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Nonlinear Adaptive Ship Course Tracking Control Based on Backstepping and Nussbaum Gain

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***Abstract*—A nonlinear adaptive controller combining adaptive Backstepping algorithm with Nussbaum gain technique is proposed for ship course tracking steering without a priori knowledge about the sign of control coefficient. It is theoretically proved that the presented adaptive controller guarantees that all signals in the uncertain nonlinear ship motion system is uniform bounded and the output of the controlled uncertain nonlinear ship motion system asymptotically tracks the output of the ship course reference model. The results of simulation for two ships' steering models show that the designed controller can be properly adopted to the ship course tracking with good compatibility.**

I. INTRODUCTION

SHIPHANDLING, in general, is a complicated control problem. The ship course control directly influences the maneuverability, economy and security of ship navigation and the combat capacity of warship [1]. Since the application of PID control law to ship navigation control in 1920's, the navigation controls have been highly emphasized due to the navigation safety, energy-saving and lowering working intensity of the crew. Many advanced control algorithms, such as model reference adaptive control [2], neural network adaptive control [3], [4], have been tried to apply to ship steering since the 1980s. So far, Nomoto linear model has been widely used in most ship motion control designs. However, under some steering conditions like a course-changing operation, it is necessary to consider the hydrodynamic nonlinearities. Hence, the ship control system model becomes nonlinear. For a certain model, a state feedback linearizing control law can be designed according to reference [5], while feedback linearization with saturating and slew rate limiting actuators is discussed by Tzeng *et al.* [6]. But it does not possess the robustness to the changes of parameter and model, because feedback linearization method requires that system parameters and structure should be accurately known. It is obvious that the mathematical model of ship steering system is uncertain when varying of speed, loading condition and environment are considered. Now, the study and development of nonlinear adaptive autopilot with robustness is a hot research topic in the field of ship motion control. A robust adaptive nonlinear control algorithm was presented for ship steering autopilot with both parametric uncertainty and unknown bound of input disturbance based on projection approach by using Lyapunov stability theory in [7]. However, the design procedure proposed in [7] requires a priori knowledge of the sign of unknown control coefficient.

Backstepping approach is a new algorithm developed in recent years, which can be used to design the adaptive controller of a large class of nonlinear control system with unknown constant parameter [8]. It is one of the most efficient ways in solving the control problem of uncertain nonlinear systems. This paper, under the condition that the nonlinearity of ship steering model is considered and the assumption that the parameters of the model are uncertain and the control direction is unknown, proposes a new adaptive control algorithm for ship course nonlinear system by incorporating the technique of Nussbaum-type function [9] into Backstepping design. The problem that the parameters are uncertain and the sign of control coefficient is unknown in course-controlling system is solved by the algorithm presented in this paper. The asymptotic stability of ship movement course error system is achieved. The goal of course-changing adaptive tracking control of vessels is realized. Finally, simulation studies on two vessels' mathematic model are performed to show the feasibility and robustness of the proposed control algorithm.

II. MATHEMADICAL MODLES OF SHIP STEERING

The linear Nomoto model has been widely accepted in designing ship course controller [10].

$$T\dot{r} + r = K\delta \tag{1}$$

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where r is the yaw rate, δ is ruder angle, T is time constant, K is rudder gain. This is mainly attributed to its simple structure and relative easiness in obtaining the model parameters from standard trial data of a ship. Despite its popularity, the Nomoto model is only valid for small rudder angles and low frequencies of rudder action. In order to better describe the ship steering dynamic behavior so that steering equation is also valid for rapid and large rudder angles, a nonlinear characteristic for r is added in equation (1). The following nonlinear model is suggested in this paper.

$$T\dot{r} + r + \alpha r^3 = K\delta \tag{2}$$

where α is called Norrbin coefficient. The value of α can be determined via a spiral test [10].

The model parameters vary significantly with operating conditions such as the forward speed. Under these varying operating conditions, it is tedious and difficult to determine properly the parameters of the controller. So we assume that model parameters T , K , α are unknown constant parameters in the design.

Noting the heading angle ψ , we have

$$\dot{\psi} = r \tag{3}$$

We select state variable as $x_1 = \psi$, $x_2 = r$, $u = \delta$ is control variable, then equations (2) and (3) are transformed into the followings:

$$\dot{x}_1 = x_2 \tag{4.1}$$

$$\dot{x}_2 = \theta_0 u + \sum_{j=1}^{2} \theta_j \varphi_{2,j}(x_2) \tag{4.2}$$

$$y = x_1 \tag{4.3}$$

where $\theta_1 = -1/T$, $\theta_2 = -\alpha/T$, $\theta_0 = K/T$, $\varphi_{2,1} = x_2$, $\varphi_{2,2} = x_2^3$. The heading angle ψ is the output of ship course control system. The model in (4) will be used for our proposed autopilot design. Evidently, this is a matching uncertain nonlinear system of single input-single output (SISO) in which nonlinear function is known. When the parameter θ_0 is known, e.g., $\theta_0 = 1$, it is just in parametric strict-feedback form [8].

$T > 0$ when a ship is line movement stable, whereas $T < 0$. In this paper, we develop an adaptive tracking control design scheme of ship course changing controller which does not require a priori knowledge about the sign of control coefficient θ_0 .

The control objective of this paper is to make the output y of the system (4) asymptotically track a desired time-variant reference trajectory ψ_d , while keeping all the closed-loop signals bounded. First, we make the following assumptions.

Assumption 1: $x_1 = \psi$, $x_2 = r$ are both measurable.

Assumption 2: The smooth reference trajectory ψ_d and its first 2 derivatives $\dot{\psi}_d, \ddot{\psi}_d$ are known and bounded.

III. DESIGN OF SHIP NONLINEAR ADAPTIVE CONTROLLER

During course-changing operation it is desirable to specify the dynamics of the desired heading instead of using a constant reference signal. One simple way to do this is by applying model reference techniques. The objective of a course-changing operation is to perform the operation quickly with minimum overshoot. The ideal performance can be given by the reference model (5) [11] such as

$$\psi_d = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \psi_r \tag{5}$$

where the input ψ_r of the reference model is the course-changing heading command, the output ψ_d of the reference model is the desired smooth course-changing signal. ξ and ω_n are the parameters that describe the closed-loop system response characteristics. In this paper, we choose $\omega_n = 0.05 rad/s$, $\xi = 0.8$.

The reference model (5) can be interpreted as a pre-filter for the commanded heading. The pre-filter ensures that numerical difficulties associated with large step inputs are avoided. A second-order model is sufficient to generate the desired smooth course signal ψ_d with known and bounded derivatives $\dot{\psi}_d, \ddot{\psi}_d$.

The adaptive controller of course tracking of the system (4) is designed as follows. Our design consists of 2 steps. The designs of both the control law and the adaptive laws are based on the change of coordinates

$$z_1 = \psi - \psi_d = x_1 - \psi_d \tag{6.1}$$

$$z_2 = x_2 - \phi_1(z_1, \dot{\psi}_d) \tag{6.2}$$

where the function ϕ_1 is referred to as intermediate control function which will be designed later using an appropriate Lyapunov function V_1 , while z_1 is just course tracking error. At the second step, the actual control u appears and the design is completed.

Step 1: Let us study the following subsystem of (4)

$$\dot{x}_1 = x_2 \tag{7}$$

In light of (6.1), equation (7) becomes

$$\dot{z}_1 = x_2 - \dot{\psi}_d \tag{8}$$

where x_2 is taken as a virtual control input.

Consider a Lyapunov function candidate V_1

$$V_1 = \frac{1}{2} z_1^2 \tag{9}$$

The time derivative of V_1 along the solution of (8)

$$\dot{V}_1 = z_1(x_2 - \dot{\psi}_d) \tag{10}$$

Let the intermediate control function ϕ_1 be

$$\phi_1(z_1, \dot{\psi}_d) = -c_1 z_1 + \dot{\psi}_d \tag{11}$$

where the design constant $c_1 > 0$ will be chosen later. A direct substitution of $x_2 = z_2 + \phi_1$ into (10) by using (11) yields

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 \tag{12}$$

The "undesired" effects of z_2 on $\dot{V}_1$ will be dealt with at the next step.

Step 2 In light of (6.2), (8) and (11), the derivative of z_2 is

$$\begin{aligned}\dot{z}_2 &= \theta_0 u + \sum_{j=1}^{2} \theta_j \varphi_{2,j} + c_1(-c_1 z_1 + z_2) - \ddot{\psi}_d \\ &= \theta_0 u + \sum_{j=1}^{2} \theta_j \varphi_{2,j} + \varphi_{2,3} \end{aligned} \tag{13}$$

where $\varphi_{2,3} = -c_1^2 z_1 + c_1 z_2 - \ddot{\psi}_d$ is a smooth function of $(z_1, z_2, \ddot{\psi}_d)$.

In order to cope with the unknown sign of control coefficient θ_0, the Nussbaum gain technique which was originally proposed in [9] is employed in this paper. A function $N(\cdot)$ is called a Nussbaum-type function [9] if it has the following properties:

$$\lim_{s\to\infty} \sup \frac{1}{s}\int_0^s N(k)dk = \infty \tag{14.1}$$

$$\lim_{s\to\infty} \inf \frac{1}{s}\int_0^s N(k)dk = -\infty \tag{14.2}$$

Commonly used Nussbaum functions include: $k^2\cos(k)$, $k^2\sin(k)$, and $\exp(k^2)\cos((\pi/2)k)$. In this paper, an even Nussbaum function $k^2\cos(k)$ is exploited.

We now give the following lemma regarding to the property of Nussbaum gain which is used in the controller design. The proof can be found in [12].

Lemma 1: Let $V(\cdot)$ and $k(\cdot)$ be smooth functions defined on $[0,\ t_f)$ with $V(t) \geq 0$, $\forall t \in [0, t_f)$, $N(\cdot)$ be an even smooth Nussbaum-type function, and θ be a nonzero constant. If the following inequality holds:

$$V(t) \leq \int_0^t (\theta N(k(\tau)) + 1)\dot{k}(\tau)d\tau + C \qquad \forall t \in [0, t_f) \tag{15}$$

where C represents some suitable constant, then $V(t)$, $k(t)$ and $\int_0^t (\theta N(k(\tau)) + 1)\dot{k}(\tau)d\tau$ must be bounded in $[0, t_f)$.

Let the control input be designed as the following actual adaptive control

$$u = N(k)(c_2 z_2 + \varphi_{2,3} + \sum_{j=1}^{2} \hat{\theta}_j \varphi_{2,j}) \tag{16}$$

with

$$N(k) = k^2 \cos(k) \tag{17}$$

$$\dot{k} = c_2 z_2^2 + \varphi_{2,3} z_2 + \sum_{j=1}^{2} \hat{\theta}_j \varphi_{2,j} z_2 \tag{18}$$

where c_2 is a positive design constant and $N(\cdot)$ is an even smooth Nussbaum-type function. $\hat{\theta}_j$, $1 \leq j \leq 2$, are the parameter estimates of the unknown parameters θ_j, $1 \leq j \leq 2$.

Let the parameter adaptation laws be

$$\dot{\hat{\theta}}_j = \varphi_{2,j} z_2, \qquad 1 \leq j \leq 2 \tag{19}$$

The distinguishing feature in (16) - (19) is that $\hat{\theta}_j$, $1 \leq j \leq 2$, are contained in the update law of $k(t)$, which is the argument of the Nussbaum-type gain $N(k)$ [12].

Consider the Lyapunov function candidate V_2

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2}\sum_{j=1}^{2} (\hat{\theta}_j - \theta_j)^2 \tag{20}$$

The time derivative of V_2 along the solution of (13), and (16) - (19) is

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + z_2(\theta_0 u + \sum_{j=1}^{2} \theta_j \varphi_{2,j} + \varphi_{2,3}) \\ &\quad + \sum_{j=1}^{2} (\hat{\theta}_j - \theta_j)\dot{\hat{\theta}}_j \\ &= -c_1 z_1^2 + z_1 z_2 \\ &\quad + \theta_0 z_2 N(k)(c_2 z_2 + \varphi_{2,3} + \sum_{j=1}^{2} \hat{\theta}_j \varphi_{2,j}) \\ &\quad + \sum_{j=1}^{2} \theta_j \varphi_{2,j} z_2 + \varphi_{2,3} z_2 + \sum_{j=1}^{2} (\hat{\theta}_j - \theta_j)\dot{\hat{\theta}}_j \\ &= -c_1 z_1^2 + z_1 z_2 - c_2 z_2^2 \\ &\quad + (\theta_0 N(k) + 1)(c_2 z_2^2 + \varphi_{2,3} z_2 + \sum_{j=1}^{2} \hat{\theta}_j \varphi_{2,j} z_2) \\ &\quad + \sum_{j=1}^{2} (\hat{\theta}_j - \theta_j)(\dot{\hat{\theta}}_j - \varphi_{2,j} z_2) \end{aligned}$$

$$= -(c_1 - \frac{1}{4})z_1^2 - (c_2 - 1)z_2^2 - (\frac{1}{2}z_1 - z_2)^2 + (\theta_0 N(k) + 1)\dot{k}$$

$$\le -(c_1 - \frac{1}{4a^2})z_1^2 - (c_2 - a^2)z_2^2 + (\theta_0 N(k) + 1)\dot{k} \tag{21}$$

So far the design procedure is complete. Due to the smoothness of the adaptive control (16), the resulting closed-loop system (4), (16)-(19) admits a solution defined on its maximum interval of existence $[0, t_f)$. Using lemma 1 to (21) when $c_1 c_2 > \frac{1}{4}$, we conclude that $V_2(t)$, $k(t)$ and $\int_0^t (\theta_0 N(k(\tau)) + 1)\dot{k}(\tau)d\tau$, hence, z_1, z_2, $\hat{\theta}_j$ and $N(k)$ are bounded. Furthermore, z_1, z_2 is square integrable according to (21), all on $[0, t_f)$. In turn ϕ_1 and the original state x_1, x_2 are also bounded on $[0, t_f)$. Therefore, no finite-time escape phenomenon may occur and $t_f = \infty$. Thus, applying Barbalat's lemma, we conclude that $\lim_{t\to\infty} z_1(t) = \lim_{t\to\infty} z_2(t) = 0$. Since $z_1 = \psi - \psi_d$, $y = \psi \to \psi_d$, the actual heading of ship asymptotically tracks the desired changing reference heading. The above facts prove the following theorem.

Theorem 1. Suppose the feedback control scheme described by (16)-(19) is applied to the uncertain strict-feedback ship motion nonlinear system (4) with completely unknown control coefficient θ_0, then all signals in the resulting closed-loop adaptive system are ultimately bounded. Furthermore, the closed-loop error system

$$\dot{z}_1 = -c_1 z_1 + z_2$$

$$\dot{z}_2 = \theta_0 u + \sum_{j=1}^{2} \theta_j \varphi_{2,j} + \varphi_{2,3}$$

achieves uniform asymptotic stability. The asymptotic track of ship course follows from $y = \psi \to \psi_d$.

IV. SIMULATION STUDIES

In this section, two simulation examples are presented, which validate control law (16) - (19). In the simulation, let the input ψ_r of the reference model be the square wave signal whose period is 400s and magnitude is $30°$.

Example 1. The ship dynamics parameters of equation (4) used in the simulation study are $T = 21$ s, $K = 0.23$ 1/s, $\alpha = 0.3$ s^2. These values are obtained from identification results of a frigate at a speed of 12 m/s [6].

We choose the control design parameters as $c_1 = 0.3$, $c_2 = 8$. The initial values are selected as $k(0) = 0.6 * \pi$, $x_1(0) = x_2(0) = \hat{\theta}_1(0) = \hat{\theta}_2(0) = 0$. The simulation results of a course-changing operation where a heading change of $60°$ is involved are plotted in Fig. 1 to Fig. 4.

Example 2. The test bed is a small ship with length of 45m. The motion of the ship described by the model of equation (4) has the following set of dynamics parameters at a forward speed of 5 m/s: $T = 31$ s, $K = 0.5$ 1/s, $\alpha = 0.4$ s^2 [4].

We adopt the same initial conditions and control design parameters as the counterparts of example 1 in the simulation. Fig. 5 to Fig. 8 depict the simulation results in this case.

It can be seen from Fig. 1 and Fig. 5 that the actual heading angle ψ (solid-line) asymptotically tracks the output ψ_d (dotted-line) of the reference model as desired, and the control rudder angles (dashdot-line) are smooth. In the Fig. 2 to Fig. 4 and Fig. 6 to Fig. 8, the estimated parameters do not converge to the true value, the envelopes of their tracking curves are however convergent. They are bounded as proven in Theorem 1. Therefore, the proposed adaptive course autopilot is effective which tracks course changes for the ship motion system (4) with uncertain parameters and completely unknown control coefficient θ_0. Furthermore, the same initial conditions and control design parameters are applied to the two different ship steering models used in the simulation studies, so the robustness of the proposed adaptive controller to the parameter changes is obviously given from Fig. 1 to Fig. 8.

V. CONCLUSION

In order to improve the performance of autopilot, this paper firstly establishes uncertain nonlinear mathematic models of ship movement course. Based on that, a new nonlinear adaptive control designing scheme is put forward, which combines Backstepping algorithm with Nussbaum-type function without the need of a priori knowledge about the sign of control gain. As to parameter uncertain nonlinear ship motion models, the proposed method simultaneously designs adaptive nonlinear ship course-tracking controller and parameter estimator which deals with the problems that the sign of control gain is unknown, and accomplishes adaptive tracking control of ship course. The control law of the adaptive algorithm is smooth. The numerical simulations illustrate that the feasibility of the designed adaptive controller for ship course tracking system.

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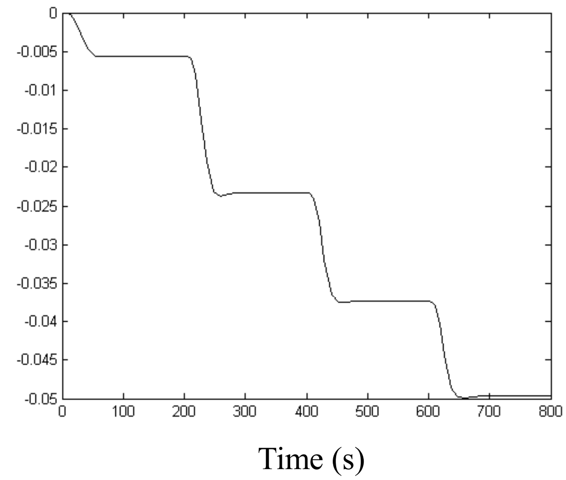

Fig.2. Adapting parameter: $\hat{\theta}_1$.

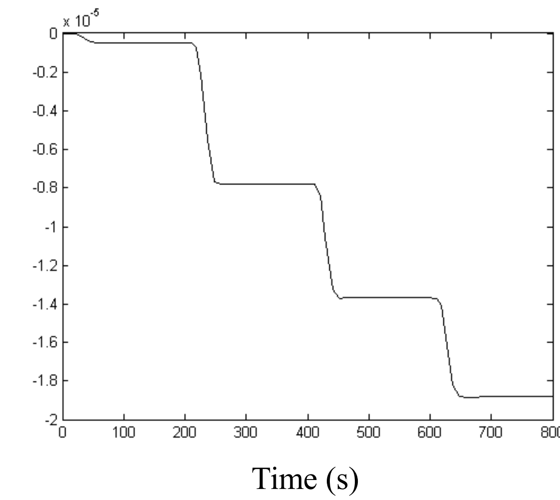

Fig. 3. Adapting parameter: $\hat{\theta}_2$.

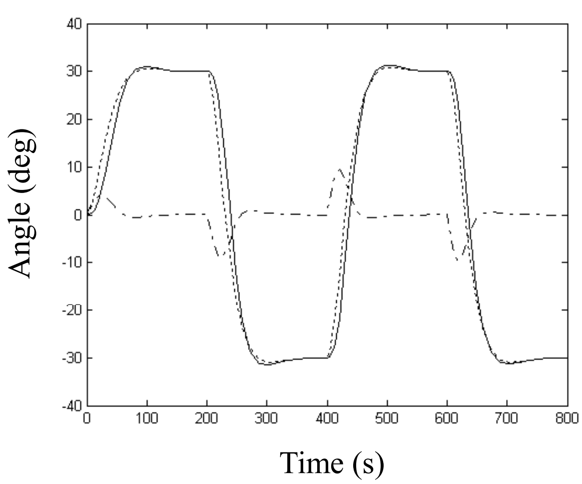

Fig. 1. Actual heading angle (—), reference heading angle (⋯), and control rudder angle (– ·).

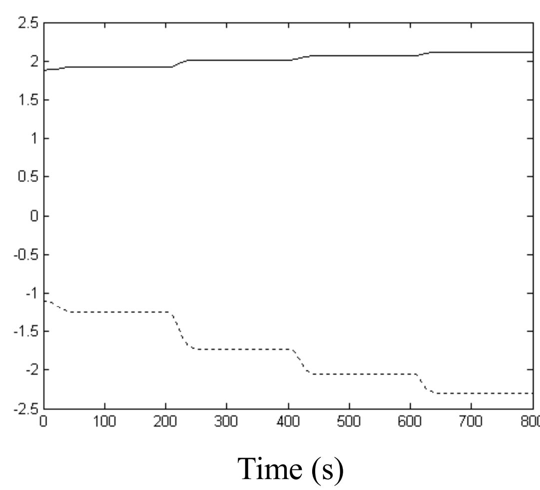

Fig. 4. Nussbaum gain $N(k)$ (⋯) and its argument $k(t)$ (—).

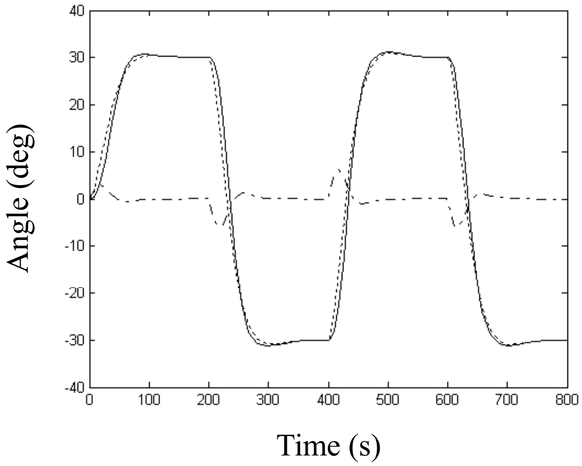

Fig. 5. Actual heading angle (—), reference heading angle (⋯), and control rudder angle (– ·).

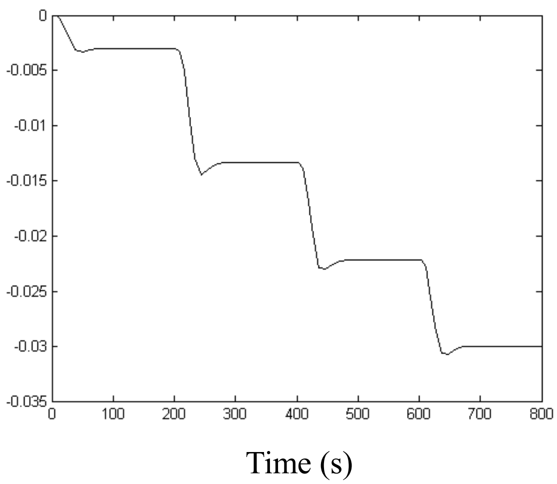

Fig. 6. Adapting parameter: $\hat{\theta}_1$.

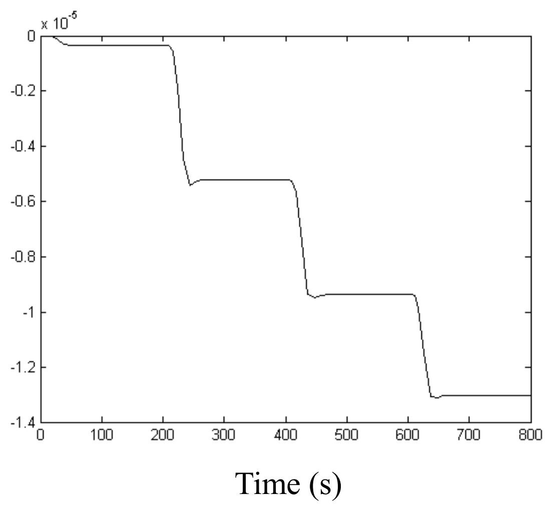

Fig. 7. Adapting parameter: $\hat{\theta}_2$.

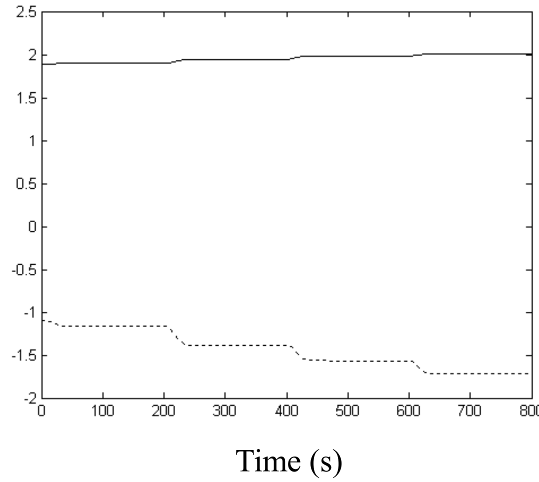

Fig. 8. Nussbaum gain $N(k)$ (⋯) and its argument $k(t)$ (—).