Hilbert Transform of the signal [11]. The analytic function $z(t)$ of a signal $s(t)$ is a complex signal having original signal $s(t)$ as a real part and Hilbert transform of original signal as its imaginary part. By representing the signal in polar coordinate form one has

$$z(t) = s(t) + jH[s(t)] = a(t).e^{jØ(t)} \tag{5}$$

where $a(t)$ is the instantaneous amplitude and $Ø(t)$ is the instantaneous phase function. Thus, the instantaneous frequency is

$$w(t) = dØ(t)/d(t) \tag{6}$$

Note that various other formulations for the instantaneous frequency of signal can be found in the literature [12]. In a special case of a single harmonic signal, the instantaneous frequency is constant and equal to the frequency of the harmonic. In general, the concept of instantaneous frequency provides an insightful description as how the frequency content of the signal varies with the time.

3. Methodology

The proposed wavelet based sifting process starts with interpolation of data with cubic spline interpolation. The interpolated data increases the time resolution of the signal which will in turn increase regularity of the decomposed components.

The interpolated data is decomposed into different components by using wavelet packet decomposition. A symmetrical wavelet is preferred in the process to guarantee symmetrical and regular shaped decomposed components. In case of the binary wavelet packet tree, decomposition at level 'n' results in 2^n components. This number may become very large at a higher decomposition levels. An optimum decomposition of the signal can be obtained based on the criteria of best entropy value [13], [14]. A particular node N in the decomposition is split into two nodes N_1 and N_2 if and only if the sum of the entropy of those decomposed nodes, N_1 and N_2 is lower than the entropy of N, thus the entropy of decomposition is kept as minimum as possible.

The percentage of energy contribution of an individual component to the total signal can be computed to sift out the significant components of the signal. Other criteria can also be applied to sift out the potential components in the signal; candidates include the minimum number of zero crossings and the minimum peak value of components.

4. Results of Numerical Validation

The wavelet packet based sifting process was validated by analyzing a vibration signal from a linear three-degree-of-freedom (3DOF) spring-mass system, as shown in Figure 1. The system natural frequencies are 1.29, 3.62 and 5.23 Hz, respectively. Without the loss of generality, zero damping was assumed in this study. An impact force was applied to the first mass element (M1). Dynamic response data were numerically simulated by subroutines in the commercial software MATLAB. The data was sampled at 100Hz.

Acceleration response data at the second mass element is selected to illustrate the concept and accuracy of the proposed approach. By applying the proposed wavelet packet based sifting process; the original signal is decomposed into three dominant components, as shown in Figure 2. The wavelet of Db36 was used as the analyzing mother wavelet in data decomposition. The time resolution of the signal is increased by using spline interpolation for interpolating the signal data with finer increment. Wavelet packet decomposition of the signal was carried out up to level 9 and the optimum decomposition tree is obtained by minimizing the entropy contribution of individual components at different levels.

To verify its accuracy the Fourier spectra of these components are plotted in Figure 3. The peak frequencies are almost identical to the natural frequencies, as expected. The three dominant components obtained by the present approach are compared with the exact solution for modal responses from the modal analysis in Figure 4 and only small errors are observed. Furthermore, the original signal is reconstructed by adding the three dominant components sifted out. Comparison between the original signal and the re-constructed signal shows only a small error.

The wavelet packet based sifting process is compared with the well known sifting process in the EMD method for different types of signal. In general, comparable results were obtained. Figure 4 shows a comparison of these two methods applied to the same acceleration data from the previous linear undamped 3DOF system. The exact modal responses from the classical modal analysis are also plotted in Figure 4 as a benchmark. Note that a shorter time interval is used in Fig. 4 for a zoom-in presentation for clarity. As observed, there are no significant differences between these two sets of results. The comparison indicates that performance of these two sifting are very similar. In certain sense, the sifting process in the EMD method may be viewed as an implicit wavelet analysis and the concept of the intrinsic mode function in the EMD method is parallel to the wavelet details in wavelet analysis.

5. Application of the Methodology for Structural Health Monitoring

The application of the wavelet packet sifting process for structural health monitoring is illustrated for two typical cases: sudden stiffness loss and progressive stiffness degradation. The former may be caused by an excess response of a structural member during a severe seismic event and the latter may be attributed to mechanical fatigue due to cyclic loading or chemical corrosion in a hazardous environment.

A dominant component of the original signal from the wavelet packet based sifting process usually has quite simple frequency characteristics and is suitable for the classical Hilbert transform. The transient frequency content or the so-called instantaneous frequency of the component can be found from the phase curve of the Hilbert transform of the component. For a healthy structure the associated instantaneous frequency is time-invariant. Any reduction in the instantaneous frequency may reflect structural damage. For a sudden stiffness loss, the change occurs in a very small time interval and for progressive stiffness degradation a gradual change in the instantaneous frequency can be observed.

In this study the same 3DOF structural model is employed and structural damage is introduced by linearly reducing the stiffness of spring K2 up to certain value. By selecting the rate of change in stiffness reduction, both cases of sudden damage and progressive damage can be simulated. For both cases, the proposed wavelet packet based sifting process is first applied to the simulated response data to sift their dominant components and the Hilbert transform is then applied to investigate their transient frequency characteristics for the purpose of structural health monitoring.

5.1 Case Study 1: Detection of Sudden Damage

In the case study of detection of sudden damage, a sudden stiffness loss is introduced at t=15sec by linearly reducing stiffness of the middle spring, i.e. K2 by 10% from t=15sec to t=15.05sec. Damage in such a small time interval may be reasonably considered as *sudden*. Without loss of generality only the dominant component of acceleration response data of M2, which is obtained by the proposed sifting process and corresponds to the highest mode of the healthy system, is selected for analysis.

Figure 5 plots the component and the associated instantaneous frequency history; the latter was obtained by Hilbert transform. An exact solution for the instantaneous frequency is also presented for comparison. A sudden change in the instantaneous frequency can be observed at t=15 sec, implying some sudden damage has occurred at that moment. The amount of frequency drop provides a global measure of damage severity of a local stiffness loss. Data analysis of other dominant components has lead to the similar conclusions.

It should be pointed out that numerical differentiation of the phase curve of Hilbert transform of a signal may generally produce fluctuated instantaneous frequency history. The associated variance is reduced in this study by filtering the phase angle curve.

5.2 Case Study 2: Monitoring Development of Stiffness Degradation

To model a progressive stiffness degradation, the value of K2 is reduced linearly by 10% from t=15sec to t=45sec. Again, the acceleration signal from the middle mass is selected for analysis. Its highest-mode component of the signal by the proposed sifting process and the associated instantaneous frequency are shown in Figure 6. A change in the instantaneous frequency is clearly observed in the same time interval as specified for the progressive damage in the data simulation. The trend and amount of change in instantaneous frequency provide valuable information as how stiffness degradation is developed. Note that despite the same trend, the change in the instantaneous frequency in Figure 6 is not linear.

6. Experimental Validation:

To examine feasibility of proposed method in real life applications, the proposed approach was applied to experimental data obtained by a Shaking Table Test of a two-story full size wooden frame performed at the Disaster Prevention Research Institute (DPRI), Kyoto University. The NS component of 1940 El Centro was used as the ground excitation. Several test runs were conducted and each test run was excited by the original records scaled at a nominal level targeted at certain intensity. Various types of damages were observed during the testing. As a preliminary study this paper analyzes the acceleration response of first floor at a load level of 6 m/s^2. For detailed information about the test, the reader is referred to [15].

The acceleration measurement of the first floor, as shown in Figure 7.1, is sifted by implementing the proposed wavelet packet sifting process. By performing the Hilbert transform of the decomposed lower frequency component in Figure 7.2, the corresponding instantaneous frequency is extracted and plotted in Figure 7.3. A permanent reduction in instantaneous frequency of structure clearly indicates that sudden damage has occurred at around t = 7sec. This result is in good agreement with the previous results using the Discrete Wavelet Transform in [15] where a wavelet damage spike, an indication of sudden damage, was clearly observed at t=7 sec. The result illustrates great promises of the proposed approach for real life applications. However, several practical issues need to be further investigated.

7. Concluding Remarks

This paper proposes an innovative sifting process based on wavelet packet analysis of a signal. Using this technique, a signal can be decomposed into its dominant components with different but simple frequency contents. The approach was validated using the simulation response data of a linear 3DOF system subjected to an impact load. It has shown that the sifted components are close to the modal responses of the system. As compared with the EMD method this new sifting process may provide comparable results on a theoretical basis. Using Hilbert Transform for each dominant component decomposed, the resulted instantaneous frequencies provide useful information for monitoring the structural health condition. A successful application of the proposed approach in two case studies of detecting a sudden stiffness loss and monitoring progressively developed stiffness degradation as well as a preliminary study using experimental data from a shaking table test of a two-story full size wooden frame structure demonstrates great promises of the proposed approach for structural health monitoring applications.

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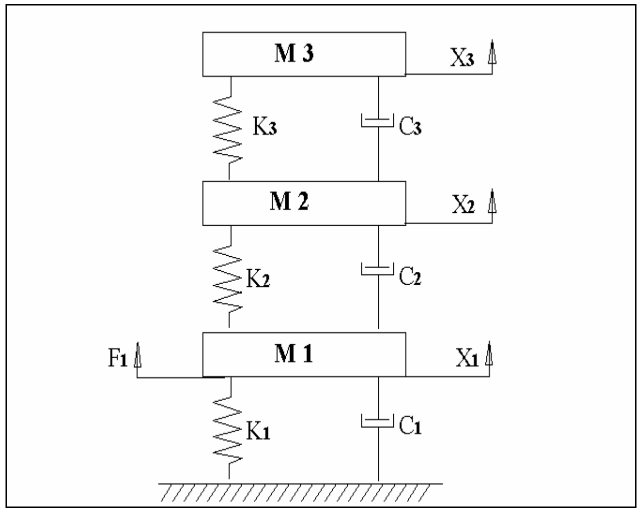

Figure1. Sketch of a 3DOF mass-spring-damper system used in simulation study

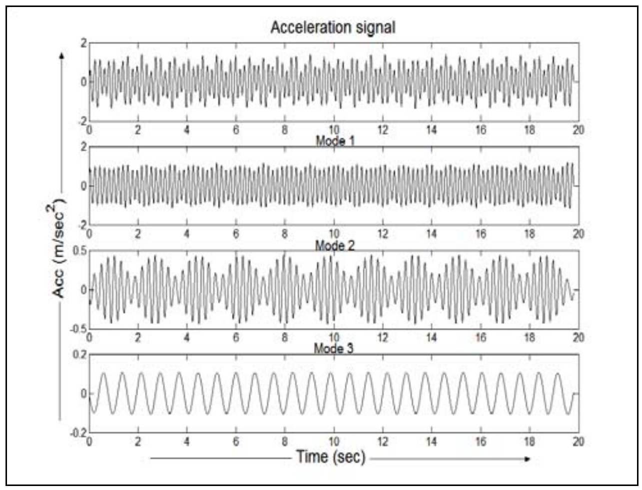

Figure 2. Decomposition of an acceleration response signal by a wavelet packet sifting process

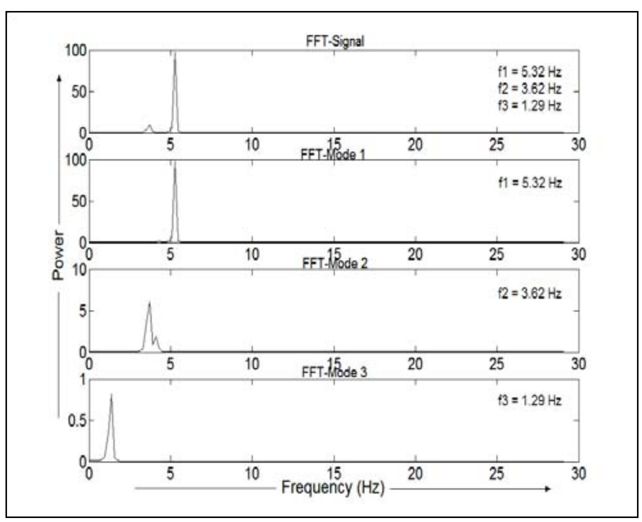

Figure 3. Fourier spectra of decomposed components in Figure.2

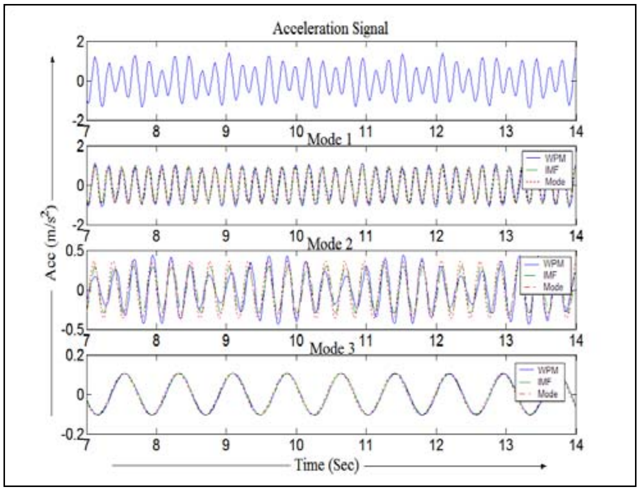

Figure 4. Comparison of wavelet packet components with the modal responses from a modal analysis and the IMFs using EMD method.

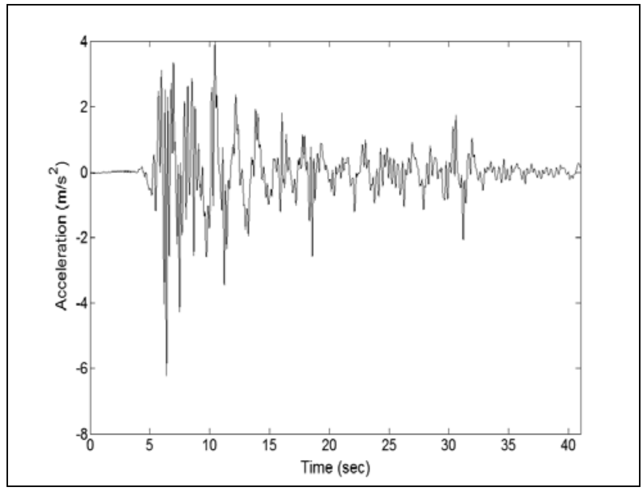

Figure 7.1. Acceleration measurement at first floor for load level of 6 m/s^2 in the Shaking Table Test

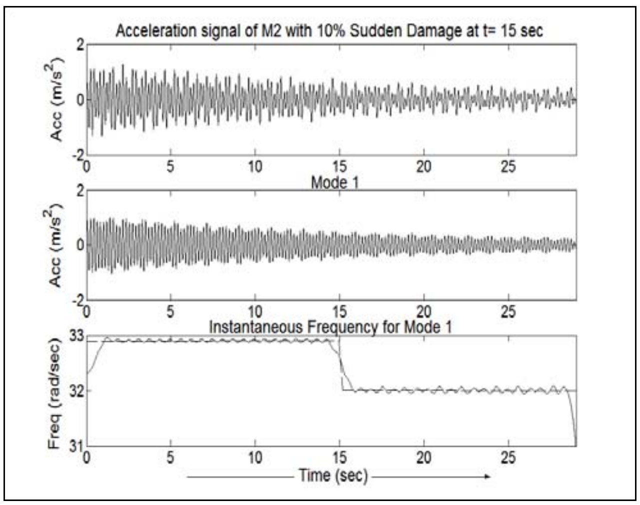

Figure 5. Results for a case study for sudden damage

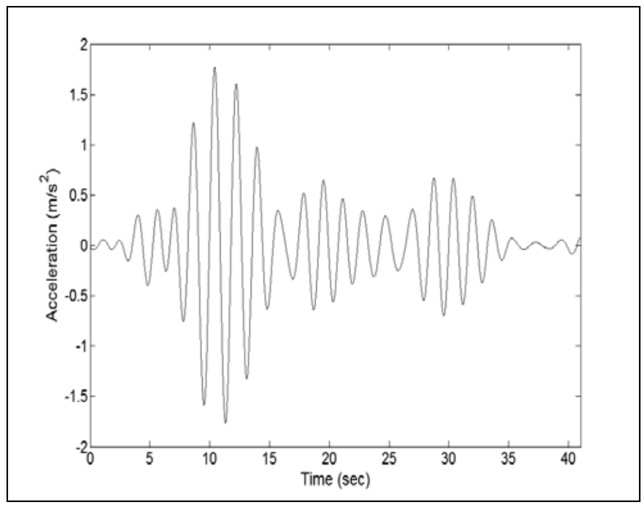

Figure 7.2. A Low Frequency Component of Acceleration Response signal at first floor

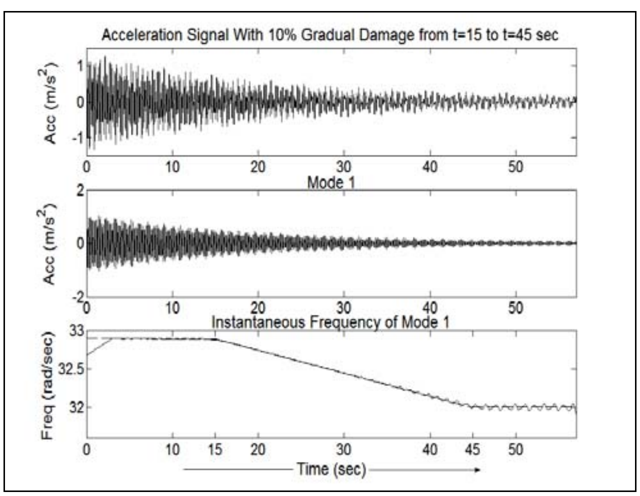

Figure 6. Results from a case study for Monitoring Progressive Damage

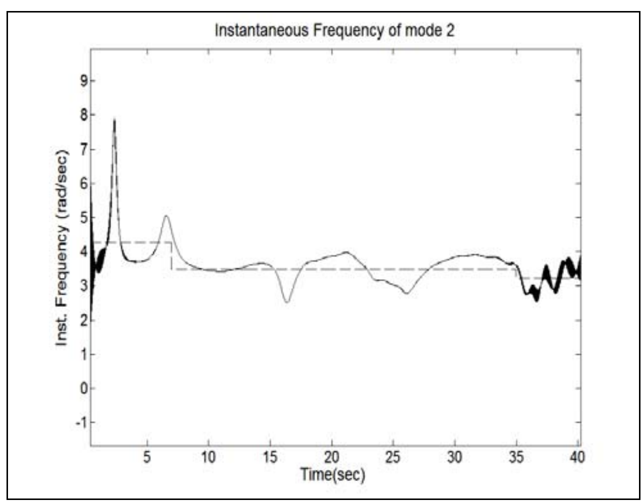

Figure 7.3. Extracted Instantaneous Frequency of the low-frequency component in Figure 7.2