Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

Identification and Tracking of Structural Parameters with Unknown Excitations

Jann N. Yang, *Fellow, ASCE,* Shuwen Pan and Silian Lin

Abstract— **System identification and damage detection for structural health monitoring of civil infrastructures have received considerable attention recently. The traditional least square estimation (LSE) method requires that the external excitation data (input data) be available, which may not be the case for many structures. In this paper, a recursive least square estimation with unknown inputs (RLSE-UI) approach is proposed to identify the structural parameters, such as the stiffness, damping and other nonlinear parameters. Analytical recursive solutions for the proposed RLSE-UI are derived and presented. An adaptive tracking technique recently developed is also implemented in the RLSE-UI approach to track the variations of structural parameters due to damages. Simulation results using an ASCE benchmark building for structural health monitoring demonstrate that the proposed RLSE-UI approach is capable of identifying the structural parameters and their variations due to damages.**

I. Introduction

AN important objective of health monitoring systems for civil infrastructures is to identify the state of the structure and to detect the damage when it occurs. When a structure is damaged, structural parameters of the damaged element will vary, e.g., a degradation of the stiffness. Hence, the identification of the structural parameters, including the stiffness, damping and other nonlinear parameters, and the tracking of the variation of these structural parameters are important tasks of structural health monitoring. Various analysis methodologies for the system identification and damage detection of structures have been available in the literature (e.g., [1]). In particular, time-domain analysis techniques including the methods of least square estimation (LSE) (e.g., [2]-[6]) and extended Kalman filter (e.g., [7]-[9]) have been developed and used for the on-line identification and tracking of structural parameters and their variations. In most of the approaches above, the external excitations (inputs) should be available from sensor measurements. Frequently, however, sensors may not be installed in the health monitoring system to measure all the excitations, such as earthquakes.

When the external excitations, such as earthquakes, are not measured or not available, iterative methodologies based on the least square estimation (LSE) and extended Kalman filter (EKF) have been proposed to identify the constant structural parameters [10]-[11]. Recently, the improvement of such iteration procedures has been made [12]. In this paper, we propose a recursive least square estimation with unknown inputs (RLSE-UI) approach to identify the structural parameters, such as the stiffness, damping and other nonlinear parameters, when the input excitations are not measured. In our proposed approach, recursive solutions for the estimates of structural parameters and unknown external excitations are derived analytically. Further, an adaptive tracking technique recently proposed [4]-[6] to track the variations of structural parameters due to damages is implemented in the proposed RLSE-UI approach. Simulation results demonstrate that the proposed approach is capable of not only identifying the structural parameters but also tracking the variations of these parameters. The proposed RLSE-UI methodology is a viable and effective technique for the system identification and damage detection of structures.

II. Least Square Estimation With Unknown Inputs

The equation of motion of a m DOF structure can be expressed as:

$$\mathbf{M}\,\ddot{\mathbf{x}}(t) + \mathbf{F}_c\,[\dot{\mathbf{x}}(t)] + \mathbf{F}_s\,[\mathbf{x}(t)] = \boldsymbol{\eta}\,\mathbf{w}(t) + \bar{\boldsymbol{\eta}}\,\bar{\mathbf{w}}(t) \qquad (1)$$

in which $\mathbf{x}(t) = [x_1, x_2, ..., x_m]^T$ = m-displacement vector; $\mathbf{M}$ = (m×m) mass matrix; $\mathbf{F}_c[\dot{\mathbf{x}}(t)]$ = m-damping force vector; $\mathbf{F}_s[\mathbf{x}(t)]$ = m-stiffness force vector; $\mathbf{w}(t) = [w_1(t), ..., w_r(t)]^T$ = r-unknown (or unmeasured) excitation vector; $\boldsymbol{\eta} = (m \times r)$ excitation influence matrix associated with $\mathbf{w}(t)$; $\bar{\mathbf{w}}(t) = [w_{r+1}(t), w_{r+2}(t), ..., w_m(t)]^T$ =(m–r)-known (or measured) excitation vector; and $\bar{\boldsymbol{\eta}} = m \times (m-r)$ excitation influence matrix associated with $\bar{\mathbf{w}}(t)$. In (1), the external excitation vector has been separated into r-unknown excitation vector **w**(t) and (m–r)-known excitation vector $\bar{\mathbf{w}}(t)$, respectively. In what follows, the bold face letter represents either a vector or a

Manuscript received March 1, 2004. This research is supported by the National Science Foundation Grant No. NSF-CMS- 0140710.

Jann N. Yang is with the University of California, Irvine, CA 92697 USA (phone: 949-824-4246; fax: 949-824-2117; e-mail: jnyang@uci.edu).

Shuwen Pan is with the University of California, Irvine, CA 92697 USA (e-mail: span1@uci.edu).

Silian Lin is with the University of California, Irvine, CA 92697 USA (e-mail: silianl@uci.edu).

matrix. Let $\boldsymbol{\theta}(t)$ be an n-parametric vector involving n unknown parameters to be estimated, including damping, stiffness, and nonlinear parameters, i.e., $\boldsymbol{\theta}(t)=[\theta_1(t),\theta_2(t),...,\theta_n(t)]^T$. The observation equation associated with the equation of motion can be expressed as

$$\boldsymbol{\varphi}[\dot{\mathbf{x}}(t),\ \mathbf{x}(t);\ t]\,\boldsymbol{\theta}(t)+\boldsymbol{\varepsilon}(t)=\boldsymbol{\eta}\mathbf{w}(t)+\mathbf{y}(t) \quad (2)$$

in which $\ddot{\mathbf{x}}$ is the measured acceleration response vector; $\dot{\mathbf{x}}$ and $\mathbf{x}$ may be obtained from $\ddot{\mathbf{x}}$ through numerical integration; $\boldsymbol{\varepsilon}(t)$ is a (m×1) model noise vector; $\boldsymbol{\varphi}[\ ]$ is a (m×n) observation matrix composed of the system response vectors; and $\mathbf{y}(t)=\overline{\boldsymbol{\eta}}\,\overline{\mathbf{w}}(t)-\mathbf{M}\ddot{x}(t)$, where $\mathbf{M}$ is assumed to be known. Consequently, $\mathbf{y}(t)$ is a known (measured) m-vector. Equation (2) involves m equations for each time instant t.

At the time instant $t=t_{k+1}=(k+1)\Delta t$ with Δt being the sampling interval, (2) can be written as

$$\boldsymbol{\varphi}_{k+1}\,\boldsymbol{\theta}_{k+1}+\boldsymbol{\varepsilon}_{k+1}-\boldsymbol{\eta}\mathbf{w}_{k+1}=\mathbf{y}_{k+1} \quad (3)$$

in which $\boldsymbol{\varphi}_{k+1}$, $\mathbf{y}_{k+1}$, $\boldsymbol{\varepsilon}_{k+1}$ and $\mathbf{w}_{k+1}$ are $\boldsymbol{\varphi}[\dot{\mathbf{x}}(t),\mathbf{x}(t);t]$, $\mathbf{y}(t)$, $\boldsymbol{\varepsilon}(t)$ and $\mathbf{w}(t)$ at $t=t_{k+1}$, respectively; and $\boldsymbol{\theta}_{k+1}=[\theta_1(k+1),\theta_2(k+1),...,\theta_n(k+1)]^T$ is the unknown parametric vector with the jth element $\theta_j(k+1)=\theta_j(t_{k+1})$.

Let us define an extended unknown vector $\boldsymbol{\theta}_{w,k+1}$ at $t=(k+1)\Delta t$, i.e.,

$$\boldsymbol{\theta}_{\mathbf{w},k+1}=[\boldsymbol{\theta}_{k+1}^T\ \vdots\ \mathbf{w}_{k+1}^T]^T \quad (4)$$

in which $\boldsymbol{\theta}_{\mathbf{w},k+1}$ is a (n+r)-unknown vector. Then, (3) can expressed as

$$\boldsymbol{\varphi}_{\mathbf{w},k+1}\,\boldsymbol{\theta}_{\mathbf{w},k+1}+\boldsymbol{\varepsilon}_{k+1}=\mathbf{y}_{k+1} \quad (5)$$

where

$$\boldsymbol{\varphi}_{\mathbf{w},k+1}=[\boldsymbol{\varphi}_{k+1}^T\ \vdots\ -\boldsymbol{\eta}^T\ ]^T \quad (6)$$

Combining all equations in (5) for k+1 time instants (from 1 to k+1), and assuming that $\boldsymbol{\theta}_{k+1}$ is a constant vector, i.e., $\boldsymbol{\theta}_k=\boldsymbol{\theta}_{k+1}$ for k = 1, 2, …, one obtains

$$\boldsymbol{\Phi}_{\mathbf{w},k+1}\boldsymbol{\theta}^*_{\mathbf{w},k+1}+\mathbf{E}_{k+1}=\mathbf{Y}_{k+1} \quad (7)$$

in which

$$\boldsymbol{\theta}^*_{\mathbf{w},k+1}=\begin{bmatrix}\boldsymbol{\theta}_{k+1}\\ \mathbf{w}_1\\ \mathbf{w}_2\\ \vdots\\ \mathbf{w}_{k+1}\end{bmatrix}=\begin{bmatrix}\boldsymbol{\theta}_{k+1}\\ \cdots\\ \mathbf{W}_{k+1}\end{bmatrix};\quad \mathbf{W}_{k+1}=\begin{bmatrix}\mathbf{w}_1\\ \mathbf{w}_2\\ \vdots\\ \mathbf{w}_{k+1}\end{bmatrix} \quad (8)$$

$$\mathbf{E}_{k+1}=\begin{bmatrix}\boldsymbol{\varepsilon}_1\\ \boldsymbol{\varepsilon}_2\\ \vdots\\ \boldsymbol{\varepsilon}_{k+1}\end{bmatrix};\quad \mathbf{Y}_{k+1}=\begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_2\\ \vdots\\ \mathbf{y}_{k+1}\end{bmatrix}$$

$$\boldsymbol{\Phi}_{\mathbf{w},k+1}=\begin{bmatrix}\boldsymbol{\varphi}_1 & -\boldsymbol{\eta} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\\ \boldsymbol{\varphi}_2 & \mathbf{0} & -\boldsymbol{\eta} & \mathbf{0} & \cdots & \mathbf{0}\\ \boldsymbol{\varphi}_3 & \mathbf{0} & \mathbf{0} & -\boldsymbol{\eta} & \cdots & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{\varphi}_{k+1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & -\boldsymbol{\eta}\end{bmatrix} \quad (9)$$

where $\boldsymbol{\theta}^*_{\mathbf{w},k+1}$ is a [n+r(k+1)]-unknown column vector, $\mathbf{Y}_{k+1}$ is a (k+1)m column vector, and $\boldsymbol{\Phi}_{\mathbf{w},k+1}$ is a [(k+1)m×(n+r(k+1))] data matrix.

Let $\hat{\boldsymbol{\theta}}_{k+1}$ be the estimate of the unknown parameter vector $\boldsymbol{\theta}$ at time $t=(k+1)\Delta t$, and $\hat{\mathbf{w}}_{i|j}$ be the estimate of unknown excitation $\mathbf{w}_i$, i.e., $\mathbf{w}_i=\mathbf{w}(t_i)$ with $t_i=i\Delta t$, at time $t=t_j=j\Delta t$, where $j\geq i$. Since the estimation of $\mathbf{w}_i$ varies from time to time (t_j for $j\geq i$), the conditional notation for the subscript of $\mathbf{w}_i$ is used. Consequently, the estimates of $\mathbf{W}_{k+1}$ and $\mathbf{W}_k$ in (8) at $t=(k+1)\Delta t$ are denoted by $\hat{\mathbf{W}}_{k+1|k+1}$ and $\hat{\mathbf{W}}_{k|k+1}$, respectively. Let $\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1}$ be the estimate of $\boldsymbol{\theta}^*_{\mathbf{w},k+1}$ at $t=t_{k+1}$ and consider an objective function

$$J[\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1},\mathbf{Q}_{k+1}]=[\mathbf{Y}_{k+1}-\boldsymbol{\Phi}_{\mathbf{w},k+1}\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1}]^T\mathbf{Q}_{k+1}\cdot[\mathbf{Y}_{k+1}-\boldsymbol{\Phi}_{\mathbf{w},k+1}\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1}] \quad (10)$$

in which $\mathbf{Q}_{k+1}$ is a (k+1)m×(k+1)m weighting matrix.

For the number of degrees of freedom m of the structure greater than the number of unknown excitations r, the least square estimation (LSE) approach can be used to estimate $\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1}$. Minimizing $J[\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1},\mathbf{Q}_{k+1}]$ with $\mathbf{Q}_{k+1}=\mathbf{I}$, one obtains the least square estimation (LSE), $\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1}$, of the parameter vector, $\boldsymbol{\theta}^*_{\mathbf{w},k+1}$, as follows

$$\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1}=[\boldsymbol{\Phi}^T_{\mathbf{w},k+1}\boldsymbol{\Phi}_{\mathbf{w},k+1}]^{-1}[\boldsymbol{\Phi}^T_{\mathbf{w},k+1}\mathbf{Y}_{k+1}] \quad (11)$$

The recursive solution for the least square estimation (LSE) can be obtained as

$$\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1}=\begin{bmatrix}\hat{\boldsymbol{\theta}}_{\mathbf{w},k|k+1}\\ \cdots\\ \hat{\mathbf{w}}_{k+1|k+1}\end{bmatrix}=\begin{bmatrix}\overline{\boldsymbol{\theta}}_{k+1}-\overline{\mathbf{P}}_{k+1}\overline{\boldsymbol{\varphi}}^T_{k+1}\boldsymbol{\eta}\mathbf{S}_{k+1}\boldsymbol{\eta}^T(\mathbf{y}_{k+1}-\overline{\boldsymbol{\varphi}}_{k+1}\overline{\boldsymbol{\theta}}_{k+1})\\ \cdots\\ -\mathbf{S}_{k+1}\boldsymbol{\eta}^T(\mathbf{y}_{k+1}-\overline{\boldsymbol{\varphi}}_{k+1}\overline{\boldsymbol{\theta}}_{k+1})\end{bmatrix} \quad (12)$$

in which $\hat{\boldsymbol{\theta}}_{\mathbf{w},k|k+1}=[\hat{\boldsymbol{\theta}}^T_{k+1}\ \vdots\ \hat{\mathbf{W}}^T_{k|k+1}]^T$ is a (n+rk) vector; and

$$\overline{\boldsymbol{\varphi}}_{k+1}=[\boldsymbol{\varphi}_{k+1}\ \vdots\ \mathbf{0}_{m\times rk}] \quad (13)$$

$$\overline{\mathbf{P}}_{k+1}=(\mathbf{I}-\mathbf{K}_{\mathbf{w},k+1}\overline{\boldsymbol{\varphi}}_{k+1})\mathbf{P}_{\mathbf{w},k} \quad (14)$$

$$\overline{\boldsymbol{\theta}}_{k+1}=\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k}+\mathbf{K}_{\mathbf{w},k+1}(\mathbf{y}_{k+1}-\overline{\boldsymbol{\varphi}}_{k+1}\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k}) \quad (15)$$

$$\mathbf{K}_{\mathbf{w},k+1}=\mathbf{P}_{\mathbf{w},k}\overline{\boldsymbol{\varphi}}^T_{k+1}(\mathbf{I}+\overline{\boldsymbol{\varphi}}_{k+1}\mathbf{P}_{\mathbf{w},k}\overline{\boldsymbol{\varphi}}^T_{k+1})^{-1} \quad (16)$$

$$\mathbf{S}_{k+1}=\left[\boldsymbol{\eta}^T(\mathbf{I}-\overline{\boldsymbol{\varphi}}_{k+1}\overline{\mathbf{P}}_{k+1}\overline{\boldsymbol{\varphi}}^T_{k+1})\boldsymbol{\eta}\right]^{-1} \quad (17)$$

$$\mathbf{P}_{\mathbf{w},k+1} = [\mathbf{\Phi}^T_{\mathbf{w},k+1}\mathbf{\Phi}_{\mathbf{w},k+1}]^{-1}$$

$$= \left[\begin{array}{c:c} (\mathbf{I}+\overline{\mathbf{P}}_{k+1}\overline{\boldsymbol{\varphi}}^T_{k+1}\boldsymbol{\eta}\mathbf{S}_{k+1}\boldsymbol{\eta}^T\overline{\boldsymbol{\varphi}}_{k+1})\overline{\mathbf{P}}_{k+1} & \overline{\mathbf{P}}_{k+1}\overline{\boldsymbol{\varphi}}^T_{k+1}\boldsymbol{\eta}\mathbf{S}_{k+1} \\ \hdashline \mathbf{S}_{k+1}\boldsymbol{\eta}^T\overline{\boldsymbol{\varphi}}_{k+1}\overline{\mathbf{P}}_{k+1} & \mathbf{S}_{k+1} \end{array}\right] \quad (18)$$

In the equations above, $\overline{\mathbf{P}}_{k+1}$, $\mathbf{K}_{\mathbf{w},k+1}$, $\mathbf{S}_{k+1}$ and $\mathbf{P}_{\mathbf{w},k+1}$ are, respectively, [(n+rk)×(n+rk)], [(n+rk)×m], (r× r), and [(n+r(k+1))×(n+r(k+1))] matrices. The major objective herein is to obtain the estimates $\hat{\boldsymbol{\theta}}_{k+1}$ and $\hat{\mathbf{w}}_{k+1|k+1}$ of $\boldsymbol{\theta}$ and $\mathbf{w}_{k+1}$ at $t=(k+1)\Delta t$. The recursive solution above for $\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1}$ involves many additional quantities and hence the computational efforts are quite significant. For the on-line identification and tracking of system parameters and their variations, it is critical to simplify the computational efforts. Hence, we re-partition the quantities of interest as follows

$$\hat{\boldsymbol{\theta}}^*_{\mathbf{w},k+1} = \left[\begin{array}{c} \hat{\boldsymbol{\theta}}_{k+1} \\ \hdashline \hat{\mathbf{W}}_{k+1|k+1} \end{array}\right] \text{ and } \hat{\boldsymbol{\theta}}^*_{\mathbf{w},k} = \left[\begin{array}{c} \hat{\boldsymbol{\theta}}_{k} \\ \hdashline \hat{\mathbf{W}}_{k|k} \end{array}\right] \quad (19)$$

where $\hat{\mathbf{W}}_{k|k}$ is the estimate of $\mathbf{W}_k$ at $t=t_k$, i.e., $\hat{\mathbf{W}}_{k|k} = [\hat{\mathbf{w}}^T_{1|k}, \hat{\mathbf{w}}^T_{2|k}, ..., \hat{\mathbf{w}}^T_{k|k}]^T$. In consistent with the decomposition (or partition) in (19), the corresponding $\mathbf{P}_{\mathbf{w},k}$ and $\mathbf{P}_{\mathbf{w},k+1}$ in (18) are re-partitioned as follows:

$$\mathbf{P}_{\mathbf{w},k} = \left[\begin{array}{c:c} \mathbf{P}_{\boldsymbol{\theta},k} & \mathbf{P}_{\boldsymbol{\theta}\mathbf{w},k} \\ \hdashline \mathbf{P}_{\mathbf{w}\boldsymbol{\theta},k} & \mathbf{P}_{\mathbf{ww},k} \end{array}\right]; \mathbf{P}_{\mathbf{w},k+1} = \left[\begin{array}{c:c} \mathbf{P}_{\boldsymbol{\theta},k+1} & \mathbf{P}_{\boldsymbol{\theta}\mathbf{w},k+1} \\ \hdashline \mathbf{P}_{\mathbf{w}\boldsymbol{\theta},k+1} & \mathbf{P}_{\mathbf{ww},k+1} \end{array}\right] \quad (20)$$

in which $\mathbf{P}_{\boldsymbol{\theta},k}$ and $\mathbf{P}_{\boldsymbol{\theta},k+1}$ are both $(n\times n)$ matrices.

Substituting (19) into (12)-(18), one obtains, after some operations, the recursive least square estimation for $\hat{\boldsymbol{\theta}}_{k+1}$

$$\hat{\boldsymbol{\theta}}_{k+1} = \hat{\boldsymbol{\theta}}_k + \mathbf{K}_{\boldsymbol{\theta},k+1}[\mathbf{y}_{k+1} - \boldsymbol{\varphi}_{k+1}\hat{\boldsymbol{\theta}}_k + \boldsymbol{\eta}\hat{\mathbf{w}}_{k+1|k+1}] \quad (21)$$

in which $\hat{\mathbf{w}}_{k+1|k+1}$ follows from (12) as

$$\hat{\mathbf{w}}_{k+1|k+1} = -\mathbf{S}_{k+1}\boldsymbol{\eta}^T[\mathbf{I} - \boldsymbol{\varphi}_{k+1}\mathbf{K}_{\boldsymbol{\theta},k+1}](\mathbf{y}_{k+1} - \boldsymbol{\varphi}_{k+1}\hat{\boldsymbol{\theta}}_k) \quad (22)$$

$$\mathbf{K}_{\boldsymbol{\theta},k+1} = \mathbf{P}_{\boldsymbol{\theta},k}\boldsymbol{\varphi}^T_{k+1}[\mathbf{I} + \boldsymbol{\varphi}_{k+1}\mathbf{P}_{\boldsymbol{\theta},k}\boldsymbol{\varphi}^T_{k+1}]^{-1} \quad (23)$$

$$\mathbf{S}_{k+1} = [\boldsymbol{\eta}^T(\mathbf{I} - \boldsymbol{\varphi}_{k+1}\mathbf{K}_{\boldsymbol{\theta},k+1})\boldsymbol{\eta}]^{-1} \quad (24)$$

$$\mathbf{P}_{\boldsymbol{\theta},k+1} = (\mathbf{I} + \mathbf{K}_{\boldsymbol{\theta},k+1}\boldsymbol{\eta}\mathbf{S}_{k+1}\boldsymbol{\eta}^T\boldsymbol{\varphi}_{k+1})(\mathbf{I} - \mathbf{K}_{\boldsymbol{\theta},k+1}\boldsymbol{\varphi}_{k+1})\mathbf{P}_{\boldsymbol{\theta},k} \quad (25)$$

where $\mathbf{K}_{\boldsymbol{\theta},k+1}$ is the LSE gain matrix for $\hat{\boldsymbol{\theta}}_{k+1}$, and $\mathbf{P}_{\boldsymbol{\theta},k+1}$ is a $(n\times n)$ adaptation gain matrix.

III. Adaptive Tracking

The recursive solution $\hat{\boldsymbol{\theta}}_{k+1}$ in (21)-(25) is derived based on the constant parametric vector $\boldsymbol{\theta}_{k+1}$. When a structure is damaged, the parameters will vary, e.g., the degradation of the stiffness. To track the parametric variation, different techniques have been proposed in the literature (e.g., [2]-[6]). Recently, an adaptive tracking technique for the least square estimation (LSE) has been proposed to identify and track the system parameters and their changes due to damages [4]-[6]. Such an adaptive tracking technique will be implemented in the currently proposed RLSE-UI approach to identify the structural damages.

To track the variation of each parameter, say the jth element $\theta_j(k+1)$ of $\boldsymbol{\theta}_{k+1}$ at t_{k+1}, the estimation error $[\hat{\theta}_j(k+1) - \theta_j(k+1)]$ is proposed to be modified as $\lambda_j(k+1)[\hat{\theta}_j(k) - \theta_j(k)]$, where $\lambda_j(k+1)$ will be determined from the current measured data, so that the residual error is contributed only by the noise, eliminating the contribution due to the parametric variation. Since the estimation error is reflected in the matrix $\mathbf{P}_{\theta,k+1}$, the recursive solution for the variable parametric vector, $\hat{\boldsymbol{\theta}}_{k+1}$, is proposed to be obtained from (21)-(25) as follows

$$\hat{\boldsymbol{\theta}}_{k+1} = \hat{\boldsymbol{\theta}}_k + \mathbf{K}_{\boldsymbol{\theta},k+1}[\mathbf{y}_{k+1} - \boldsymbol{\varphi}_{k+1}\hat{\boldsymbol{\theta}}_k + \boldsymbol{\eta}\hat{\mathbf{w}}_{k+1|k+1}] \quad (26)$$

$$\hat{\mathbf{w}}_{k+1|k+1} = -\mathbf{S}_{k+1}\boldsymbol{\eta}^T[\mathbf{I} - \boldsymbol{\varphi}_{k+1}\mathbf{K}_{\boldsymbol{\theta},k+1}](\mathbf{y}_{k+1} - \boldsymbol{\varphi}_{k+1}\hat{\boldsymbol{\theta}}_k) \quad (27)$$

in which

$$\mathbf{K}_{\boldsymbol{\theta},k+1} = (\boldsymbol{\Lambda}_{k+1}\mathbf{P}_{\boldsymbol{\theta},k}\boldsymbol{\Lambda}^T_{k+1})\boldsymbol{\varphi}^T_{k+1} \cdot [\mathbf{I} + \boldsymbol{\varphi}_{k+1}(\boldsymbol{\Lambda}_{k+1}\mathbf{P}_{\boldsymbol{\theta},k}\boldsymbol{\Lambda}^T_{k+1})\boldsymbol{\varphi}^T_{k+1}]^{-1} \quad (28)$$

$$\mathbf{S}_{k+1} = [\boldsymbol{\eta}^T(\mathbf{I} - \boldsymbol{\varphi}_{k+1}\mathbf{K}_{\boldsymbol{\theta},k+1})\boldsymbol{\eta}]^{-1} \quad (29)$$

$$\mathbf{P}_{\boldsymbol{\theta},k+1} = (\mathbf{I} + \mathbf{K}_{\boldsymbol{\theta},k+1}\boldsymbol{\eta}\mathbf{S}_{k+1}\boldsymbol{\eta}^T\boldsymbol{\varphi}_{k+1}) \cdot (\mathbf{I} - \mathbf{K}_{\boldsymbol{\theta},k+1}\boldsymbol{\varphi}_{k+1})(\boldsymbol{\Lambda}_{k+1}\mathbf{P}_{\boldsymbol{\theta},k}\boldsymbol{\Lambda}^T_{k+1}) \quad (30)$$

In (28) and (30), $\boldsymbol{\Lambda}_{k+1}$ is a diagonal matrix, referred to as the adaptive factor matrix, with diagonal elements $\lambda_1(k+1)$, $\lambda_2(k+1)$, …, $\lambda_n(k+1)$, where $\lambda_j(k+1)$ is referred to as the adaptive factor for the unknown parameter $\theta_j(k+1)$ at $t_{k+1} = (k+1)\Delta t$.

The adaptive factor matrix is determined by adapting the current data at t_{k+1} as follows [4]-[6]. Let

$$\overline{\boldsymbol{\gamma}}_{k+1} = \mathbf{y}_{k+1} - \boldsymbol{\varphi}_{k+1}\hat{\boldsymbol{\theta}}_{k+1} + \boldsymbol{\eta}\hat{\mathbf{w}}_{k+1|k+1} \quad (31)$$

$$\boldsymbol{\gamma}_{k+1} = \mathbf{y}_{k+1} - \boldsymbol{\varphi}_{k+1}\hat{\boldsymbol{\theta}}_k + \boldsymbol{\eta}\hat{\mathbf{w}}_{k+1|k+1} \quad (32)$$

in which $\overline{\boldsymbol{\gamma}}_{k+1}$ = m – residual error vector, and $\boldsymbol{\gamma}_{k+1}$ = m – predicted output error vector. Substituting (26) into (31), one obtains

$$\overline{\boldsymbol{\gamma}}_{k+1} = (\mathbf{I} - \boldsymbol{\varphi}_{k+1}\mathbf{K}_{\boldsymbol{\theta},k+1})\boldsymbol{\gamma}_{k+1} \quad (33)$$

Taking the expectation of $\overline{\boldsymbol{\gamma}}_{k+1}\overline{\boldsymbol{\gamma}}^T_{k+1}$, one obtains

$$E[\overline{\boldsymbol{\gamma}}_{k+1}\overline{\boldsymbol{\gamma}}^T_{k+1}] = (\mathbf{I} - \boldsymbol{\varphi}_{k+1}\mathbf{K}_{\boldsymbol{\theta},k+1})\mathbf{V}_{k+1}(\mathbf{I} - \boldsymbol{\varphi}_{k+1}\mathbf{K}_{\boldsymbol{\theta},k+1})^T \quad (34)$$

in which $\mathbf{V}_{k+1}$ is the covariance matrix of the predicted output error, i.e.,

$$\mathbf{V}_{k+1} = E[\gamma_{k+1}\gamma_{k+1}^T] \quad (35)$$

As $\hat{\boldsymbol{\theta}}_{k+1}$ and $\hat{\mathbf{w}}_{k+1|k+1}$ approach, respectively, $\boldsymbol{\theta}_{k+1}$ and $\mathbf{w}_{k+1}$, it follows from (5) and (31) that

$$E[\overline{\gamma}_{k+1}\overline{\gamma}_{k+1}^T] = E[\boldsymbol{\varepsilon}_{k+1}\boldsymbol{\varepsilon}_{k+1}^T] = \boldsymbol{\sigma}_{k+1}^2 \quad (36)$$

where $\boldsymbol{\sigma}_{k+1}^2$ is the variance of the noise at $t_{k+1} = (k+1)\Delta t$.

Substituting (36) into (34), one obtains

$$\mathbf{V}_{k+1} = \left[\mathbf{I} + \boldsymbol{\varphi}_{k+1}(\boldsymbol{\Lambda}_{k+1}\mathbf{P}_{\boldsymbol{\theta},k}\,\boldsymbol{\Lambda}_{k+1}^T)\,\boldsymbol{\varphi}_{k+1}^T\right]\boldsymbol{\sigma}_{k+1}^2 \cdot\left[\mathbf{I} + \boldsymbol{\varphi}_{k+1}(\boldsymbol{\Lambda}_{k+1}\mathbf{P}_{\boldsymbol{\theta},k}\,\boldsymbol{\Lambda}_{k+1}^T)\,\boldsymbol{\varphi}_{k+1}^T\right]^T \quad (37)$$

It is noticed from (37) that $\mathbf{P}_{\boldsymbol{\theta},k}$ and $\boldsymbol{\sigma}_{k+1}^2$ are computed and $\boldsymbol{\varphi}_{k+1}$ is measured at $t_{k+1} = (k+1)\Delta t$. Equation (37) is referred to as the adaptive tracking condition, from which $\boldsymbol{\Lambda}_{k+1}$ may be estimated. Due to space limitation, the reader is referred to [4]-[6] for the determination of $\boldsymbol{\Lambda}_{k+1}$ based on (37).

IV. ASCE Phase I Benchmark Structure

Recently, an ASCE benchmark building for structural health monitoring has been established [13] in order to compare the efficacy of various system identification and damage detection techniques. This 4-story steel frame benchmark building was modeled by: (i) a 12 DOF shear-beam model, and (ii) a 120 DOF model based on the finite-element approach. Various cases and damage patterns were considered (see details in [13]). For simplicity of demonstration, we only consider the 12 DOF model and the external loading (white noise) is applied only to the 4th floor similar to Case 3 in [13]; however, the loading is applied in the weak direction (y-direction). Due to symmetry, the system is reduced to a 4 DOF shear-beam model similar to Case 1 in [13]. The masses of floors are $m_1 = 3.4524$ tons, $m_2 = m_3 = 2.6524$ tons and $m_4 = 1.8099$ tons, and the stiffness for each story is identical, i.e., $k_1 = k_2 = k_3 = k_4 = 67.90$ MN/m. The damping matrix $\mathbf{C}$ is assumed to be proportional to the stiffness matrix $\mathbf{K}$, i.e., $\mathbf{C} = \alpha\mathbf{K}$ with $\alpha = 10^{-4}$ sec. Hence, the damping coefficient of each story unit is $c_1 = c_2 = c_3 = c_4 = 6.79$ kN.sec/m.

Two damage patterns in [13] will be considered. For Damage Pattern 1, all braces in the first story unit are removed at $t = 6$ seconds, i.e., k_1 reduces from 67.90 MN/m to 19.68 MN/m, and c_1 reduces from 6.79 kN.sec/m to 1.968 kN.sec/m. For Damage Pattern 2, all braces in the first and third stories are removed at $t = 6$ seconds, i.e., both k_1 and k_3 reduce from 67.90 MN/m to 19.68 MN/m, and c_1 and c_3 reduce from 6.79 kN.sec/m to 1.968 kN.sec/m.

We first consider Case I in which $\ddot{x}_j(t)$ and $x_j(t)$ are measured for each floor. The measurement of a response signal, say $\ddot{x}_j(t)$, is simulated by theoretically computing the response and then superimposed by a 2% stationary noise process. The velocity $\dot{x}_j(t)$ of each floor is obtained by a direct numerical integration of the noise-polluted acceleration. The sampling frequency for all data is 1kHz. The transient response in the first 0.5 seconds are not used, i.e., the initial time starts at t = 0.5 sec. Based on the proposed recursive solution with unknown excitations, the identified results for constant k_j and c_j (j = 1, 2 ,3, 4) are presented in Fig. 1 as solid curves. The dashed curves shown in Fig. 1 are the theoretical results for comparison. The estimated excitation, $\hat{\mathbf{w}}(t)$, and the difference between the estimated and theoretical excitations, $\hat{\mathbf{w}}(t) - \mathbf{w}(t)$, are presented in Figs.2(a) and 2(b), respectively.

When the structure is damaged at t = 6 sec., the predicted results are presented in Figs. 3 and 4 as solid curves, respectively, for Damage Patterns 1 and 2. Again the dashed curves in these figures are the theoretical results.

For Case II, only the acceleration responses $\ddot{x}_j(t)$ (j = 1,2,3,4) are measured. Here, $\dot{x}_j(t)$ are computed by the numerical integration of noise-polluted $\ddot{x}_j(t)$, and $x_j(t)$ are computed by a second numerical integration. In this case, the drift of $x_j(t)$ is quite serious and hence it is removed by the EMD method (see [4]-[5]). The identified results for the Damage Pattern 1 are presented in Fig. 5 as solid curves, whereas the theoretical results are shown by dashed curves. Since the predicted results for Damage Pattern 2 are similar to that for Damage Pattern 1 in Fig. 5, they are not presented. It is observed from Figs. 1-5 that the proposed recursive least square estimation with unknown inputs (RLSE-UI) along with the tracking technique is capable of not only identifying structural parameters but also tracking their changes due to damages. It is further observed from these figures that the tracking technique is better for Case I than for Case II. This is due to the numerical errors introduced in the double integration for Case II.

V. Conclusion

A recursive least square estimation with unknown inputs (RLSE-UI) approach has been proposed to identify the structural parameters when the external excitations are not measured. Analytical recursive solutions for such an approach have been derived and presented. The proposed RLSE-UI method is applicable to linear and nonlinear structures. Further, an adaptive tracking technique recently proposed has been implemented in the proposed RLSE-UI approach to track the variations of structural parameters, such as stiffness, damping, etc., due to damages. Simulation results, based on an ASCE benchmark building for structural health monitoring, demonstrate that the proposed RLSE-UI approach is capable of identifying the structural parameters and their variations due to damages.

REFERENCES

[1] F.-K. Chang, ed., *Structural Health Monitoring*, Proc. 1st, 2nd, 3rd, and 4th Inter. Workshop on Structural Health Monitoring, Stanford Univ., Stanford, CA. (1997, 1999, 2001, 2003).

[2] J. W., Lin, R. Betti, A. W, Smyth, and R. W. Longman, "On-line Identification of Nonlinear Hysteretic Structural Systems Using A Variable Trace Approach", *Earthquake Engineering and Structural Dynamics*, vol. 30, pp. 1279-1303, 2001.

[3] A. W. Smyth, S. F Masri, E. B. Kosmatopoulos, A. G. Chassiakos, and T. K. Caughey, "Development of Adaptive Modeling Techniques For Non-linear Hysteretic Systems", *Int. J. of Non-Linear Mechanics*, vol. 37, No. 8, pp. 1435-1451, 2002.

[4] J. N. Yang and S. Lin, "Parametric Identification of Structures Using an Adaptive Tracking Technique", *Proc. 16th ASCE Engrg. Mech. Conf.*, CD ROM, 12 pages, Univ. of Washington, Seattle, 2003.

[5] J. N. Yang and S. Lin, "A Tracking Technique For Parametric Identification of Nonlinear Structures", *Structural Health Monitoring 2003*, pp. 615-622, DEStech Publications, Inc., Lancaster, PA., *Proc. 4th Inter. Workshop on Structural Health Monitoring*, Stanford University, CA., 2003.

[6] J. N. Yang and S. Lin, "On-line Identification of Nonlinear Hysteretic Structures Using An Adaptive Tracking Technique", *International Journal of Non-Linear Mechanics*, to be published.

[7] M. Shinozuka and R. Ghanem, "Structural System Identification. II: Experimental Verification", *Journal of Engineering Mechanics*, vol. 121, No. 2, pp. 265-273, 1995.

[8] O. Maruyama and M. Hoshiya, "System Identification of An Experimental Model by Extended Kalman Filter", *Proc. Structural Safety and Reliability*, ICOSSA2001, CD-ROM, 2001.

[9] Jann N. Yang, S. Lin, and L. Zhou, "Identification of Parametric Changes for Civil Engineering Structures Using An Adaptive Kalman Filter", *Proc. SPIE*, 2004, to be published.

[10] D. Wang and A. Haldar, "Element-Level System Identification with Unkown Input*", Journal of Engineering Mechanics*, vol. 120, No.1, pp. 159-176, 1994.

[11] D. Wang and A. Haldar, "System Identification With Limited Observations and Without Input", *Journal of Engineering Mechanics,* vol. 123, No. 5, pp. 504-511, 1997.

[12] X. Ling and A. Haldar, "System Identification Using The Localized Structural Model", *Proc. Structural Safety and Reliability*, ICOSSA2001, CD-ROM, 2001.

[13] E. A. Johnson, H. F. Lam, L. S., Katafygiotis, and J. L. Beck, "The Phase I IASC-ASCE Structural Health Monitoring Benchmark Problem using Simulated Data", ASCE *Journal of Engineering Mechanics*, Special Issue, vol. 31, No. 1, pp. 3-15, 2004.

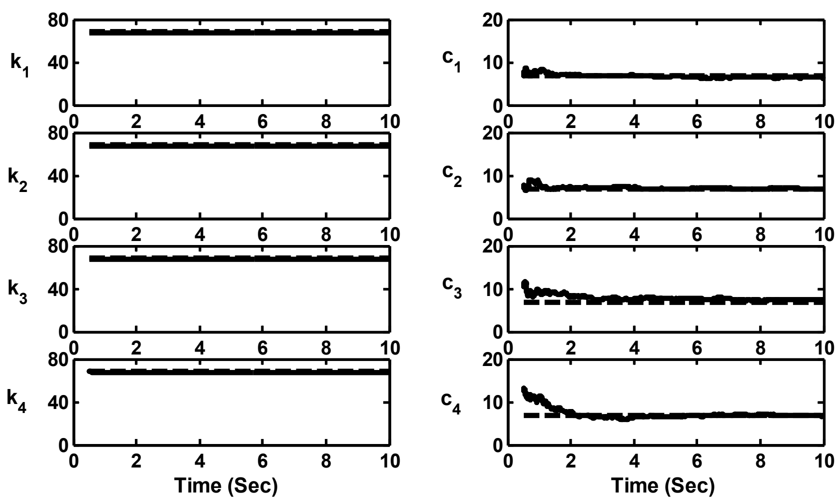

Fig. 1. Identified results for undamaged structure, Case I; k_i in MN/m and c_i in kN.sec/m.

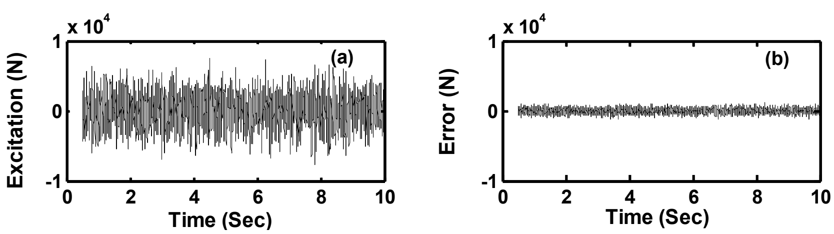

Fig. 2. Estimated excitation and error: (a) estimated excitation, and (b) error of estimated excitation.

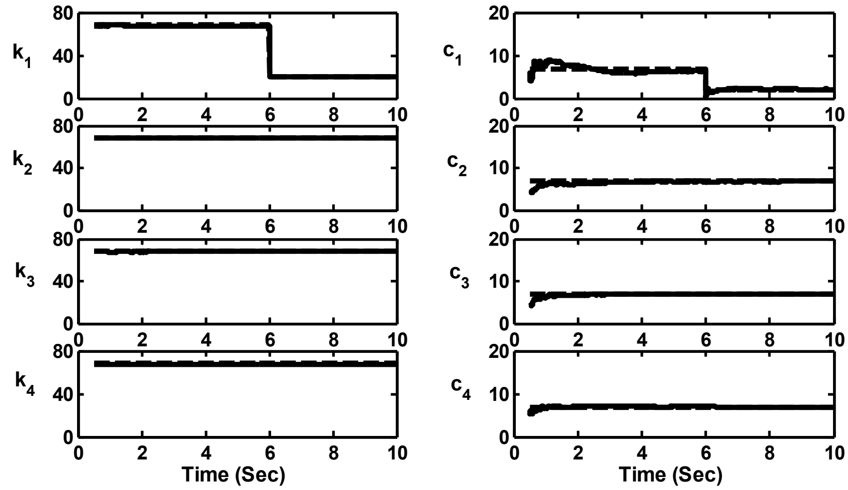

Fig. 3. Identified results for Damage Pattern 1, Case I; k_i in MN/m and c_i in kN.sec/m.

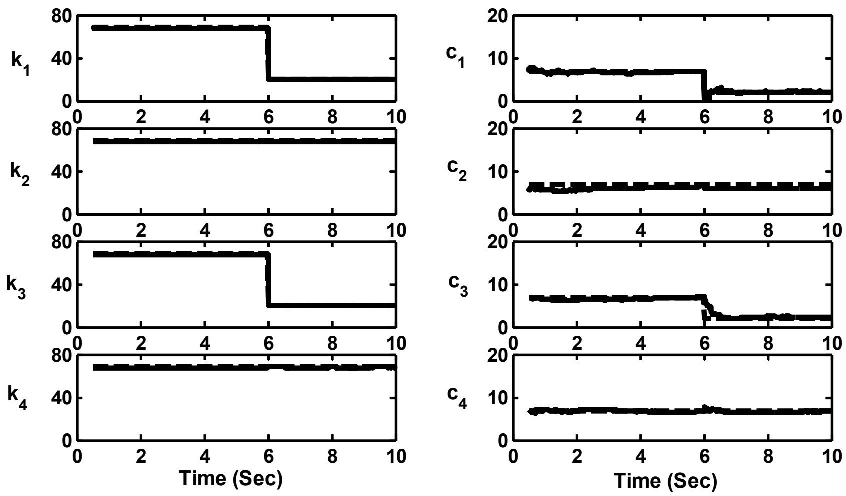

Fig. 4. Identified results for Damage Pattern 2, case I; k_i in MN/m and c_i in kN.sec/m.

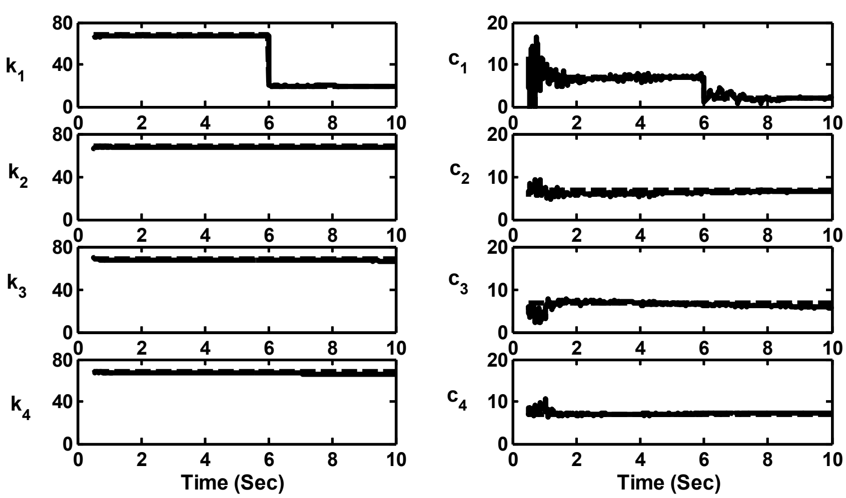

Fig. 5. Identified results for Damage Pattern 1, case II; k_i in MN/m and c_i in kN.sec/m.