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Damage Localization for Offshore Structures by Modal Strain Energy Decomposition Method

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***Abstract*— A newly developed damage localization method applicable to two-dimensional and three-dimensional frame structures is presented. This method is based on decomposing the modal strain energy into two parts, one associated with element's axial coordinates and the other transverse coordinates. The method requires only a small number of mode shapes identified from damaged and undamaged structures. Numerical studies are conducted based on synthetic data generated from finite element models. This study demonstrates that the newly developed method is capable of localizing damage for template offshore structures no matter of the damage located either at a vertical pile, a horizontal beam or a slanted brace.**

I. INTRODUCTION

Offshore structures, during their service life, continually accumulate damage that results from the action of various environmental forces. The cumulative damage may cause the change of the modal properties of the structural system, such as natural frequencies, damping ratios and mode shapes. In practice, modal parameters could be extracted from structural response data even without any knowledge of the excitation, such as using the Natural Excitation Technique (NExT) [1] in conjunction with the Eigensystem Realization Algorithm (ERA) [2]. Upon a few mode shapes for damaged and undamaged structures becoming available, a damage index method developed by Stubbs *et al.* [3] could have been applied to localize the damage of the structure. However, while this damage index method [3] had been successfully applied to beam-type (one-dimensional) structures for damage localization, its applications to two- and three-dimensional frame-type structures were not as promising. The present study develops an improved damage index method to localize the damage for a three-dimensional frame structure, specifically, a template offshore structure. This new approach is based on defining two damage indices by decomposing element's modal strain energy into two parts. One index is computed from the modal strain energy associated with the *axial* coordinates, and the other is from modal strain energy associated with *transverse* coordinates.

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II. DAMAGE LOCALIZATION METHODS

A. Overview of an Existent Damage Index Method

Developed in [3], a damage index for each element of a structure system, β_j, is computed as:

$$\beta_j = \frac{E_j}{E_j^*} = \frac{\sum_{i=1}^{N_m} \left(\gamma_{ij}^* + \gamma_i^*\right) \gamma_i}{\sum_{i=1}^{N_m} \left(\gamma_{ij} + \gamma_i\right) \gamma_i^*} \qquad j = 1, \cdots, N_e \tag{1}$$

where E_j, E_j^* = Young's modulus for the jth element before and after damage, respectively (throughout the paper, superscript $*$ is used to indicate a damage version), N_m = the number of modes being considered, N_e = the number of elements of the structural system, $\gamma_i = \sum_{k=1}^{N_e} \gamma_{ik}$, $\gamma_i^* = \sum_{k=1}^{N_e} \gamma_{ik}^*$, $\gamma_{ij} = \Phi_i^T K_{j0} \Phi_i$, and $\gamma_{ij}^* = \Phi_i^{*T} K_{j0} \Phi_i^*$, in which Φ_i, Φ_i^* = the ith mode of the undamaged and damaged system, respectively, the superscript "T" = transpose operator, K_{j0} = K_j/E_j, and K_j = the global version of the stiffness matrix of the jth element for undamaged system. One can interpret γ_{ij} as a quantity for the contribution of the jth element to the ith modal strain energy for the undamaged structure, and γ_i as the total for the ith modal strain energy.

Furthermore, [3] defined the damage indicator of the jth member as:

$$Z_j = \frac{\beta_j - \overline{\beta}}{\sigma_\beta} \tag{2}$$

where $\overline{\beta}$ and σ_β represent the sample mean and standard deviation of β_j, respectively. It is realized that Z_j is nothing more than a statistically normalized quantity for β_j.

B. Development of the Modal Strain Energy Decomposition Method

The major concept of the new damage localization algorithm is to separate the jth modal strain energy of the structure into two groups according to local element coordinates.

To explain this decomposition method, a beam element in a plane (two-dimensional) is chosen for illustration purpose. For a beam with length L, cross section area A and moment of inertia I, its local stiffness matrix (a 6-by-6 matrix), associated with the jth element, is given as:

$k_j = E_j/L^3 \times$

$$\begin{bmatrix} AL^2 & 0 & 0 & -AL^2 & 0 & 0 \\ 0 & 12I & 6IL & 0 & -12L & 6IL \\ 0 & 6IL & 4IL^2 & 0 & -6IL & 2IL^2 \\ -AL^2 & 0 & 0 & AL^2 & 0 & 0 \\ 0 & -12I & -6IL & 0 & 12L & -6IL \\ 0 & 6IL & 2IL^2 & 0 & -6IL & 4IL^2 \end{bmatrix}$$

in which columns (rows) 1 & 4 correspond to axial coordinates, 2 & 5 transverse coordinates and 3 & 6 rotational coordinates.

The above element stiffness matrix can be decomposed into:

$$k_j = k_j^a + k_j^t + k_j^r + k_j^{tr} \quad (3)$$

where superscripts a, t, r and tr stand for axial, transverse, rotational, and transverse-rotational, respectively. In particular, one has

$$k_j^a = \frac{E_j A}{L} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$k_j^t = \frac{12E_j I}{L^3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Likewise, k_j^r is the matrix containing rotational terms only, and k_j^{tr} is associated with the cross transverse-rotational stiffness terms. It is recognized that the measurements associated with rotational coordinates are difficult to obtain practically, so most damage detection methods use mode shapes that include only translational coordinates.

The *axial* modal strain energy of the jth element corresponding to the ith mode is defined as:

$$\mathcal{E}_{ij}^a = \Phi_i^T K_j^a \Phi_i \quad (4)$$

where K_j^a is the global version of the matrix k_j^a. In turn, the total axial modal strain energy of the structure corresponding to the ith mode is obtained as:

$$\mathcal{E}_i^a = \Phi_i^T K^a \Phi_i \quad (5)$$

where K^a is the combined stiffness matrix assembled by all individual K_j^a, $j = 1, \cdots, N_e$. For the ith mode, the fractional contribution to the total axial modal strain energy (or generalized stiffness) by the jth member is denoted as:

$$F_{ij}^a = \frac{\mathcal{E}_{ij}^a}{\mathcal{E}_i^a} \quad (6)$$

Similarly, for a damaged structure, the counterpart of F_{ij}^a is defined as:

$$F_{ij}^{a\,*} = \frac{\mathcal{E}_{ij}^{a\,*}}{\mathcal{E}_i^{a\,*}} \quad (7)$$

where

$$\mathcal{E}_i^{a\,*} = \Phi_i^{*\,T} K^{a\,*} \Phi_i^* \quad (8)$$

and

$$\mathcal{E}_{ij}^{a\,*} = \Phi_i^{*\,T} K_j^{a\,*} \Phi_i^* \quad (9)$$

The quantities K_j^a and $K_j^{a\,*}$ may also be expressed as:

$$K_j^a = E_j K_{j0}^a \quad (10)$$

and

$$K_j^{a\,*} = E_j^* K_{j0}^a \quad (11)$$

where the scalars E_j and E_j^* are Young's modulus representing material strength of the undamaged and damaged j^{th} members, respectively. Clearly, the matrix K_{j0}^a involves only geometric quantities.

For a given mode i, the terms F_{ij}^a and $F_{ij}^{a\,*}$ have the following properties:

$$\sum_{j=1}^{N_e} F_{ij}^a = \sum_{j=1}^{N_e} F_{ij}^{a\,*} = 1 \quad (12)$$

When N_e is large, both F_{ij}^a and $F_{ij}^{a\,*}$ tend to be much less than unity. Similar to the procedure used in [3], an assumption is made between F_{ij}^a and $F_{ij}^{a\,*}$ as:

$$\frac{1 + F_{ij}^{a\,*}}{1 + F_{ij}^a} = 1 \quad (13)$$

Substituting (6) and (7) into (13) yields

$$\frac{\left(\mathcal{E}_{ij}^{a\,*} + \mathcal{E}_i^{a*}\right)\mathcal{E}_i^a}{\left(\mathcal{E}_{ij}^a + \mathcal{E}_i^a\right)\mathcal{E}_i^{a*}} = 1 \quad (14)$$

Define β_{ij}^a to be the ratio E_j/E_j^*, computed based on the axial modal strain energy associated with the ith mode. Substituting (4) – (5) and (8) – (11) into (14), and imposing the approximations $\mathcal{E}_i^a \approx E_j {\Phi_i}^T K_0^a \Phi_i$ and $\mathcal{E}_i^{a*} \approx E_j^* \Phi_i^{*\,T} K_0^a \Phi_i^*$ where K_0^a is the assemblage of K_{j0}^a, one obtains

$$\beta_{ij}^a = \frac{E_j}{E_j^*} = \frac{\left(\Phi_i^{*\,T} K_{j0}^a \Phi_i^* + \Phi_i^{*\,T} K_0^a \Phi_i^*\right)\mathcal{E}_i^a}{\left(\Phi_i^T K_{j0}^a \Phi_i + \Phi_i^T K_0^a \Phi_i\right)\mathcal{E}_i^{a*}} \quad (15)$$

Furthermore, substituting the following approximation

$$\frac{\mathcal{E}_i^a}{\mathcal{E}_i^{a*}} \approx \frac{\Phi_i^T K_0^a \Phi_i}{\Phi_i^{*\,T} K_0^a \Phi_i^*} \quad (16)$$

into (15), one obtains

$$\beta_{ij}^a = \frac{E_j}{E_j^*} = \frac{\left(\Phi_i^{*\,T} K_{j0}^a \Phi_i^* + \Phi_i^{*\,T} K_0^a \Phi_i^*\right)\Phi_i^T K_0^a \Phi_i}{\left(\Phi_i^T K_{j0}^a \Phi_i + \Phi_i^T K_0^a \Phi_i\right)\Phi_i^{*\,T} K_0^a \Phi_i^*} \quad (17)$$

To take N_m modes into consideration, one can take the average of β_{ij}^a for $i = 1, \cdots, N_m$:

$$\beta_j^a = \frac{1}{N_m} \sum_{i=1}^{N_m} \frac{\left(\Phi_i^{*T} K_{j0}^a \Phi_i^* + \Phi_i^{*T} K_0^a \Phi_i^*\right) \Phi_i^T K_0^a \Phi_i}{\left(\Phi_i^T K_{j0}^a \Phi_i + \Phi_i^T K_0^a \Phi_i\right) \Phi_i^{*T} K_0^a \Phi_i^*} \tag{18}$$

where β_j^a can be interpreted as the damage index associated with the jth member based on the variation of axial modal strain energy.

Alternatively, one can also calculate β_j^a according to

$$\beta_j^a = \frac{\sum_{i=1}^{N_m} \left(\Phi_i^{*T} K_{j0}^a \Phi_i^* + \Phi_i^{*T} K_0^a \Phi_i^*\right) \Phi_i^T K_0^a \Phi_i}{\sum_{i=1}^{N_m} \left(\Phi_i^T K_{j0}^a \Phi_i + \Phi_i^T K_0^a \Phi_i\right) \Phi_i^{*T} K_0^a \Phi_i^*} \tag{19}$$

Equation (19) is viewed as the counterpart of (1) while only the axial modal strain energy is under consideration.

By the same token, if only the transverse modal strain energy is considered, one should obtain the corresponding index as

$$\beta_j^t = \frac{\sum_{i=1}^{N_m} \left(\Phi_i^{*T} K_{j0}^t \Phi_i^* + \Phi_i^{*T} K_0^t \Phi_i^*\right) \Phi_i^T K_0^t \Phi_i}{\sum_{i=1}^{N_m} \left(\Phi_i^T K_{j0}^t \Phi_i + \Phi_i^T K_0^t \Phi_i\right) \Phi_i^{*T} K_0^t \Phi_i^*} \tag{20}$$

Following the normalization procedure as (2), one can define two damage localization indicators as:

1) the axial damage indicator or axial modal strain energy change ratio (AMSECR):

$$Z_j^a = \frac{\beta_j^a - \overline{\beta^a}}{\sigma_{\beta^a}} \tag{21}$$

2) the transverse damage indicator or transverse modal strain energy change ratio (TMSECR):

$$Z_j^t = \frac{\beta_j^t - \overline{\beta^t}}{\sigma_{\beta^t}} \tag{22}$$

where the over-line represents the mean value and σ represents the standard deviation of the corresponding variable.

C. Estimate of Damage Severity

Based on the definition of β_j^a or β_j^t which measures the ratio E_j/E_j^*, literally one should be able to measure the severity of the damage, or the degree of strength loss, occurred at the jth member. Defining the loss of the strength at jth member as

$$\alpha_j = \frac{{E_j}^* - E_j}{E_j} \tag{23}$$

one shows that the estimate of α_j from β_j^a should be

$$\alpha_j^a = \frac{1}{\beta_j^a} - 1 \tag{24}$$

Similarly, the estimate of α_j from β_j^t is calculated as

$$\alpha_j^t = \frac{1}{\beta_j^t} - 1 \tag{25}$$

One should realize that when $\alpha_j = 0$ it stands for a no damage situation, when $\alpha_j = -1$ it suggests a complete loss of strength at the member j. Theoretically, it must hold that $-1 \leq \alpha_j \leq 0$.

D. Rationale for the Modal Strain Energy Decomposition

Structural members of a typical template offshore platform consist of vertical pile members, horizontal beams and slanted braces. When the vibration modes under consideration are mainly lateral (horizontal) motion, instead of up-down (vertical) motion, the modal strain energy of the pile members would be dominated by their transverse modal strain energy. On the other hand, the modal strain energy of the horizontal members would be dominated by their axial modal strain energy.

When a member of an offshore structure suffers the loss of strength, the entries of the global stiffness matrix that correspond to the nodal coordinates of the member would lower their values. In turn, changes on the vibration modes are expected to be more significant at those nodal coordinates. Because those nodal coordinates are shared by the damaged member and members connected to it, the variation on the element's modal strain energy due to this damage is expected to be noticeable not only at the damaged member itself, also at those members that are connected to the damaged member.

In view of the statements above, if the damaged member is a horizontal beam, it is not possible to detect this damage based on the one-index modal strain energy method, *i.e.*, using (2), which calculates the modal strain energy without decomposition, because the largest modal strain energy change would always take place at vertical members. In contrast, if the two-index method is applied, it would expect that the largest axial damage indicator Z_j^a occurs at the damaged beam, together with larger values of transverse damage indicator Z_j^t at pile members connected the damaged beam. If the damaged element is a vertical pile member, applying the 2-index method, one expects that the largest transverse damage indicator Z_j^t is at this particular pile member, together with larger values of axial damage indicator Z_j^a at those horizontal or slanted braces adjacent to the damaged pile member.

III. NUMERICAL STUDIES

A. Offshore Platform Model

The structure studied here is a template offshore platform located at a water depth of 97.4 m. The offshore platform, which consists of vertical pile members, horizontal beams and slanted braces, is modelled with 207 elements (see Fig. 1). A commercial finite element package has been employed to produce synthetic data. Several special kinds of elements to account for various physics have been utilized, including

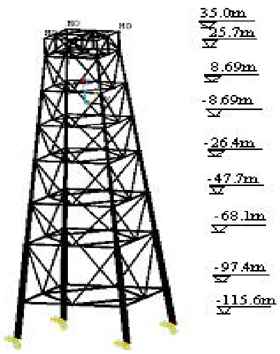

Fig. 1. Sketch of the offshore platform under study

TABLE I

A SUMMARY OF THE DAMAGE CASES

Case	Damaged Member	Member Number	Reduction on E_j
A	horizontal beam	14	5%
B	slanted brace	105	5%
C	vertical pile	78 & 79	10%

the simulation of external forces due to ocean wave and current, the buoyant effect of the water, and the element mass containing added mass of the water and the pipe internals, etc. Additionally, the buildings and equipments at the top of the offshore platform are also modelled accordingly.

B. Synthesized Damage Cases

The aforementioned finite element model is taken as the undamaged baseline model. For facilitating the following presentation, each structural member of the offshore platform is distinguished by assigning a unique number. Three damage cases are synthesized for numerical studies, covering the cases with damage occurred at a vertical, horizonal and slanted member, respectively. A brief summary of the three cases is given in Table I.

1) Case A — damaged horizontal beam: The first damage scenario is with a damaged beam (member number 14) having 5% loss on Young's modulus. Following the newly developed two-index method, Fig. 2 shows the results of the axial damage indicator, Z_j^a, and Fig. 3 the transverse damage indicator, Z_j^t. The numerical result of Z_j^a indicates that the horizontal member 14 and slanted brace 106 (their positions are shown in Fig. 4) have significantly larger values on Z_j^a, thus these two members are likely to be the damaged elements. Similarly, from the numerical results of Z_j^t, the vertical elements 79, 78 and 55 (their positions are shown in Fig. 5) are the potentially damaged elements. If the vertical element 79 was damaged, one would expect that several horizontal/slanted members connected to member 79 must exhibit larger values on Z_j^a. Obviously this is not the case. On the other hand, if beam 14 was damaged, one could anticipate a larger Z_j^t value on vertical elements 78 and 79. It is also reasonable to have a larger Z_j^t value on

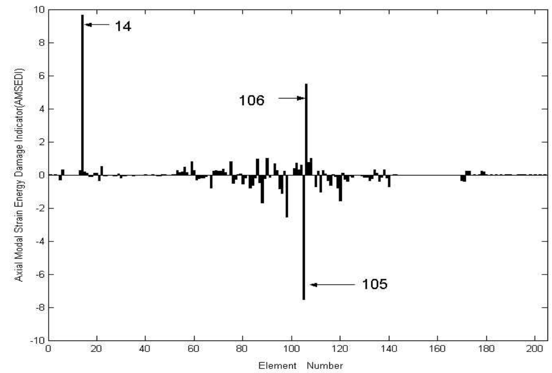

Fig. 2. Results of axial damage indicator (Case A)

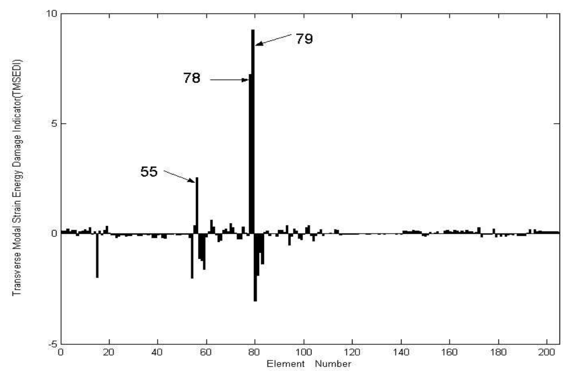

Fig. 3. Results of transverse damage indicator (Case A)

element 55 because of its relative position to element 79. Therefore, one could conclude that beam 14 is the damaged member.

2) Case B — damaged slanted brace: The second damage scenario considered herein has a damaged slanted brace (member number 105) with 5% loss on Young's modulus. Numerical results of the axial damage indicator, Z_j^a, and the transverse damage indicator, Z_j^t, are shown in Fig. 6 and Fig. 7, respectively. From Fig. 6, the slanted braces 105 and 106 are most likely damaged (positions shown in Fig. 8). Similarly, from Fig. 7, the vertical elements 56 and 80 are probably damaged (positions shown in Fig. 9). If the damaged element was one of the pile elements 56 and 80, one would expect its surrounding elements must have a larger Z_j^a value. This does not happen. While both elements 105 or 106 could be the damaged element, element 105 indeed has the largest Z_j^a, and thus is most likely to be the damaged member. Certainly, when the damage occurs at element 105, it is reasonable to have larger Z_j^t values on vertical elements 56 and 80.

3) Case C — damaged vertical pile: Vertical pile members 78 and 79 with 10% loss of Young's modulus is the third damage scenario. The results of Z_j^a and Z_j^t are provided in Fig. 10 and Fig. 11, respectively. As shown

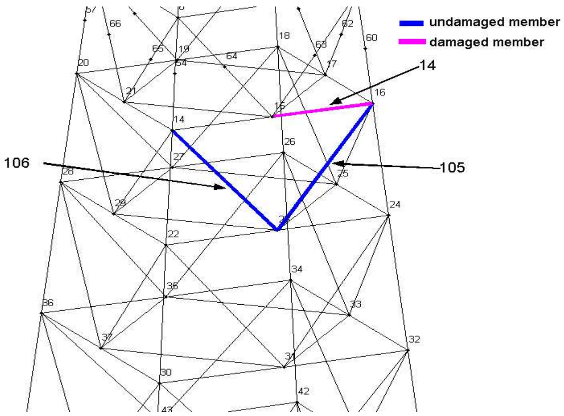

Fig. 4. Member positions with significant Z_j^a value (Case A)

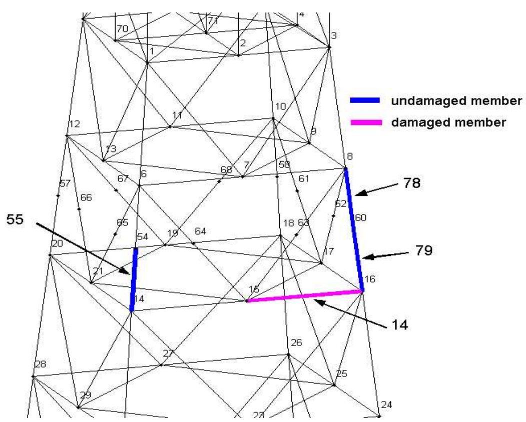

Fig. 5. Member positions with significant Z_j^t value (Case A)

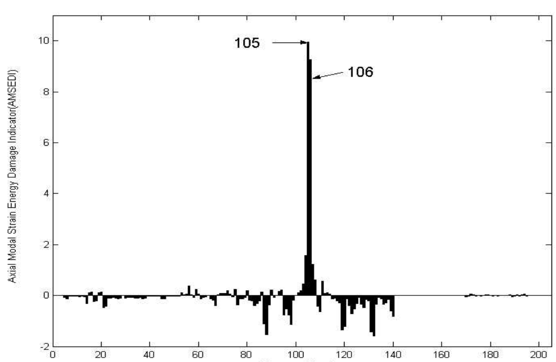

Fig. 6. Results of axial damage indicator (Case B)

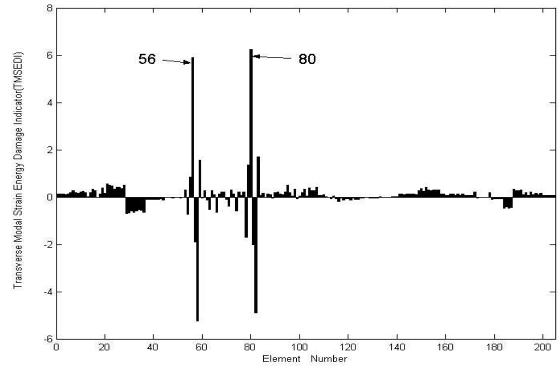

Fig. 7. Results of transverse damage indicator (Case B)

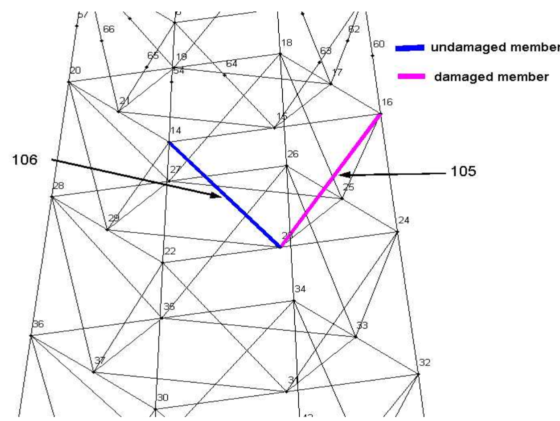

Fig. 8. Member positions with significant Z_j^a value (Case B)

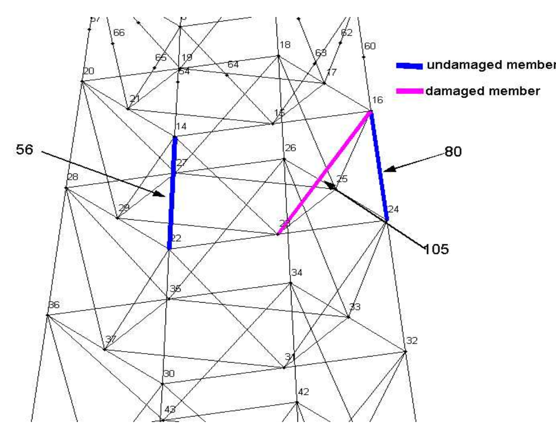

Fig. 9. Member positions with significant Z_j^t value (Case B)

in Fig. 10, slanted braces 94 and 96 (positions shown in Fig. 12), together with many other members are having comparably large Z_j^a values. It suggests that the information about Z_j^a might not be useful to identify the damaged member. From the numerical results of Z_j^t, the vertical elements 78 and 79 are most likely damaged (positions shown in Fig. 13). Applying the rationales presented earlier, one reaches the conclusion that the damaged element is most likely to be the vertical element 78, and possible element 79 as well, since the surrounding beams/braces of element 78 indeed possess larger Z_j^a values.

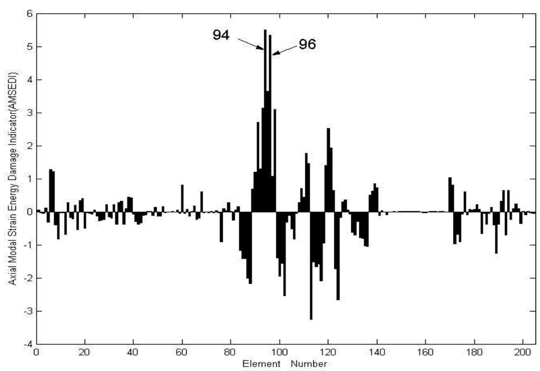

Fig. 10. Results of axial damage indicator (Case C)

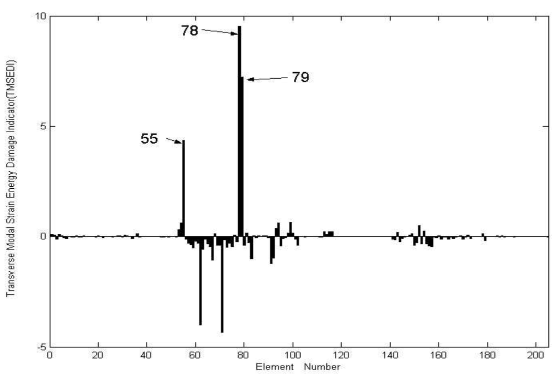

Fig. 11. Results of transverse damage indicator (Case C)

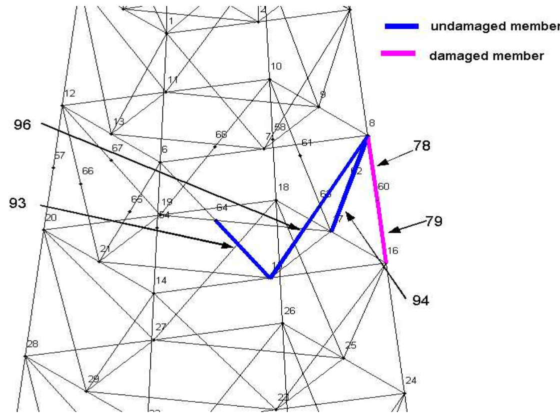

Fig. 12. Member positions with significant Z_j^a value (Case C)

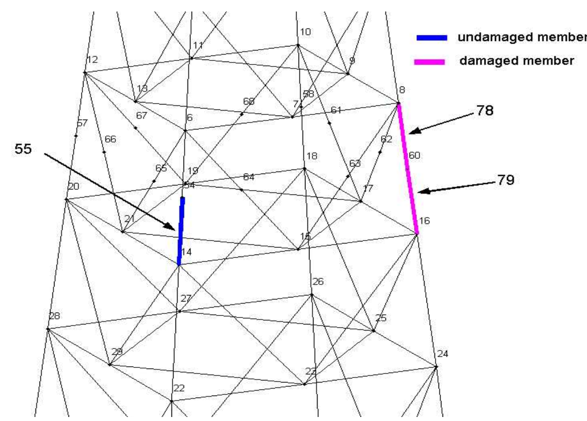

Fig. 13. Member positions with significant Z_j^t value (Case C)

IV. CONCLUDING REMARKS

The ultimate goal of this study has been set to improve the damage localization for template offshore platforms under ambient excitation. This study extends an existent one-damage-index modal strain energy method to a two-damage-index method. This newly developed damage localization method calculates two damage indicators, termed as axial damage indicator and transverse damage indicator, for each element of the structure. The essence is to separate the total modal strain energy into two parts, one corresponding to axial coordinates and the other transverse coordinates for each element. Numerical studies have been conducted based on synthetic data generated from finite element models. While the existent one-damage-index method fails to locate damage location for three-dimensional frame structures, this study demonstrates that the two-damage-index method is capable of localizing damage for template offshore structures no matter of the damage located either at a vertical pile, a horizontal beam or a slanted brace.

V. ACKNOWLEDGMENTS

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